RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.6

Algebra of Vectors Ex 23.6 Q1

Magnitude of a vector $x\hat{i} + y\hat{j} + z\hat{k}$ is given by $\sqrt{(x)^2 + y^2 + z^2}$.

So,

$$|\vec{a}| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$= 7$$

$$\therefore |\vec{a}| = 7$$

Algebra of Vectors Ex 23.6 Q2

Unit vector of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

Unit vector of $3\hat{i} + 4\hat{j} - 12\hat{k} = \dfrac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}$

$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{9 + 16 + 144}}$$
$$= \frac{3\hat{i} + 4\hat{j} - 12\hat{k}}{\sqrt{169}}$$

Unit vector of $\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) = \dfrac{1}{13}\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right)$

Algebra of Vectors Ex 23.6 Q3

Let $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let d be the resultant of $\vec{a}, \vec{b},$ and $\vec{c}$,

$$\vec{d} = \vec{a} + \vec{b} + \vec{c}$$

$$= \left(\hat{i} - \hat{j} + 3\hat{k}\right) + \left(2\hat{i} + \hat{j} - 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

$$\vec{d} = 4\hat{i} + 2\hat{j} - \hat{k}$$

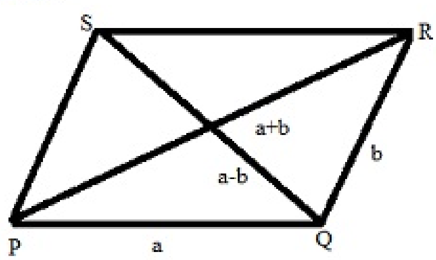
$$\text{Unit vector } \vec{d} = \frac{\vec{d}}{\left|\vec{d}\right|}$$

$$= \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{(4)^2 + (2)^2 + (-1)^2}}$$

$$\vec{d} = \frac{4\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{16 + 4 + 1}}$$

Algebra of Vectors Ex 23.6 Q4

Let $PQRS$ be a parallelogram such that $PQ = \vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $QR = \vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$



In $\triangle PQR$

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b} = \hat{i} + \hat{j} - \hat{k} + \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{PR} = -\hat{i} + 2\hat{j} + \hat{k}$$

In $\triangle PSQ$

$$\overrightarrow{PS} + \overrightarrow{SQ} = \overrightarrow{PQ}$$

$$\overrightarrow{SQ} = \vec{a} - \vec{b} = \hat{i} + \hat{j} - \hat{k} - \left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\overrightarrow{SQ} = 3\hat{i} + 0\hat{j} - 3\hat{k}$$

The unit vector along $\overrightarrow{PR} = \frac{\overrightarrow{PR}}{\left|\overrightarrow{PR}\right|} = \frac{-\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1+4+1}} = \frac{1}{\sqrt{6}}\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

The unit vector along $\overrightarrow{SQ} = \frac{\overrightarrow{SQ}}{\left|\overrightarrow{SQ}\right|} = \frac{3\hat{i} + 0\hat{j} - 3\hat{k}}{\sqrt{9+0+9}} = \frac{1}{\sqrt{2}}\left(\hat{i} - \hat{k}\right)$

Algebra of Vectors Ex 23.6 Q5

$$3\vec{a} - 2\vec{b} + 4\vec{c} = 3\left(3\hat{i} - \hat{j} - 4\hat{k}\right) - 2\left(-2\hat{i} + 4\hat{j} - 3\hat{k}\right) + 4\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} - 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$= 17\hat{i} - 3\hat{j} - 10\hat{k}$$

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{(17)^2 + (-3)^2 + (10)^2}$$

$$= \sqrt{289 + 9 + 100}$$

$$= \sqrt{398}$$

$$\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right| = \sqrt{398}$$

Algebra of Vectors Ex 23.6 Q6

Here, $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - \hat{k}$

Position vector of $P = \hat{i} - \hat{j} + 2\hat{k}$

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$

$$3\hat{i} + 2\hat{j} - \hat{k} = \text{Position vector of } Q - \left(\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$\text{Position vector of } Q = \left(3\hat{i} + 2\hat{j} - \hat{k}\right) + \left(\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$= 4\hat{i} + \hat{j} + \hat{k}$$

Coordinates of $Q = (4, 1, 1)$

Algebra of Vectors Ex 23.6 Q7

Let $\vec{A} = \hat{i} - \hat{j}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 4\hat{i} + 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$
$$= 3\hat{i} + 4\hat{j} + \hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(3)^2 + (4)^2 + (1)^2} = \sqrt{9 + 16 + 1} = \sqrt{26}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$$
$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$$
$$= -2\hat{i} - 7\hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(2)^2 + (-7)^2 + (4)^2} = \sqrt{4 + 49 + 16} = \sqrt{69}$$

$$\overrightarrow{CA} = \vec{A} - \vec{C}$$
$$= \hat{i} - \hat{j} - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$$
$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$
$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\left|\overrightarrow{CA}\right| = \sqrt{(-1)^2 + (3)^2 + (-5)^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$$

Here, $\left|AB\right|^2 + \left|\overrightarrow{CA}\right|^2 = \left|\overrightarrow{BC}\right|^2$

$$26 + 35 = 69$$
$$61 \neq 69$$
$$\text{LHS} \neq \text{RHS}$$

Since sum of square of two sides is not equal to the square of third sides. So, ΔABC is not a right triangle

Algebra of Vectors Ex 23.6 Q8

Here,

Let vertex $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

vertex $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

vertex $\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Side $\overrightarrow{AB} = \vec{B} - \vec{A}$

$$= \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\overrightarrow{AB} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$

$$= \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$$

$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - b_1\hat{i} - b_2\hat{j} - b_3\hat{k}$$

$$\overrightarrow{BC} = (c_1 - b_1)\hat{i} + (c_2 - b_2)\hat{j} + (c_3 - b_3)\hat{k}$$

$$\overrightarrow{AC} = \left(c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\right) - \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} - a_1\hat{i} - a_2\hat{j} - a_3\hat{k}$$

$$\overrightarrow{AC} = (c_1 - a_1)\hat{i} + (c_2 - a_2)\hat{j} + (c_3 - a_3)\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(c_1 - b_1)^2 + (c_2 - b_2)^2 + (c_3 - b_3)^2}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(c_1 - a_1)^2 + (c_2 - a_2)^2 + (c_3 - a_3)^2}$$

Algebra of Vectors Ex 23.6 Q9

Here, given vertex $A = (1, -1, 2)$

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$$

vertex $B = (2, 1, 3)$

$$\vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$$

vertex $C = (-1, 2, -1)$

$$\vec{C} = -\hat{i} + 2\hat{j} - \hat{k}$$

Centroid $\vec{O} = \dfrac{\vec{A} + \vec{B} + \vec{C}}{3}$

$$= \frac{\left(\hat{i} - \hat{j} + 2\hat{k}\right) + \left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \left(-\hat{i} + 2\hat{j} - \hat{k}\right)}{3}$$

$$= \frac{2\hat{i} + 2\hat{j} + 4\hat{k}}{3}$$

Centroid $\vec{O} = \dfrac{2\hat{i}}{3} + \dfrac{2\hat{j}}{3} + \dfrac{4\hat{k}}{3}$

Algebra of Vectors Ex 23.6 Q10

The position vector of point R dividing the line segment joining two points

P and Q in the ratio $m: n$ is given by:

i. Internally:

$$\frac{m\vec{b}+n\vec{a}}{m+n}$$

ii. Externally:

$$\frac{m\vec{b}-n\vec{a}}{m-n}$$

Position vectors of P and Q are given as:

$$\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k} \text{ and } \overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\overrightarrow{OR}=\frac{2\left(-\hat{i}+\hat{j}+\hat{k}\right)+1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2+1}=\frac{\left(-2\hat{i}+2\hat{j}+2\hat{k}\right)+\left(\hat{i}+2\hat{j}-\hat{k}\right)}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

Algebra of Vectors Ex 23.6 Q11

Here, $P\left(2\hat{i}-3\hat{j}+4\hat{k}\right)$ and

$Q\left(4\hat{i}+\hat{j}-2\hat{k}\right)$

We know that,

If A and B are two points with position vector $\vec{a}$ and $\vec{b}$ then the position vector of mid point C is given by

$$\frac{\vec{a}+\vec{b}}{2}$$

Let R is the mid point of PQ.

$$\text{Position vector of } R = \frac{\vec{P}+\vec{Q}}{2}$$

$$\vec{R}=\frac{2\hat{i}-3\hat{j}+4\hat{k}+4\hat{i}+\hat{j}-2\hat{k}}{2}$$

$$=\frac{6\hat{i}-2\hat{j}+2\hat{k}}{2}$$

$$=\frac{2\left(3\hat{i}-\hat{j}+\hat{k}\right)}{2}$$

Position vector of mid point $= 3\hat{i}-\hat{j}+\hat{k}$

Algebra of Vectors Ex 23.6 Q12

Here, point $P = (1,2,3)$

$$\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Point $Q = (4,5,6)$

$$\vec{Q} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$
$$= \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$= 3\hat{i} + 3\hat{j} + 3\hat{k}$$
$$= 3\left(\hat{i} + \hat{j} + k\right)$$

$$\left|\overrightarrow{PQ}\right| = 3\sqrt{(1)^2 + (1)^2 + (1)^2}$$
$$= 3\sqrt{1+1+1}$$
$$\left|\overrightarrow{PQ}\right| = 3\sqrt{3}$$

$$\text{Unit vector in the direction of } \overrightarrow{PQ} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|}$$
$$= \frac{3\left(\hat{i} + \hat{j} + \hat{k}\right)}{3\sqrt{3}}$$

$$\text{Unit vector is the direction of } \overrightarrow{PQ} = \frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

Algebra of Vectors Ex 23.6 Q13

The position vectors of A, B and C are $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively.
Therefore,
$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$
Clearly, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$.
So, $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ form a triangle.
Now

$$\left|\overrightarrow{AB}\right| = \sqrt{1+4+36} = \sqrt{41}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{4+1+1} = \sqrt{6}$$
$$\left|\overrightarrow{CA}\right| = \sqrt{1+9+25} = \sqrt{35}$$

Clearly,

$$\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{CA}\right|^2$$
$$AB^2 = BC^2 + CA^2$$

Hence ΔABC is a right triangle right angle at C.

Algebra of Vectors Ex 23.6 Q14

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, − 2).

Solution 16:

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, − 2) is given by,

$$\overline{OR} = \frac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2} = \frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i}+2\hat{j}+\hat{k}$$

Algebra of Vectors Ex 23.6 Q15

$x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector if $\left|x(\hat{i}+\hat{j}+\hat{k})\right| = 1$.

Now,

$$\left|x(\hat{i}+\hat{j}+\hat{k})\right| = 1$$

$$\Rightarrow \sqrt{x^2+x^2+x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

$$\Rightarrow \sqrt{3}\,x = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.6 Q16

We have,

$$\vec{a} = \hat{i}+\hat{j}+\hat{k},\ \vec{b} = 2\hat{i}-\hat{j}+3\hat{k} \text{ and } \vec{c} = \hat{i}-2\hat{j}+\hat{k}$$

$$2\vec{a}-\vec{b}+3\vec{c} = 2(\hat{i}+\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})+3(\hat{i}-2\hat{j}+\hat{k})$$

$$= 2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}$$

$$= 3\hat{i}-3\hat{j}+2\hat{k}$$

$$\left|2\vec{a}-\vec{b}+3\vec{c}\right| = \sqrt{3^2+(-3)^2+2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

Algebra of Vectors Ex 23.6 Q17

Here, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$$= 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(4\hat{i} - 2\hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$$= \hat{i} - 2\hat{j} + 2\hat{k}$$

Let $\vec{d}$ is a vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$

So, $\vec{d} = \lambda\left(2\vec{a} - \vec{b} + 3\vec{c}\right)$

Where λ is any scalar

$$= \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\vec{d} = \lambda\hat{i} - \lambda 2\hat{j} + \lambda 2\hat{k} \qquad (i)$$

Given that $\left|\vec{d}\right| = 6$

$$\sqrt{(\lambda)^2 + (-2\lambda)^2 + (2\lambda)^2} = 6$$

$$\sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 6$$

$$\sqrt{9\lambda^2} = 6$$

$$3\lambda = 6$$

$$\lambda = \frac{6}{3}$$

$$\lambda = 2$$

Put the value of λ in equation (i)

$$\vec{d} = 2\hat{i} - 2(2)\hat{j} + 2(2)\hat{k}$$

$$= 2\hat{i} - 4\hat{j} + 4\hat{k}$$

A vector of magnitude 6 which is parallel to $2\vec{a} - \vec{b} + 3\vec{c}$ is given by

$2\hat{i} - 4\hat{j} + 4\hat{k}$

Algebra of Vectors Ex 23.6 Q18

Given that

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

and

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Thus, Find a vector of magnitude of 5 *units parallel to the resultant of the vectors*

$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} - \hat{k} + \hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + \hat{j}$

$\Rightarrow \left|\vec{a} + \vec{b}\right| = \sqrt{9+1} = \sqrt{10}$

Thus, the unit vector along the resultant vector $\vec{a} + \vec{b}$ *is*

$$\frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

The vector of magnitude of 5 *units parallel to the resultant*

$$vector = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} \times 5 = \sqrt{\frac{5}{2}}\left(3\hat{i} + \hat{j}\right)$$

Algebra of Vectors Ex 23.6 Q19

Let D be the point at which median drawn from A touches side BC.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of the vertices A, B and C.

Position vector of D $= \dfrac{\vec{b} + \vec{c}}{2}$ ....[Since D is midpoint of B and C]

$$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{a} = \frac{\vec{b} + \vec{c} - 2\vec{a}}{2} = \frac{\vec{b} - \vec{a} + \vec{c} - \vec{a}}{2} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \frac{\hat{j} + \hat{i} + 3\hat{i} - \hat{j} + 4\hat{k}}{2}$$

$\overrightarrow{AD} = 2\hat{i} + 2\hat{k}$

$\left|\overrightarrow{AD}\right| = \sqrt{4+4} = 4\sqrt{2}$ units

Note : Answer given in the book is incorrect.