RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.8

Algebra of Vectors Ex 23.8 Q1

(i) Let P, Q, R be the points whose position vectors are $2\hat{i} + \hat{j} - \hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + 4\hat{j} - 3\hat{k}$ respectively.

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$
$$= \left(3\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} - \hat{k}\right)$$
$$= 3\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - \hat{j} + \hat{k}$$
$$\overrightarrow{PQ} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\overrightarrow{QR} = \text{Position vector of } R - \text{Position vector of } Q$$
$$= \left(\hat{i} + 4\hat{j} - 3\hat{k}\right) - \left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$
$$= \hat{i} + 4\hat{j} - 3\hat{k} - 3\hat{i} + 2\hat{j} - \hat{k}$$
$$= -2\hat{i} + 6\hat{j} - 4\hat{k}$$
$$\overrightarrow{QR} = -2\overrightarrow{PQ}$$

Therefore, $\overrightarrow{QR}$ is parallel to $\overrightarrow{PQ}$ but there is a common point Q. So, P, Q, R are collinear.

(ii) Let P, Q, R be the points represented be the vectors are $3\hat{i} - 2\hat{j} + 4\hat{k}$, $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + 4\hat{j} - 2\hat{k}$ respectively.

$$\overrightarrow{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$
$$= \left(-\hat{i} + 4\hat{j} - 2\hat{k}\right) - \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$$
$$= \hat{i} + \hat{j} + \hat{k} - 3\hat{i} + 2\hat{j} - 4\hat{k}$$
$$= -2\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\overrightarrow{QR} = \text{Position vector of } R - \text{Position vector of } Q$$
$$= \left(-\hat{i} + 4\hat{j} - 2\hat{k}\right) - \left(\hat{i} + \hat{j} + \hat{k}\right)$$
$$= -\hat{i} + 4\hat{j} - 2\hat{k} - \hat{i} - \hat{j} - \hat{k}$$
$$= -2\hat{i} + 3\hat{j} - 3\hat{k}$$
$$\overrightarrow{PQ} = \overrightarrow{QR}$$

So, $\overrightarrow{PQ}$ is parallel to $\overrightarrow{QR}$ but Q is the common point Q. So, P, Q, R are collinear.

Algebra of Vectors Ex 23.8 Q2(i)

Here, $\vec{A} = 6\hat{i} - 7\hat{j} - \hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{C} = 4\hat{i} - 5\hat{j} - 0 \times \hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(2\hat{i} - 3\hat{j} + \hat{k}\right) - \left(6\hat{i} - 7\hat{j} - \hat{k}\right)$$
$$= 2\hat{i} - 3\hat{j} + \hat{k} - 6\hat{i} + 7\hat{j} + \hat{k}$$
$$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(4\hat{i} - 5\hat{j} - 0 \times \hat{k}\right) - \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$$
$$= 4\hat{i} - 5\hat{j} - 0 \times \hat{k} - 2\hat{i} + 3\hat{j} - \hat{k}$$
$$\overrightarrow{BC} = 2\hat{i} - 2\hat{j} - \hat{k}$$

$$\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is the common point. So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(ii)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(4\hat{i} + 3\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$
$$= 4\hat{i} + 3\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(3\hat{i} + \hat{j} + 2\hat{k}\right) - \left(4\hat{i} + 3\hat{j} + \hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 2\hat{k} - 4\hat{i} - 3\hat{j} - \hat{k}$$
$$\overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}$$

So, $\overrightarrow{AB} = -2\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.
Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iii)

Here, $\vec{A} = \hat{i} + 2\hat{j} + 7\hat{k}$
$\vec{B} = 2\hat{i} + 6\hat{j} + 3\hat{k}$
$\vec{C} = 3\hat{i} + 10\hat{j} - \hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$
$= \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 7\hat{k}\right)$
$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k}$
$\overrightarrow{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$
$= \left(3\hat{i} + 10\hat{j} - \hat{k}\right) - \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$
$= 3\hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} - 6\hat{j} - 3\hat{k}$
$\overrightarrow{BC} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = \overrightarrow{BC}$
So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.
So, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(iv)

Here, $\vec{A} = -3\hat{i} - 2\hat{j} - 5\hat{k}$
$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{C} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$
$= \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right)$
$= \hat{i} + 2\hat{j} + 3\hat{k} + 3\hat{i} + 2\hat{j} + 5\hat{k}$
$\overrightarrow{AB} = 4\hat{i} + 4\hat{j} + 8\hat{k}$

$\overrightarrow{BC} = \vec{C} - \vec{B}$
$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$
$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$
$\overrightarrow{BC} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, $\overrightarrow{AB} = 2\overrightarrow{BC}$
Hence, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.
Therefore, A, B, C are collinear.

Algebra of Vectors Ex 23.8 Q2(v)

Here, $\vec{A} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$

$\vec{C} = -\hat{i} + 11\hat{j} + 9\hat{k}$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(3\hat{i} - 5\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} - 5\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(-\hat{i} + 11\hat{j} + 9\hat{k}\right) - \left(3\hat{i} - 5\hat{j} + \hat{k}\right)$$
$$= -\hat{i} + 11\hat{j} + 9\hat{k} - 3\hat{i} + 5\hat{j} - \hat{k}$$
$$= -4\hat{i} + 16\hat{j} + 8\hat{k}$$

So, $\overrightarrow{AB} = -4\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to vector $\overrightarrow{BC}$ but $\vec{B}$ is a common vector.

So, A, B, C are collinear

Algebra of Vectors Ex 23.8 Q3(i)

We know that,

Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two.

Let,

$$5\vec{a} + 6\vec{b} + 7\vec{c} = x\left(7\vec{a} - 8\vec{b} + 9\vec{c}\right) + y\left(3\vec{a} + 20\vec{b} + 5\vec{c}\right)$$
$$5\vec{a} + 6\vec{b} + 7\vec{c} = 7\vec{a}x - 8\vec{b}x + 9\vec{c}x + 3\vec{a}y + 20\vec{b}y + 5\vec{c}y$$
$$5\vec{a} + 6\vec{b} + 7\vec{c} = \left(7x + 3y\right)\vec{a} + \left(-8x + 20y\right)\vec{b} + \left(9x + 5y\right)\vec{c}$$

Comparing the LHS and RHS,

$7x + 3y = 5 \qquad \text{(i)}$

$-8x + 20y = 6 \qquad \text{(ii)}$

$9x + 5y = 7 \qquad \text{(iii)}$

For solving (i) and (ii),

Subtract $-8 \times \text{(i)}$ from $7 \times \text{(ii)}$,

$$\begin{array}{l} -56x + 140y = 42 \\ \underset{(+)}{-}\,56x \underset{(+)}{-} 24y = \underset{(+)}{-}\,40 \\ \hline \qquad 164y = 82 \end{array}$$

$$y = \frac{82}{164}$$

$$y = \frac{1}{2}$$

Put $y = \frac{1}{2}$ in equation (i),

$$7x + 3y = 5$$

$$7x + 3\left(\frac{1}{2}\right) = 5$$

$$7x + \frac{3}{2} = 5$$

$$7x = \frac{5}{1} - \frac{3}{2}$$

$$7x = \frac{10 - 3}{2}$$

$$7x = \frac{7}{2}$$

$$x = \frac{7}{14}$$

$$x = \frac{1}{2}$$

Now, put $x = \frac{1}{2}$ and $y = \frac{1}{2}$ in equation (iii),

$$9x + 5y = 7$$

$$9\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right) = 7$$

$$\frac{9}{2} + \frac{5}{2} = 7$$

$$\frac{14}{2} = 7$$

$$7 = 7$$

$$\text{LHS} = \text{RHS}$$

$\therefore$ The value of x,y satisfy equation (iii).

So,

$5\vec{a} + 6\vec{b} + 7\vec{c}$, $7\vec{a} - 8\vec{b} + 9\vec{c}$, $3\vec{a} + 20\vec{b} + 5\vec{c}$ are coplanar.

Algebra of Vectors Ex 23.8 Q3(ii)

We know that,
Three vectors are coplanar if one of them can be expressed as the linear combination of other two.

Let

$$\vec{a} - 2\vec{b} + 3\vec{c} = x\left(-3\vec{b} + 5\vec{c}\right) + y\left(-2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$

$$\vec{a} - 2\vec{b} + 3\vec{c} = -3\vec{b}x + 5\vec{c}x + 2\vec{a}y + 3\vec{b}y - 4\vec{c}y$$

$$\vec{a} - 2\vec{b} + 3\vec{c} = (-2y)\vec{a} + (-3x + 3y)\vec{b} + (5x - 4y)\vec{c}$$

Comparing the LHS and RHS,

$$-2y = 1 \qquad \text{(i)}$$

$$-3x + 3y = -2 \qquad \text{(ii)}$$

$$5x - 4y = 3 \qquad \text{(iii)}$$

From solving (i) and $y = -\frac{1}{2}$

Put value of y in equation (ii),

$$-3x + 3y = -2$$

$$-3x + 3\left(-\frac{1}{2}\right) = -2$$

$$-3x - \frac{3}{2} = -2$$

$$-3x = \frac{-2}{1} + \frac{3}{2}$$

$$-3x = \frac{-4+3}{2}$$

$$-3x = \frac{-1}{2}$$

$$x = \frac{-1}{-6}$$

$$x = \frac{1}{6}$$

Put value of x and y in equation (iii)

$$5x - 4y = 3$$

$$5\left(\frac{1}{6}\right) - 4\left(-\frac{1}{2}\right) = 3$$

$$\frac{5}{6} + \frac{4}{2} = 3$$

$$\frac{5+12}{6} = 3$$

$$\frac{17}{6} = 3$$

$$\text{LHS} \neq \text{RHS}$$

So, value of x and y do not satisfy the equation (iii).

So,
vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$, and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are not coplanar.

Algebra of Vectors Ex 23.8 Q4

Here,

Position vector of $P = 6\hat{i} - 7\hat{j}$

Position vector of $Q = 16\hat{i} - 19\hat{j} - 4\hat{k}$

Position vector of $R = 3\hat{j} - 6\hat{k}$

Position vector of $S = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left(16\hat{i} - 19\hat{j} - 4\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$

$$= 16\hat{i} - 19\hat{j} - 4\hat{k} - 6\hat{i} + 7\hat{j}$$

$$= 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$\overrightarrow{PR}$ = Position vector of R – Position vector of P

$$= \left(3\hat{j} - 6\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$

$$= 3\hat{j} - 6\hat{k} - 6\hat{i} + 7\hat{j}$$

$$= -6\hat{i} + 10\hat{j} - 6\hat{k}$$

$\overrightarrow{PS}$ = Position vector of S – Position vector of P

$$= \left(2\hat{i} - 5\hat{j} + 10\hat{k}\right) - \left(6\hat{i} - 7\hat{j}\right)$$

$$= 2\hat{i} - 5\hat{j} + 10\hat{k} - 6\hat{i} + 7\hat{j}$$

$$= -4\hat{i} + 2\hat{j} + 10\hat{k}$$

Let $\overrightarrow{PQ} = x\overrightarrow{PR} + y\overrightarrow{PS}$

$$10\hat{i} - 12\hat{j} - 4\hat{k} = x\left(-6\hat{i} + 10\hat{j} - 6\hat{k}\right) + \left(-4\hat{i} + 2\hat{j} + 10\hat{k}\right)$$

$$= -6x\hat{i} + x10\hat{j} - 6x\hat{k} - 4y\hat{i} + 2y\hat{j} + 10y\hat{k}$$

$$10\hat{i} - 12\hat{j} - 4\hat{k} = \left(-6x - 4y\right)\hat{i} + \left(10x + 2y\right)\hat{j} + \left(-6x + 10y\right)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$ of LHS and RHS,

$-6x - 4y = 10$

$3x + 2y = -5$ (i)

$10x + 2y = -12$ (ii)

$-6x + 10y = -4$ (iii)

Substracting (i) from (ii),

$$\begin{array}{rcl} 10x + 2y &=& -12 \\ \underset{(-)}{3x} + \underset{(-)}{2y} &=& \underset{(+)}{-5} \\ \hline 7x &=& -7 \end{array}$$

$x = \frac{-7}{7}$

$x = -1$

Put $x = -1$ in equation (i)

$$3x + 2y = -5$$
$$3(-1) + 2y = -5$$
$$-3 + 2y = -5$$
$$2y = -5 + 3$$
$$2y = -2$$
$$y = \frac{-2}{2}$$
$$y = -1$$

Put $x = -1$ and $y = -1$ in equation (iii),

$$-6x + 10y = -4$$
$$-6(-1) + 10(-1) = -4$$
$$6 - 10 = -4$$
$$-4 = -4$$
$$\text{LHS} = \text{RHS}$$

Therefore,
P, Q, R, S are coplanar.

Algebra of Vectors Ex 23.8 Q5(i)

We know that, three vectors are coplanar if one of the vector can be expressed as linear combination of other two.

Let,

$2\hat{i} - \hat{j} + \hat{k} = x\left(\hat{i} - 3\hat{j} - 5\hat{k}\right) + y\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$

$2\hat{i} - \hat{j} + \hat{k} = x\hat{i} - 3x\hat{j} - 5x\hat{k} + 3y\hat{i} - 4y\hat{j} - 4y\hat{k}$

$2\hat{i} - \hat{j} + \hat{k} = (x + 3y)\hat{i} + (-3x - 4y)\hat{j} + (-5x - 4y)\hat{k}$

Comparing the coefficients of LHS and RHS,

$x + 3y = 2 \qquad \text{(i)}$

$-3x - 4y = -1 \qquad \text{(ii)}$

$-5x - 4y = 1 \qquad \text{(iii)}$

For solving equation (i) and (ii),

Add $3 \times$(i) with equation (ii),

$$\begin{array}{r} 3x + 9y = 6 \\ -3x - 4y = -1 \\ \hline 5y = 5 \end{array}$$

$y = \frac{5}{5}$

$y = 1$

Put y in equation (i),

$x + 3y = 2$

$x + 3(1) = 2$

$x + 3 = 2$

$x = 2 - 3$

$x = -1$

Put the value of x and y in equation (iii),

$-5x - 4y = 1$

$-5(-1) - 4(1) = 1$

$5 - 4 = 1$

$1 = 1$

LHS = RHS

So, the value of x and y satisfy equation (iii).

Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q5(ii)

We know that,
Three vectors are coplanar if one of the vector can be expressed as the linear combination of other two vectors.

Let,

$$\hat{i}+\hat{j}+\hat{k} = x\left(2\hat{i}+3\hat{j}-\hat{k}\right)+y\left(-\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$\hat{i}+\hat{j}+\hat{k} = 2x\hat{i}+3x\hat{j}-x\hat{k}+-y\hat{i}-2y\hat{j}+2y\hat{k}$$

$$\hat{i}+\hat{j}+\hat{k} = \left(2x-y\right)\hat{i}+\left(3x-2y\right)\hat{j}+\left(-x+2y\right)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$2x - y = 1$ (i)

$3x - 2y = 1$ (ii)

$-x + 2y = 1$ (iii)

For solving (i) and (ii),
Subtracting $2\times$(i) from (ii),

$$\begin{array}{r} 3x - 2y = 1 \\ \underset{(-)}{4x} \underset{(+)}{-2y} = \underset{(-)}{2} \\ \hline -x \quad = -1 \end{array}$$

$x = 1$

Put the value of x in equation (i),

$2x - y = 1$

$2(1) - y = 1$

$2 - y = 1$

$-y = 1 - 2$

$-y = -1$

$y = 1$

Put the value of x and y in equation (iii),

$-x + 2y = 1$

$-(1) + 2(1) = 1$

$-1 + 2 = 1$

$1 = 1$

LHS = RHS

The value of x and y satisfy equation (iii).
Hence, vectors are coplanar.

Algebra of Vectors Ex 23.8 Q6(i)

We know that,
Three vectors are coplanar if one of them vector can be expressed as the linear combination of the other two.

Let,

$$\left(3\hat{i}+\hat{j}-\hat{k}\right) = x\left(2\hat{i}-\hat{j}+7\hat{k}\right)+y\left(7\hat{i}-\hat{j}+23\hat{k}\right)$$

$$= 2x\hat{i}-x\hat{j}+7x\hat{k}+7y\hat{i}-y\hat{j}+23y\hat{k}$$

$$\left(3\hat{i}+\hat{j}-\hat{k}\right) = \left(2x+7y\right)\hat{i}+\left(-x-y\right)\hat{j}+\left(7x+23y\right)\hat{k}$$

Equating the coefficients of LHS and RHS,

$2x+7y = 3 \qquad (i)$

$-x-y = 1 \qquad (ii)$

$7x+23y = -1 \qquad (iii)$

For solving (i) and (ii),
Add (i) and $2\times(ii)$,

$$\begin{array}{r} 2x+7y = 3 \\ \underline{-2x-2y = 2} \\ 5y = 5 \end{array}$$

$y = \dfrac{5}{5}$

$y = 1$

Put the value of y in equation (i),

$2x+7y = 3$

$2x+7(1) = 3$

$2x+7 = 3$

$2x = 3-7$

$2x = -4$

$x = \dfrac{-4}{2}$

$x = -2$

Put the value of x and y in equation (iii),

$7x+23y = -1$

$7(2)+23(1) = -1$

$14+23 = -1$

$37 = -1$

$\text{LHS} \neq \text{RHS}$

The value of x and y do not satisfy the equation (iii).
Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q6(ii)

We know that,
Three vectors are coplanar if any one of the vector can be expressed as the linear combination of other two vectors.

Let,

$$\hat{i}+2\hat{j}+3\hat{k}=x\left(2\hat{i}+\hat{j}+3\hat{k}\right)+y\left(\hat{i}+\hat{j}+\hat{k}\right)=2x\hat{i}+x\hat{j}+3x\hat{k}+y\hat{i}+y\hat{j}+y\hat{k}$$

$$\therefore\ \hat{i}+2\hat{j}+3\hat{k}=(2x+y)\hat{i}+(x+2y)\hat{j}+(3x+y)\hat{k}$$

Comparing the coefficients of LHS and RHS,

$$2x+y=1 \qquad (i)$$

$$x+2y=2 \qquad (ii)$$

$$3x+y=3 \qquad (iii)$$

Subtracting $2\times(ii)$ from equation (i),

$$\begin{array}{r} 2x+4y=4 \\ \underset{(-)}{2x}\ \underset{(-)}{+y}=\underset{(-)}{1} \\ \hline 3y=3 \end{array}$$

$$y=\frac{3}{3}$$

$$y=1$$

Put the value of y in equation (i),

$$2x+y=1$$

$$2x+1=1$$

$$2x=1-1$$

$$2x=0$$

$$x=\frac{0}{2}$$

$$x=0$$

Put the value of x and y in equation (iii),

$$3x+y=3$$

$$3(0)+1=3$$

$$0+1=3$$

$$1=3$$

$$\text{LHS}\neq\text{RHS}$$

The value of x and y do not satisfy the equation (iii).
Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(i)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let,

$$\left(2\vec{a}-\vec{b}+3\vec{c}\right) = x\left(\vec{a}+\vec{b}-2\vec{c}\right)+y\left(\vec{a}+\vec{b}-3\vec{c}\right)$$
$$= \vec{a}x+\vec{b}x-2\vec{c}x+\vec{a}y+\vec{b}y-3\vec{c}y$$
$$\left(2\vec{a}-\vec{b}+3\vec{c}\right) = \left(x+y\right)\vec{a}+\left(x+y\right)\vec{b}+\left(-2x-3y\right)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$x+y=2$ (i)

$x+y=-1$ (ii)

$-2x-3y=3$ (iii)

For solving the equation (i) and (ii),
Subtracting (ii) from (i),

$$\begin{array}{l} x+y=2 \\ x+y=-1 \\ (-)(-)\quad(+) \\ \hline 0\quad=3 \end{array}$$

There is no value of x and y that can satisfy the equation (iii).
Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q7(ii)

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$$\vec{a}+2\vec{b}+3\vec{c} = x\left(2\vec{a}+\vec{b}+3\vec{c}\right)+y\left(\vec{a}+\vec{b}+\vec{c}\right)$$
$$= 2\vec{a}x+\vec{b}x+3\vec{c}x+\vec{a}y+\vec{b}y+\vec{c}y$$
$$\vec{a}+2\vec{b}+3\vec{c} = \left(2x+y\right)\vec{a}+\left(x+y\right)\vec{b}+\left(3x+y\right)\vec{c}$$

Comparing the coefficients of LHS and RHS,

$2x+y=1$ (i)

$x+y=2$ (ii)

$3x+y=3$ (iii)

For solving the equation (i) and (ii),
Subtracting equation (i) from equation (ii),

$$\begin{array}{l} x+y=2 \\ 2x+y=1 \\ (-)\quad(-)\quad(-) \\ \hline -x\quad=1 \end{array}$$

$x = -1$

Put the value of x in equation (i)

$x + y = 2$

$-1 + y = 2$

$y = 2 + 1$

$y = 3$

Put the x and y in equation (iii),

$3x + y = 3$

$3(-1) + 3 = 3$

$-3 + 3 = 3$

$0 = 3$

$\text{LHS} \neq \text{RHS}$

The value of x and y do not satisfy the equation (iii).

Hence, vectors are non-coplanar.

Algebra of Vectors Ex 23.8 Q8

We know that,
Three vectors are coplanar if any one of them can be expressed as the linear combination of other two vectors.

Let

$$\vec{a} = x\vec{b} + y\vec{c}$$
$$= x\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + y\left(\hat{i} + \hat{j} + \hat{k}\right)$$
$$= 2\hat{i}x + \hat{j}x + 3\hat{k}x + \hat{i}y + \hat{j}y + \hat{k}y$$

$$\hat{i} + 2\hat{j} + 3\hat{k} = \left(2x + y\right)\hat{i} + \left(x + y\right)\hat{j} + \left(3x + y\right)\hat{k}$$

Comparing the coefficient of LHS and RHS,

$2x + y = 1$ (i)

$x + y = 2$ (ii)

$3x + y = 3$ (iii)

For solving (i) and (ii),

Subtracting (i) from (ii),

$$\begin{array}{rcl} x + y &=& 2 \\ 2x + y &=& 1 \\ (-)\quad (-) && (-) \\ \hline -x \quad &=& 1 \\ x &=& -1 \end{array}$$

Put the value of x in equation (i),

$$x + y = 2$$
$$-1 + y = 2$$
$$y = 2 + 1$$
$$y = 3$$

Put the values of x and y in equation (iii)

$$3x + y = 3$$
$$3\left(-1\right) + 3 = 3$$
$$-3 + 3 = 3$$
$$0 = 3$$
$$\text{LHS} \neq \text{RHS}$$

The values of x and y do not satisfy equation (iii).

Hence,

$\vec{a}, \vec{b}, \vec{c}$ are non coplanar.

Let,

$$\vec{d} = x\vec{b} + y\hat{j} + z\hat{k}$$

$$= x\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + y\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + z\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$= x\hat{i} + 2x\hat{j} + 3x\hat{k} + 2y\hat{i} + \hat{j}y + 3y\hat{k} + z\hat{i} + z\hat{j} + z\hat{k}$$

$$2\hat{i} - \hat{j} - 3\hat{k} = \left(x + 2y + z\right)\hat{i} + \left(2x + y + z\right)\hat{j} + \left(3x + 3y + z\right)\hat{k}$$

Comparing the coefficient of LHS and RHS,

$$x + 2y + z = 2 \qquad (i)$$

$$2x + y + z = -1 \qquad (ii)$$

$$3x + 3y + z = -3 \qquad (iii)$$

Subtracting equation (i) from (ii),

$$\begin{array}{l} 2x + y + z = -1 \\ \underset{(-)}{x} \underset{(-)}{+} 2y \underset{(-)}{+} z \underset{(-)}{=} 2 \\ \hline x - y \quad = -3 \qquad (iv) \end{array}$$

Subtracting equation (ii) from (iii),

$$\begin{array}{l} 3x + 3y + z = -3 \\ \underset{(-)}{2x} \underset{(-)}{+ y} \underset{(-)}{+ z} = \underset{(+)}{-1} \\ \hline x + 2y \quad = -2 \qquad (v) \end{array}$$

Subtracting (iv) from (v),

$$\begin{array}{l} x + 2y = -2 \\ \underset{(-)}{x} \underset{(+)}{-} y = \underset{(+)}{-3} \\ \hline 3y = 1 \end{array}$$

$$y = \frac{1}{3}$$

Put y in equation (v),

$$x + 2y = -2$$

$$x + 2\left(\frac{1}{3}\right) = -2$$

$$2 + \frac{2}{3} = -2$$

$$x = \frac{-2}{1} - \frac{2}{3}$$

$$= \frac{-6 - 2}{3}$$

$x = \frac{-8}{3}$

Put value of x and y in equation (i),

$x + 2y + z = 2$

$\frac{-8}{3} + 2\left(\frac{1}{3}\right) + z = 2$

$\frac{-8}{3} + \frac{2}{3} + z = 2$

$z = \frac{2}{1} + \frac{8}{3} - \frac{2}{3}$

$z = \frac{6 + 8 - 2}{3}$

$z = \frac{14 - 2}{3}$

$z = \frac{12}{3}$

$z = 4$

So,

$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$

$\vec{d} = \left(\frac{-8}{3}\right)\vec{a} + \left(\frac{1}{3}\right)\vec{b} + (4)\vec{c}$

Algebra of Vectors Ex 23.8 Q9

Necessary Condition: Let $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors. Then one of them can be expressed as the linear combination of other two vectors.

Let, $\vec{c} = x\vec{a} + y\vec{b}$

$x\vec{a} + y\vec{b} - \vec{c} = 0$

Put $x = l$, $y = m$, $(-1) = n$

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Thus, if $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors,
then there exist scalars l, m, n

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Such that l,m,n are not all zero simultaneously.

Sufficient Condition: Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that there exist scalars l, m, n not all zero simultaneously satisfying $l\vec{a} + m\vec{b} + n\vec{c} = 0$

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

$n\vec{c} = -l\vec{a} - m\vec{b}$

Dividing by n, both the sides

$\frac{n\vec{c}}{n} = \frac{-l\vec{a}}{n} - \frac{m\vec{b}}{n}$

$\vec{c} = \left(-\frac{l}{n}\right)\vec{a} + \left(-\frac{m}{n}\right)\vec{b}$

$\vec{c}$ is a linear combination of $\vec{a}$ and $\vec{b}$

Hence, $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors.

Algebra of Vectors Ex 23.8 Q10

Given that, A, B, C and D are four points with position vector $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively.

Let A, B, C, D are coplanar.

If so, there exists x, y, z, u not all zero such that

$x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = 0$

$x + y + z + u = 0$

Let, $x = 3,\ y = -2,\ z = 1,\ u = -2$

$3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

$$\begin{aligned} \text{and, } x + y + z + u &= 3 + (-2) + 1 + (-2) \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

Thus, A, B, C, D are coplanar.

if $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

$3\vec{a} + \vec{c} = 2\vec{b} + 2\vec{d}$

Divide by sum of the coefficients that is by 4 on both sides,

$$\frac{3\vec{a} + \vec{c}}{4} = \frac{2\vec{b} + 2\vec{d}}{4}$$

$$\frac{3\vec{a} + \vec{c}}{3 + 1} = \frac{2\vec{b} + 2\vec{d}}{2 + 2}$$

It shows that P is the point which divides AC in ratio $1 : 3$ internally as well as BD in ratio $2 : 2$ internally.

Thus, P is the point of intersection of AC and BD.

Hence,

A, B, C, D are coplanar.

We can say that,

A, B, C, D are coplanar if and only if Let $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$