RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.9

Algebra of Vectors Ex 23.9 Q1

We know that, If l, m, n are the direction cosine of a vector and α, β, γ can the direction angle, then

$$l = \cos\alpha,\ m = \cos\beta\ n = \cos\gamma$$

and, $\quad l^2 + m^2 + n^2 = 1 \qquad$ (i)

$\therefore \quad l = \cos 45°,\ m = \cos 60°,\ n = \cos 120°$

$$l = \frac{1}{\sqrt{2}},\ m = \frac{1}{2}, n = -\frac{1}{2}$$

Put l, m, n in equation (i)

$$l^2 + m^2 + n^2 = 1$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\frac{2+1+1}{4} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1$$

LHS = RHS

Therefore, a vector can have direction angle 45°, 60°, 120°.

Algebra of Vectors Ex 23.9 Q2

Here, $l = 1, m = 1, n = 1$

Put it in

$$l^2 + m^2 + n^2 = 1$$
$$(1)^2 + (1)^2 + (1)^2 = 1$$
$$1 + 1 + 1 = 1$$
$$3 = 1$$
$$\text{LHS} \neq \text{RHS}$$

Therefore,
1,1,1 can not be direction cosines of a straight line.

Algebra of Vectors Ex 23.9 Q3

Here, $\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{4}, \gamma = ?$

$$l = \cos\alpha = \cos\frac{\pi}{4}$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta = \cos\frac{\pi}{4}$$
$$m = \frac{1}{\sqrt{2}}$$

$$n = \cos\gamma$$

Put value of $l, m,$ and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$$
$$\frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$$
$$1 + \cos^2\gamma = 1$$
$$\cos^2\gamma = 1 - 1$$
$$\cos^2\gamma = 0$$
$$\cos\gamma = 0$$
$$\gamma = \cos^{-1}(0)$$
$$\gamma = \frac{\pi}{2}$$

The angle made by the vector with the z-axis $= \frac{\pi}{2}$

Algebra of Vectors Ex 23.9 Q4

Here, $\alpha = \beta = \gamma$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n = x$ (say)

We know that,

$l^2 + m^2 + n^2 = 1$

$x^2 + x^2 + x^2 = 1$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

$l = \pm\frac{1}{\sqrt{3}}, \ m = \pm\frac{1}{\sqrt{3}}, \ n = \pm\frac{1}{\sqrt{3}}$

Hence, direction cosiner of $\vec{r}$ are,

$\pm\frac{1}{\sqrt{3}}, \ \pm\frac{1}{\sqrt{3}}, \ \pm\frac{1}{\sqrt{3}}$

Vector $\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$= 6\left(\pm\frac{1}{\sqrt{3}}\hat{i} + \pm\frac{1}{\sqrt{3}}\hat{j} + \pm\frac{1}{\sqrt{3}}\hat{k}\right)$

$= \frac{\pm 6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$ [Rationalizing the denominator]

$= \frac{\pm 6\sqrt{3}}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

$\vec{r} = \pm 2\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Algebra of Vectors Ex 23.9 Q5

Here, $\alpha = 45°$, $\beta = 60°$, $\gamma = \theta$ (say)

$$l = \cos\alpha$$
$$= \cos 45°$$
$$l = \frac{1}{\sqrt{2}}$$

$$m = \cos\beta$$
$$= \cos 60°$$
$$m = \frac{1}{2}$$

$$n = \cos\theta$$

Put l, m, and n in

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$
$$\frac{2+1}{4} + \cos^2\theta = 1$$
$$\frac{3}{4} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{1}{1} - \frac{3}{4}$$
$$= \frac{4-3}{4}$$
$$\cos^2\theta = \frac{1}{2}$$

So, $l = \frac{1}{\sqrt{2}}$, $m = \frac{1}{2}$, $n = \pm\frac{1}{2}$

The required,

vector $\vec{r} = |\vec{r}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$$= 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$$
$$= 8\,\frac{\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}}{2}$$

$$\vec{r} = 4\left(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k}\right)$$

Algebra of Vectors Ex 23.9 Q6

(i)

Here, the direction ratios of the vector

$2\hat{i} + 2\hat{j} - \hat{k} = 2, 2, -1$

The direction cosines of the vector

$$= \frac{2}{|\vec{r}|}, \frac{2}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$$

$$= \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{2}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}, \frac{-1}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}$$

$$= \frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$$

$$= \frac{2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$$

(ii)

Here, let $\vec{r} = 6\hat{i} - 2\hat{j} - 3\hat{k}$

and, $|\vec{r}| = \sqrt{(6)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

$|\vec{r}| = 7$

The direction cosines of $\vec{r}$ are given by

$$= \frac{6}{|\vec{r}|}, \frac{-2}{|\vec{r}|}, \frac{-3}{|\vec{r}|}$$

$$= \frac{6}{7}, \frac{-2}{7}, \frac{-3}{7}$$

Algebra of Vectors Ex 23.9 Q7(i)

Let, $\vec{r} = \hat{i} - \hat{j} + \hat{k}$

The direction ratios of the vector $\vec{r} = 1, -1, 1$

And, $|\vec{r}| = \sqrt{(1)^2 + (-1)^2 + (1)^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

The direction cosines of the vector $\vec{r}$

$= \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

So, $l = \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$m = \cos\beta = \frac{-1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Thus, angles made by $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right), \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Algebra of Vectors Ex 23.9 Q7(ii)

Let, $\vec{r} = \hat{j} - \hat{k}$

$\vec{r} = 0 \times \hat{i} + \hat{j} - \hat{k}$

The direction ratios of $\vec{r} = 0, 1, -1$

and, $\left|\vec{r}\right| = \sqrt{(0)^2 + (1)^2 + (-1)^2}$

$= \sqrt{0 + 1 + 1}$

$\left|\vec{r}\right| = \sqrt{2}$

The direction cosines of the $\vec{r}$ are given by

$$= \frac{0}{\left|\vec{r}\right|}, \quad \frac{1}{\left|\vec{r}\right|}, \quad \frac{-1}{\left|\vec{r}\right|}$$

$$= \frac{0}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}}, \quad \frac{-1}{\sqrt{2}}$$

So, $l = \cos\alpha = 0$

$\alpha = \cos^{-1}(0)$

$\alpha = \frac{\pi}{2}$

$m = \cos\beta = \frac{1}{\sqrt{2}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\beta = \frac{\pi}{4}$

$n = \cos\gamma = -\frac{1}{\sqrt{2}}$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\lambda = \pi - \frac{\pi}{7}$

$\gamma = \frac{3\pi}{4}$

So, angles made by the vector $\vec{r}$ with coordinate axes are given by

$\frac{\pi}{2}, \quad \frac{\pi}{4}, \quad \frac{3\pi}{4}$

Algebra of Vectors Ex 23.9 Q7(iii)

Let, $4\hat{i} + 8\hat{j} + \hat{k} = \vec{r}$

The direction ratios of $\vec{r} = 4, 8, 1$

And, $|\vec{r}| = \sqrt{(4)^2 + (8)^2 + (1)^2}$

$= \sqrt{16 + 64 + 1}$

$= \sqrt{81}$

$|\vec{r}| = 9$

The direction cosines of the $\vec{r}$ are given by

$= \frac{4}{|\vec{r}|}, \frac{8}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{4}{9}, \frac{8}{9}, \frac{1}{9}$

Now, $l = \cos\alpha = \frac{4}{9}$

$\alpha = \cos^{-1}\left(\frac{4}{9}\right)$

$m = \cos\beta = \frac{8}{9}$

$\beta = \cos^{-1}\left(\frac{8}{9}\right)$

$n = \cos\gamma = \frac{1}{9}$

$\gamma = \cos^{-1}\left(\frac{1}{9}\right)$

The angles made by the vector $\vec{r}$ with the coordinate axes are given by

$\cos^{-1}\left(\frac{4}{9}\right), \quad \cos^{-1}\left(\frac{8}{9}\right), \quad \cos^{-1}\left(\frac{1}{9}\right)$

Algebra of Vectors Ex 23.9 Q8

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α, β, and γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z axes.

Then, we have $\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$.

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Algebra of Vectors Ex 23.9 Q9

Let a vector be equally inclined to axes OX, OY, and OZ at angle α.

Then, the direction cosines of the vector are cos α, cos α, and cos α.

Now,

$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Algebra of Vectors Ex 23.9 Q10

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_1 \qquad [|\vec{a}| = 1]$$

$$\cos\frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \qquad [|\vec{a}| = 1]$$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$$\Rightarrow a_3 = \cos\theta$$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$.

The Plane Ex 23.9 Q11

Let l, m, n be the direction cosines of the vector $\vec{r}$.

$l = \cos\alpha$, $m = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $n = \cos\left(\frac{\pi}{2}\right) = 0$

$l^2 + m^2 + n^2 = 1$

$l^2 + \frac{1}{2} + 0 = 1$

$l = \pm\frac{1}{\sqrt{2}}$

$\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$

$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\right)$

$\vec{r} = \pm 3\hat{i} + 3\hat{j}$

The Plane Ex 23.9 Q12

Let l, m, n be the direction cosines of the vector $\vec{r}$.

Vector $\vec{r}$ is indined at equal angles to the three axes.

$l = \cos\alpha$, $m = \cos\alpha$ and $n = \cos\alpha$

$\Rightarrow l = m = n$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \pm\frac{1}{\sqrt{3}}$

$\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$

$\vec{r} = 2\sqrt{3}\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$

$\vec{r} = \pm 2\hat{i} \pm 2\hat{j} \pm 2\hat{k}$