RD Sharma Solutions
Class 12 Maths
Chapter 24
Ex 24.1

Scalar or Dot Product Ex 24.1 Q1

(i)

$$\vec{a} \cdot \vec{b}$$
$$= \left(\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} - 4\hat{j} + 7\hat{k}\right)$$
$$= (1)(4) + (-2) \cdot (-4) + (1)(7)$$
$$= 4 + 8 + 7$$
$$= 19$$

$$\vec{a} \cdot \vec{b} = 19$$

(ii)

$$\vec{a} \cdot \vec{b} = \left(\hat{j} + 2\hat{k}\right) \cdot \left(2\hat{i} + \hat{k}\right)$$
$$= \left(0 \times \hat{i} + \hat{j} + 2\hat{k}\right)\left(2\hat{i} + 0 \times \hat{j} + \hat{k}\right)$$
$$= (0)(2) + (1)(0) + (2)(1)$$
$$= 0 + 0 + 2$$

$$\vec{a} \cdot \vec{b} = 2$$

(iii)

$$\vec{a} \cdot \vec{b} = \left(\hat{j} - \hat{k}\right) \cdot \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$$
$$= \left(0 \times \hat{i} + \hat{j} - \hat{k}\right)\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$$
$$= (0)(2) + (1)(3) + (-1)(-2)$$
$$= 0 + 3 + 2$$

$$\vec{a} \cdot \vec{b} = 5$$

Scalar or Dot Product Ex 24.1 Q2

(i)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 2\hat{j} + \hat{k}\right).\left(4\hat{i} - 9\hat{j} + 2\hat{k}\right) = 0$

$\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 4\lambda - 18 + 2 = 0$

$\Rightarrow 4\lambda - 16 = 0$

$\Rightarrow 4\lambda = 16$

$\Rightarrow \lambda = \dfrac{16}{4}$

$\Rightarrow \lambda = 4$

(ii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 2\hat{j} + \hat{k}\right).\left(5\hat{i} - 9\hat{j} + 2\hat{k}\right) = 0$

$\Rightarrow (\lambda)(5) + (2)(-9) + (1)(2) = 0$

$\Rightarrow 5\lambda - 18 + 2 = 0$

$\Rightarrow 5\lambda - 16 = 0$

$\Rightarrow 5\lambda = 16$

$\Rightarrow \lambda = \dfrac{16}{5}$

(iii)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \lambda\hat{k}\right) = 0$

$\Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) = 0$

$\Rightarrow 6 + 6 - 4\lambda = 0$

$\Rightarrow 12 - 4\lambda = 0$

$\Rightarrow -4\lambda = -12$

$\Rightarrow \lambda = \dfrac{-12}{-4}$

$\Rightarrow \lambda = 3$

(iv)

$\vec{a}$ and $\vec{b}$ are prependicular

$\Rightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow \left(\lambda\hat{i} + 3\hat{j} + 2\hat{k}\right)\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$

$\Rightarrow (\lambda)(1) + (3)(-1) + (2)(3) = 0$

$\Rightarrow \lambda - 3 + 6 = 0$

$\Rightarrow \lambda + 3 = 0$

$\Rightarrow \lambda = -3$

Scalar or Dot Product Ex 24.1 Q3

We know that, if θ is the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{6}{4\times 3}$$

$$= \frac{6}{12}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{3}$

Scalar or Dot Product Ex 24.1 Q4

$$\left(\vec{a}-2\vec{b}\right) = \left(\hat{i}-\hat{j}\right)-2\left(-\hat{j}+2\hat{k}\right)$$

$$= \left(\hat{i}-\hat{j}\right)+2\hat{j}-4\hat{k}$$

$$= \left(\hat{i}+\hat{j}-4\hat{k}\right)$$

$$\left(\vec{a}+\vec{b}\right) = \left(\hat{i}-\hat{j}\right)+\left(-\hat{j}+2\hat{k}\right)$$

$$= \hat{i}-\hat{j}-\hat{j}+2\hat{k}$$

$$= \left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

Now,

$$\left(\vec{a}-2\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)$$

$$= \left(\hat{i}+\hat{j}-4\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$= (1)(1)+(1)(-2)+(-4)(2)$$

$$= 1-2-8$$

$$= -9$$

$$\left(\vec{a}-2\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right) = -9$$

Scalar or Dot Product Ex 24.1 Q5(i)

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\vec{a}.\vec{b} = \left(\hat{i}-\hat{j}\right)\left(\hat{j}+\hat{k}\right)$$

$$= \left(\hat{i}-\hat{j}+0\times\hat{k}\right)\left(0\times\hat{i}+\hat{j}+\hat{k}\right)$$

$$= (1)(0)+(-1)(1)+(0)(1)$$

$$= 0-1+0$$

$$\vec{a}.\vec{b} = -1$$

$$|\vec{a}| = |\hat{i}-\hat{j}|$$

$$= |\hat{i}-\hat{j}+0\times\hat{k}|$$

$$= \sqrt{(1)^2+(-1)^2+(0)^2}$$

$$= \sqrt{1+1+0}$$

$$|\vec{a}| = \sqrt{2}$$

$$|\vec{b}| = |\hat{j}+\hat{k}|$$

$$= |0\times\hat{i}+\hat{j}+\hat{k}|$$

$$= \sqrt{(0)^2+(1)^2+(1)^2}$$

$$= \sqrt{0+1+1}$$

$$|\vec{b}| = \sqrt{2}$$

Put $\vec{a}\times\vec{b}, |\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-1}{\sqrt{2}\times\sqrt{2}}$$

$$\cos\theta = \frac{-1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \pi - \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

Angle between $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$

Scalar or Dot Product Ex 24.1 Q5(ii)

Let θ be the angle between two vector $\vec{a} = 3\hat{i} - 2\hat{j} - 6\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \cdots (1)$$

$$\begin{aligned}
\vec{a}.\vec{b} &= \left(3\hat{i} - 2\hat{j} - 6\hat{k}\right)\left(4\hat{i} - \hat{j} + 8\hat{k}\right) \\
&= 3*4 + (-2)(-1) + (-6)8 \\
&= 12 + 2 - 48 \\
&= -34
\end{aligned}$$

$$\begin{aligned}
|\vec{a}| &= \sqrt{3^2 + (-2)^2 + (-6)^2} \\
&= \sqrt{49} \\
&= 7
\end{aligned}$$

$$\begin{aligned}
|\vec{b}| &= \sqrt{4^2 + (-1)^2 + 8^2} \\
&= \sqrt{81} \\
&= 9
\end{aligned}$$

Putting value of $|\vec{a}|, |\vec{b}|$ and $\vec{a}.\vec{b}$ in equation (1)

$$\begin{aligned}
\cos\theta &= \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \\
&= \frac{-34}{7*9} \\
&= \frac{-34}{63} \\
\theta &= \cos^{-1}\left(\frac{-34}{63}\right) \\
&= 122.66^\circ
\end{aligned}$$

Scalar or Dot Product Ex 24.1 Q5(iii)

Let the angle between $\vec{a}$ and $\vec{b}$ be θ, then

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \quad ---(i)$$

$$\begin{aligned}
\vec{a} \cdot \vec{b} &= \left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(4\hat{i} + 4\hat{j} - 2\hat{k}\right) \\
&= (2)(4) + (-1)(4) + (2)(-2) \\
&= 8 - 4 - 4
\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\begin{aligned}
|\vec{a}| &= \left|2\hat{i} - \hat{j} + 2\hat{k}\right| \\
&= \sqrt{(2)^2 + (-1)^2 + (2)^2} \\
&= \sqrt{4 + 1 + 4} \\
&= \sqrt{9}
\end{aligned}$$

$$|\vec{a}| = 3$$

$$\begin{aligned}
|\vec{b}| &= \left|4\hat{i} + 4\hat{j} - 2\hat{k}\right| \\
&= \sqrt{(4)^2 + (4)^2 + (-2)^2} \\
&= \sqrt{16 + 16 + 4} \\
&= \sqrt{36}
\end{aligned}$$

$$|\vec{b}| = 6$$

Put $\vec{a} \cdot \vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i)

$$\begin{aligned}
\cos\theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \\
&= \frac{0}{3 \times 6} \\
&= \frac{0}{18}
\end{aligned}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$

Scalar or Dot Product Ex 24.1 Q5(iv)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \quad\quad ---(i)$$

$$\vec{a}\ \vec{b} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right)\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$
$$= (2)(1) + (-3)(1) + (1)(-2)$$
$$= 2 - 3 - 2$$
$$\vec{a}\cdot\vec{b} = -3$$

$$|\vec{a}| = |2\hat{i} - 3\hat{j} + \hat{k}|$$
$$= \sqrt{(2)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{4+9+1}$$
$$= \sqrt{14}$$

$$|\vec{b}| = |\hat{i} + \hat{j} - 2\hat{k}|$$
$$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-2)^2}$$
$$= \sqrt{1+1+4}$$
$$|\vec{b}| = \sqrt{6}$$

Put $\vec{a}\cdot\vec{b}$, $|\vec{a}|$ and $|\vec{b}|$ in equation (i),

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$
$$= \frac{-3}{\sqrt{14} \times \sqrt{6}}$$
$$\cos\theta = \frac{-3}{\sqrt{84}}$$
$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ $= \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

Scalar or Dot Product Ex 24.1 Q5(v)

Let θ be the angle between vector $\vec{a}$ and $\vec{b}$, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \qquad ---(i)$$

$$\begin{aligned}
\vec{a}.\vec{b} &= \left(\hat{i} + 2\hat{j} - \hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right) \\
&= (1)(1) + (2)(-1) + (-1)(1) \\
&= 1 - 2 - 1
\end{aligned}$$
$$\vec{a}.\vec{b} = -2$$

$$\begin{aligned}
|\vec{a}| &= |\hat{i} + 2\hat{j} - \hat{k}| \\
&= \sqrt{(1)^2 + (2)^2 + (-1)^2} \\
&= \sqrt{1 + 4 + 1} \\
&= \sqrt{6}
\end{aligned}$$

$$\begin{aligned}
|\vec{b}| &= |\hat{i} - \hat{j} + \hat{k}| \\
|\vec{b}| &= \sqrt{(1)^2 + (-1)^2 + (1)^2} \\
&= \sqrt{1 + 1 + 1} \\
|\vec{b}| &= \sqrt{3}
\end{aligned}$$

Put $\vec{a}.\vec{b}$, $|\vec{a}|$, $|\vec{b}|$ in equation (i),

$$\begin{aligned}
\cos\theta &= \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \\
&= \frac{-2}{\sqrt{6}\sqrt{3}} \\
&= \frac{-2}{\sqrt{18}} \\
&= \frac{-2 \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} \\
&= \frac{-2\sqrt{2}}{3 \times 2}
\end{aligned}$$
$$\cos\theta = \frac{-\sqrt{2}}{3}$$
$$\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$$

Angle between vector $\vec{a}$ and $\vec{b}$ = $\cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$

Scalar or Dot Product Ex 24.1 Q6

Component along x-, y- and z-axis are $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

Let θ_1 be the angle between $\vec{a}$ and $\hat{i}$.

$$\cos\theta_1 = \frac{\vec{a}.\hat{i}}{|\vec{a}||\hat{i}|}$$

$$= \frac{\left(\hat{i} - \hat{j} + \sqrt{2}\hat{k}\right)\left(\hat{i} + 0.\hat{j} + 0.\hat{k}\right)}{\left|\hat{i} - \hat{j} + \sqrt{2}\hat{k}\right|\left|\hat{i} + 0.\hat{j} + 0.\hat{k}\right|}$$

$$= \frac{(1)(1) + (-1)(0) + \left(\sqrt{2}\right)(0)}{\sqrt{(1)^2 + (-1)^2 + \left(\sqrt{2}\right)^2}.\sqrt{(1)^2 + (0)^2 + (0)^2}}$$

$$= \frac{1 + 0 + 0}{\sqrt{4}\sqrt{1}}$$

$$\cos\theta_1 = \frac{1}{2}$$

$$\theta_1 = \frac{\pi}{3}$$

Let θ_2 be the angle between $\vec{a}$ and $\hat{j}$.

$$\cos\theta_2 = \frac{\vec{a}.\hat{j}}{|\vec{a}||\hat{j}|}$$

$$= \frac{\left(\hat{i} - \hat{j} + \sqrt{2}\hat{k}\right)\left(0.\hat{j} + \hat{j} + 0\hat{k}\right)}{\sqrt{(1)^2 + (-1)^2 + \left(\sqrt{2}\right)^2}.\sqrt{(0)^2 + (1)^2 + (0)^2}}$$

$$= \frac{(1)(0) + (-1)(1) + \left(\sqrt{2}\right)(0)}{\sqrt{1+1+2}\ \sqrt{1}}$$

$$= \frac{-1}{\sqrt{4}\sqrt{1}}$$

$$= \frac{-1}{2}$$

$$\cos\theta_2 = -\frac{1}{2}$$

$$\theta_2 = \pi - \frac{\pi}{3}$$

$$\theta_2 = \frac{2\pi}{3}$$

Let θ_3 be the angle between $\vec{a}$ and $\hat{k}$, then

$$\cos\theta_3 = \frac{\vec{a}.\hat{k}}{|\vec{a}||\hat{k}|}$$

$$= \frac{\left(\hat{i} - \hat{j} + \sqrt{2}\hat{k}\right)\left(0\hat{i} + 0\hat{j} + \hat{k}\right)}{\sqrt{(1)^2 + (-1)^2 + \left(\sqrt{2}\right)^2}\ \sqrt{(0)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(1)(0) + (-1)(0) + \left(\sqrt{2}\right)(1)}{\sqrt{1+1+2}\ \sqrt{1}}$$

$$= \frac{\sqrt{2}}{\sqrt{4}\ \sqrt{1}}$$

$$\cos\theta_3 = \frac{1}{\sqrt{2}}$$

$$\theta_3 = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta_3 = \frac{\pi}{4}$$

So, the angle between vector $\vec{a}$ and x-axis is $\frac{\pi}{3}$, vector $\vec{a}$ and y-axis is $\frac{2\pi}{3}$, vector $\vec{a}$ and z-axis is $\frac{\pi}{4}$.

Scalar or Dot Product Ex 24.1 Q7(i)

Let the requird vector be $x\hat{i} + y\hat{j} + z\hat{k}$

According to question,

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} - 3\hat{k}\right) = 0$$
$$(x)(1) + (y)(1) + (z)(-3) = 0$$
$$x + y - 3z = 0 \qquad\qquad ---(i)$$

And,

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 3\hat{j} - 2\hat{k}\right) = 5$$
$$(x)(1) + (y)(3) + (z)(-2) = 5$$
$$x + 3y - 2z = 5 \qquad\qquad ---(ii)$$

And,

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + \hat{j} + 4\hat{k}\right) = 8$$
$$(x)(2) + (y)(1) + (z)(4) = 8$$
$$2x + y + 4z = 8 \qquad\qquad ---(iii)$$

Subtracting (i) from (ii),

$$x + 3y - 2z = 5$$
$$\underset{(-)(-)\ (+)}{x + y - 3z\ = 0}$$
$$\overline{\qquad 2y + z = 5 \qquad ---(iv)}$$

Subtracting $2 \times$ (ii) from (iii),

$$2x + y + 4z\ = 8$$
$$\underset{(-)\ (-)\ (+)\ (-)}{2x + 6y - 4z = 10}$$
$$\overline{\qquad -5y + 8z = -2 \qquad ---(v)}$$

Subtracting $8 \times$ (iv) from (v),

$$-5y + 8z = -2$$
$$\underset{(-)\ (-)\ (-)}{1\ 8y + 8z = 40}$$
$$\overline{-21y \qquad = -42 \qquad\qquad ---(vi)}$$
$$y = \frac{-42}{-21}$$
$$y = 2$$

Put $y = 2$ in equation (iv),

$2y + z = 5$

$2(2) + z = 5$

$4 + z = 5$

$z = 5 - 4$

$z = 1$

Put $y = 2$ and $z = 1$ in equation (i),

$x + y - 3z = 0$

$x + (2) - 3(1) = 0$

$x + 2 - 3 = 0$

$x - 1 = 0$

$x = 1$

The required vector $= x\hat{i} + y\hat{j} + z\hat{k}$

The required vector $= \hat{i} + 2\hat{j} + \hat{k}$

Scalar or Dot Product Ex 24.1 Q8(i)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors, then

$$\left|\hat{a}\right| = \left|\hat{b}\right| = 1$$

$$\left|\hat{a} + \hat{b}\right|^2 = \left(\hat{a} + \hat{b}\right)^2$$

$$= \left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 + 2\hat{a} \cdot \hat{b}$$

$$= \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\hat{a} \cdot \hat{b}$$

$$= (1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b}$$

$$\left|\hat{a} + \hat{b}\right|^2 = 2 + 2\hat{a} \times \hat{b}$$

$$\left|\hat{a} + \hat{b}\right|^2 = 2 + 2 \times \left|\hat{a}\right|\left|\hat{b}\right|\cos\theta \qquad \left[\text{Since } \vec{a} \cdot \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right]$$

$$\left|\hat{a} + \hat{b}\right|^2 = 2 + 2 \times 1 \times 1 \times \cos\theta$$

$$= 2 + 2\cos\theta$$

$$\left|\hat{a} + \hat{b}\right|^2 = 2(1 + \cos\theta)$$

$$\left|\hat{a} + \hat{b}\right|^2 = 2\left(2\cos^2\frac{\theta}{2}\right) \qquad \left[\text{Since } 1 + \cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\left|\hat{a} + \hat{b}\right|^2 = 4\cos^2\frac{\theta}{2}$$

$$\left|\hat{a} + \hat{b}\right| = \sqrt{4\cos^2\frac{\theta}{2}}$$

$$\left|\hat{a} + \hat{b}\right| = 2\cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} + \hat{b}\right|$$

Scalar or Dot Product Ex 24.1 Q8(ii)

Here, $\hat{a}$ and $\hat{b}$ are unit vectors

$$\left|\hat{a}\right| = \left|\hat{b}\right| = 1$$

$$\frac{\left|\hat{a} - \hat{b}\right|^2}{\left|\hat{a} + \hat{b}\right|^2} = \frac{\left(\hat{a} - \hat{b}\right)^2}{\left(\hat{a} + \hat{b}\right)^2}$$

$$= \frac{\left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 - 2\hat{a}.\hat{b}}{\left(\hat{a}\right)^2 + \left(\hat{b}\right)^2 + 2\hat{a}.\hat{b}}$$

$$= \frac{\left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 - 2\hat{a}.\hat{b}}{\left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\hat{a}.\hat{b}}$$

$$\frac{\left|\hat{a} - \hat{b}\right|^2}{\left|\hat{a} + \hat{b}\right|^2} = \frac{(1)^2 + (1)^2 - 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta}{(1)^2 + (1)^2 + 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta} \qquad \left[\text{Since } \vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right]$$

$$\frac{\left|\hat{a} - \hat{b}\right|^2}{\left|\hat{a} + \hat{b}\right|^2} = \frac{1 + 1 - 2(1)(1)\cos\theta}{1 + 1 + 2(1)(1)\cos\theta}$$

$$= \frac{2 - 2\cos\theta}{2 + 2\cos\theta}$$

$$= \frac{2(1 - \cos\theta)}{2(1 + \cos\theta)}$$

$$= \frac{2 \times \sin^2\frac{\theta}{2}}{2 \times \cos^2\frac{\theta}{2}} \qquad \left[\text{Since } 1 - \cos\theta = 2\sin^2\frac{\theta}{2},\ 1 + \cos\theta = 2\cos^2\frac{\theta}{2}\right]$$

$$\frac{\left|\hat{a} - \hat{b}\right|^2}{\left|\hat{a} + \hat{b}\right|^2} = \tan^2\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{\left|\hat{a} - \hat{b}\right|}{\left|\hat{a} + \hat{b}\right|}$$

Scalar or Dot Product Ex 24.1 Q9

Let $\hat{a}$ and $\hat{b}$ are two unit vectors

Then, $|\hat{a}| = |\hat{b}| = 1$

And sum of $\hat{a}$ and $\hat{b}$ is a unit vector, then

$$|\hat{a} + \hat{b}| = 1$$

Taking square of both the sides,

$$|\hat{a} + \hat{b}|^2 = (1)^2$$

$$(\hat{a} + \hat{b})^2 = 1$$

$$(\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$(1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$2 + 2\hat{a} \ \hat{b} = 1$$

$$2\hat{a} \cdot \hat{b} = 1 - 2$$

$$2\hat{a} \cdot \hat{b} = -1$$

$$\hat{a} \cdot \hat{b} = \frac{-1}{2} \quad - - - (i)$$

$$|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 - 2\hat{a} \cdot \hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 - 2 \times \hat{a} \cdot \hat{b}$$

$$= (1)^2 + (1)^2 - 2 \times \left(\frac{-1}{2}\right)$$

Using equation (i)

$$= 1 + 1 + \frac{2}{2}$$

$$= 1 + 1 + 1$$

$$|\hat{a} - \hat{b}|^2 = 3$$

$$|\hat{a} - \hat{b}| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q10

Given that $\vec{a}, \vec{b}, \vec{c}$ are mutually prependicular, so,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

and $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, so

$$\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1$$

Now,

$$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left(\vec{a} + \vec{b} + \vec{c}\right)^2$$

$$= \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a}$$

$$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2(0) + 2(0) + 2(0)$$

$$= (1)^2 + (1)^2 + (1)^2 + 0$$

$$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 1 + 1 + 1$$

$$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 3$$

$$\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{3}$$

Scalar or Dot Product Ex 24.1 Q11

Here, $\left|\vec{a}+\vec{b}\right| = 60$

Squaring both the sides,

$$\left|\vec{a}+\vec{b}\right|^2 = (60)^2$$
$$\left(\vec{a}+\vec{b}\right) = (60)^2$$
$$\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + 2\vec{a}\vec{b} = 3600$$
$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\vec{b} = 3600 \qquad ---(i)$$

Now, $\left|\vec{a}-\vec{b}\right| = 40$

Squaring both the sides,

$$\left|\vec{a}-\vec{b}\right|^2 = (40)^2$$
$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\vec{b} = 1600 \qquad ---(ii)$$

Adding (i) and (ii),

$$2\left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2 + 2\vec{a}\vec{b} - 2\vec{a}\vec{b} = 3600 - 1600$$
$$2\left|\vec{a}\right|^2 + 2(46)^2 = 5200$$
$$2\left|\vec{a}\right|^2 = 5200 - 4232$$
$$2\left|\vec{a}\right|^2 = 968$$
$$\left|\vec{a}\right|^2 = \frac{968}{2}$$
$$\left|\vec{a}\right|^2 = 484$$
$$\left|\vec{a}\right| = \sqrt{484}$$
$$\left|\vec{a}\right| = 22$$

Scalar or Dot Product Ex 24.1 Q12

Let θ be the angle between $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i}$

Then,

$$\cos\theta = \frac{\left(\hat{i} + \hat{j} + \hat{k}\right).\left(\hat{i}\right)}{\left|\hat{i} + \hat{j} + \hat{k}\right|\left|\hat{i}\right|}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

Similarly, if α and γ are angles that $\hat{i} + \hat{j} + \hat{k}$ make with $\hat{j}$ and $\hat{k}$

Then,

$$\cos\alpha = \sqrt{3}$$

and $\quad \cos\gamma = \sqrt{3}$

Therefore, $\hat{i} + \hat{j} + \hat{k}$ is equally inclined the three axes.

Scalar or Dot Product Ex 24.1 Q13

We have,

$$\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$

$$\vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

Then,

$$\vec{a} \cdot \vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) \times \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$

$$= \frac{1}{49}\left(6 - 18 + 12\right) = 0$$

Similarly,

$$\vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0$$

∴ $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular

Scalar or Dot Product Ex 24.1 Q14

Let $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right| = \left|\vec{b}\right|$$

Let $\left|\vec{a}\right| = \left|\vec{b}\right|$

Squaring both the sides.

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2$$

$$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$$

$$\left(\vec{a}\right)^2 - \left(\vec{b}\right)^2 = 0$$

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$$

Thus,

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0 \Leftrightarrow \left|\vec{a}\right| = \left|\vec{b}\right|$$

Scalar or Dot Product Ex 24.1 Q15

If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, find λ

Given that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$\therefore \vec{a} \bullet \left(\lambda\vec{b} + \vec{c}\right) = 0$

$\lambda\vec{a} \bullet \vec{b} + \vec{a} \bullet \vec{c} = 0$

$\lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \bullet \left(\hat{i} + \hat{j} - 2\hat{k}\right) + \left(2\hat{i} - \hat{j} + \hat{k}\right) \bullet \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

$\lambda\left(2 - 1 - 2\right) + \left(2 - 3 - 1\right) = 0$

$-\lambda - 2 = 0$

$\lambda = 2$

Scalar or Dot Product Ex 24.1 Q16

$\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{p} + \vec{q}$

$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$

$= 6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}$

$\vec{p} - \vec{q}$

$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$

$= 4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}$

$(\vec{p} + \vec{q}).(\vec{p} - \vec{q}) = 0$

$\Rightarrow [6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}].[4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}] = 0$

$\Rightarrow 24 + (\lambda^2 - 9) - 16 = 0$

$\Rightarrow \lambda^2 - 9 + 8 = 0$

$\Rightarrow \lambda^2 - 1 = 0$

$\therefore \lambda = \pm 1$

Scalar or Dot Product Ex 24.1 Q17

According to question $\overline{\beta}_1$ is parallel to $\overline{\alpha}$. So

$$\overline{\beta}_1 = \gamma \overline{\alpha}$$
$$= \gamma \left(3\hat{i} + 4\hat{j} + 5\hat{k} \right)$$

$$\overline{\beta} = \overline{\beta}_1 + \overline{\beta}_2$$
$$2\hat{i} + \hat{j} - 4\hat{k} = \gamma \left(3\hat{i} + 4\hat{j} + 5\hat{k} \right) + \overline{\beta}_2 \qquad \left(putting\ \overline{\beta}\ and\ \overline{\beta}_1 \right)$$
$$\overline{\beta}_2 = \left(2 - 3\gamma \right)\hat{i} + \left(1 - 4\gamma \right)\hat{j} - \left(4 + 5\gamma \right)\hat{k}$$

Again $\overline{\beta}_2$ is perpendicular to $\overline{\alpha}$. So

$$\overline{\beta}_2.\overline{\alpha} = 0$$
$$\left[\left(2 - 3\gamma \right)\hat{i} + \left(1 - 4\gamma \right)\hat{j} - \left(4 + 5\gamma \right)\hat{k} \right].\left(3\hat{i} + 4\hat{j} + 5\hat{k} \right) = 0$$
$$6 - 9\gamma + 4 - 16\gamma - 20 - 25\gamma = 0$$
$$-50\gamma = 10$$
$$\gamma = -\frac{1}{5}$$

$$\overline{\beta}_1 = -\frac{1}{5}\left(3\hat{i} + 4\hat{j} + 5\hat{k} \right)$$
$$\overline{\beta} = \overline{\beta}_1 + \overline{\beta}_2$$
$$2\hat{i} + \hat{j} - 4\hat{k} = -\frac{1}{5}\left(3\hat{i} + 4\hat{j} + 5\hat{k} \right) + \overline{\beta}_2 \qquad \left(putting\ \overline{\beta}\ and\ \overline{\beta}_1 \right)$$

$$\overline{\beta}_2 = \frac{1}{5}\left(13\hat{i} + 9\hat{j} - 15\hat{k} \right)$$
$$\overline{\beta} = -\frac{1}{5}\left(3\hat{i} + 4\hat{j} + 5\hat{k} \right) + \frac{1}{5}\left(13\hat{i} + 9\hat{j} - 15\hat{k} \right)$$

Scalar or Dot Product Ex 24.1 Q18

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then,

$$\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3\left(-6 \right) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$
$$\therefore \vec{a} \neq \vec{0}$$
$$|\vec{b}| = \sqrt{3^2 + 3^2 + \left(-6 \right)^2} = \sqrt{54}$$
$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Scalar or Dot Product Ex 24.1 Q19

Here,

$$\vec{b} + \vec{c} = \left(\hat{i} - 3\hat{j} + 5\hat{k}\right)\left(2\hat{i} + \hat{j} - 4\hat{k}\right)$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b} + \vec{c} = \vec{a}$$

∴ $\vec{a}, \vec{b}, \vec{c}$ are represents the sides of a triangle.

$$|\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (1)^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= \sqrt{14}$$

$$|\vec{b}| = \sqrt{(1)^2 + (-3)^2 + (5)^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$|\vec{b}| = \sqrt{35}$$

$$|\vec{c}| = \sqrt{(2)^2 + (1)^2 + (-4)^2}$$

$$= \sqrt{4 + 1 + 16}$$

$$= \sqrt{21}$$

$$\left(\sqrt{21}\right)^2 + \left(\sqrt{14}\right)^2 = \left(\sqrt{35}\right)^2$$

$$21 + 14 = 35$$

$$35 = 35$$

$$|\vec{c}|^2 + |\vec{a}|^2 = |\vec{b}|^2$$

∴   By the pythagorous theorem,

Triangle formed by $\vec{a}, \vec{b}, \vec{c}$ is a right angled triangled.

Scalar or Dot Product Ex 24.1 Q20

The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$$\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If $\left(\vec{a} + \lambda\vec{b}\right)$ is perpendicular to $\vec{c}$, then

$$\left(\vec{a} + \lambda\vec{b}\right) \cdot \vec{c} = 0.$$

$$\Rightarrow \left[(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}\right] \cdot \left(3\hat{i} + \hat{j}\right) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of λ is 8.

Scalar or Dot Product Ex 24.1 Q21

$$\vec{A} = 0.\hat{i} - \hat{j} - 2\hat{k}$$
$$\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$$
$$\vec{C} = 5\hat{i} + 7\hat{j} + \hat{k}$$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(3\hat{i} + \hat{j} + 4\hat{k}\right) - \left(0.\hat{i} - \hat{j} - 2\hat{k}\right)$$
$$= 3\hat{i} + \hat{j} + 4\hat{k} - 0.\hat{i} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} - 3\hat{i} - \hat{j} - 4\hat{k}$$
$$\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$
$$= \left(5\hat{i} + 7\hat{j} + \hat{k}\right) - \left(-\hat{j} - 2\hat{k}\right)$$
$$= 5\hat{i} + 7\hat{j} + \hat{k} + \hat{j} + 2\hat{k}$$
$$\overrightarrow{AC} = 5\hat{i} + 8\hat{j} + 3\hat{k}$$

Angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$,

$$\cos A = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|}$$

$$= \frac{\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)\left(5\hat{i} + 8\hat{j} + 3\hat{k}\right)}{\sqrt{(3)^2 + (2)^2 + (6)^2}\sqrt{(5)^2 + (8)^2 + (3)^2}}$$

$$= \frac{(3)(5) + (2)(8) + (6)(3)}{\sqrt{9 + 4 + 36}\sqrt{25 + 64 + 9}}$$

$$= \frac{15 + 16 + 18}{\sqrt{49}\sqrt{98}}$$

$$= \frac{49}{\sqrt{49}\sqrt{49 \times 2}}$$

$$\cos A = \frac{49}{49\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

Angle between $\vec{BC}$ and $\vec{BA}$

$$\cos B = \frac{\vec{BC}.\vec{BA}}{|\vec{BC}||\vec{BA}|}$$

$$= \frac{\left(2\hat{i} + 6\hat{j} - 3\hat{k}\right)\left(-3\hat{i} - 2\hat{j} - 6\hat{k}\right)}{\sqrt{(2)^2 + (6)^2 + (-3)^2}\sqrt{(-3)^2 + (-2)^2 + (-6)^2}}$$

$$= \frac{(2)(-3) + (6)(-2) + (-3)(-6)}{\sqrt{4 + 36 + 9}\sqrt{9 + 4 + 36}}$$

$$= \frac{-6 - 12 + 18}{\sqrt{49}\sqrt{98}}$$

$$\cos B = \frac{-18 + 18}{49}$$

$$= \frac{0}{49}$$

$$\cos B = 0$$

$$B = \cos^{-1}(0)$$

$$\angle B = \frac{\pi}{2}$$

We know that,

$$\angle A + \angle B + \angle C = \pi$$

$$\frac{\pi}{4} + \frac{\pi}{2} + \angle C = \pi$$

$$\frac{3\pi}{4} + \angle C = \pi$$

$$\angle C = \frac{\pi}{1} - \frac{3\pi}{4}$$

$$\angle C = \frac{4\pi - 3\pi}{4}$$

$$\angle C = \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

$$\angle B = \frac{\pi}{2}$$

Scalar or Dot Product Ex 24.1 Q22

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \dfrac{1}{2}$, and $\theta = 60°$.   ...(1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60° \qquad \left[\text{Using }(1)\right]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q23

Given

$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{c} = \hat{i} - \hat{j}$$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$$

$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$$

$\overrightarrow{AB} = -2\hat{i} - \hat{j} + 4\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(\hat{i} - \hat{j}\right) - \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$$

$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$\overrightarrow{CA}$ = Position vector of A – Position vector of C

$$= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

Now, $\overrightarrow{AB}.\overrightarrow{CA}$

$$= \left(-2\hat{i} - \hat{j} + 4\hat{k}\right).\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= (-2)(3) + (-1)(-2) + (4)(1)$$

$$= -6 + 2 + 4$$

$$= -6 + 6$$

$$= 0$$

So, $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CA}$
    $\angle A$ is right angle.

Hence, ABC is a right triangle

Scalar or Dot Product Ex 24.1 Q24

Given,

$$A = (1,2,3)$$
$$B = (-1,0,0)$$
$$C = (0,1,2)$$

Position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$

Position vector of $B = -\hat{i} + 0\hat{j} + 0\hat{k}$

Position vector of $C = 0\hat{i} + \hat{j} + 2\hat{k}$

$\overline{AB}$ = Position vector of B - Position vector of A

$$= \left(-\hat{i} + 0\hat{j} + 0\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= -2\hat{i} - 2\hat{j} - 3\hat{k}$$

$\overline{BC}$ = Position vector of C - Position vector of B

$$= \left(0\hat{i} + \hat{j} + 2\hat{k}\right) - \left(-\hat{i} + 0\hat{j} + 0\hat{k}\right)$$
$$= \hat{i} + \hat{j} + 2\hat{k}$$

$\overline{AC}$ = Position vector of C - Position vector of A

$$= \left(0\hat{i} + \hat{j} + 2\hat{k}\right) - \left(1\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= -\hat{i} - \hat{j} - \hat{k}$$

$$\overline{AB}.\overline{BC} = \left(-2\hat{i} - 2\hat{j} - 3\hat{k}\right).\left(\hat{i} + \hat{j} + 2\hat{k}\right)$$
$$= -2 - 2 - 6$$
$$= -10$$

$$\angle ABC = \frac{\overrightarrow{AB}.\overrightarrow{BC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|}$$

$$= \frac{-10}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2}\sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{-10}{\sqrt{17}\sqrt{6}}$$

$$= \frac{-10}{\sqrt{102}}$$

$$\angle ABC = \cos^{-1}\left(\frac{-10}{\sqrt{102}}\right)$$

Scalar or Dot Product Ex 24.1 Q25

Given

$$A = (0, 1, 1)$$
$$B = (3, 1, 5)$$
$$C = (0, 3, 3)$$

Position vector of $A = 0\hat{i} + \hat{j} + \hat{k}$

Position vector of $B = 3\hat{i} + \hat{j} + 5\hat{k}$

Position vector of $C = 0\hat{i} + 3\hat{j} + 3\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$\quad = \left(3\hat{i} + \hat{j} + 5\hat{k}\right) - \left(0\hat{i} + \hat{j} + \hat{k}\right)$

$\quad = 3\hat{i} + \hat{j} + 5\hat{k} - \hat{j} - \hat{k}$

$\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$\quad = \left(0\hat{i} + 3\hat{j} + 3\hat{k}\right) - \left(3\hat{i} + \hat{j} + 5\hat{k}\right)$

$\overrightarrow{BC} = 3\hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} - 5\hat{k}$

$\quad = -3\hat{i} + 2\hat{j} - 2\hat{k}$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$\quad = \left(-3\hat{j} + 3\hat{k}\right) - \left(\hat{j} + \hat{k}\right)$

$\quad = 3\hat{j} + 3\hat{k} - \hat{j} - \hat{k}$

$\quad = 2\hat{j} + 2\hat{k}$

$\overrightarrow{BC}.\overrightarrow{AC}$

$= \left(-3\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{j} + 2\hat{k}\right)$

$= (-3)(0) + (2)(2) + (-2)(+2)$

$= 0 + 4 - 4$

$= 0$

So, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ is perpendicular

$\Rightarrow$ $\angle C$ is right angle.

Scalar or Dot Product Ex 24.1 Q26

Projection of $\left(\vec{b}+\vec{c}\right)$ on $\vec{a}$

$$= \frac{\left(\vec{b}+\vec{c}\right).\vec{a}}{\left|\vec{a}\right|}$$

$$= \frac{\vec{b}.\vec{a}+\vec{c}.\vec{a}}{\sqrt{(2)^2+(-2)^2+(1)^2}}$$

$$= \frac{\left(\hat{i}+2\hat{j}-2\hat{k}\right)\left(2\hat{i}-2\hat{j}+\hat{k}\right)\left(2\hat{i}-\hat{j}+4\hat{k}\right)\left(2\hat{i}-2\hat{j}+\hat{k}\right)}{\sqrt{4+4+1}}$$

$$= \frac{(1)(2)+(2)(-2)+(-2)(1)+(2)(2)+(-1)(-2)+(4)(1)}{\sqrt{9}}$$

$$= \frac{2-4-2+4+2+4}{3}$$

$$= \frac{12-6}{3} = \frac{6}{3} = 2$$

Projection of $\left(\vec{b}+\vec{c}\right) = 2$

Scalar or Dot Product Ex 24.1 Q27

$$\vec{a}+\vec{b} = \left(5\hat{i}-\hat{j}-3\hat{k}\right)+\left(\hat{i}+3\hat{j}-5\hat{k}\right)$$
$$= 5\hat{i}-\hat{j}-3\hat{k}+\hat{i}+3\hat{j}-5\hat{k}$$
$$\vec{a}+\vec{b} = 6\hat{i}+2\hat{j}-8\hat{k} \qquad ---\text{(i)}$$

$$\vec{a}-\vec{b} = \left(5\hat{i}-\hat{j}-3\hat{k}\right)-\left(\hat{i}+3\hat{j}-5\hat{k}\right)$$
$$= 5\hat{i}-\hat{j}-3\hat{k}-\hat{i}-3\hat{j}+5\hat{k}$$
$$\vec{a}-\vec{b} = 4\hat{i}-4\hat{j}+2\hat{k} \qquad ---\text{(ii)}$$

Now, $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)$

$$= \left(6\hat{i}+2\hat{j}-8\hat{k}\right).\left(4\hat{i}-4\hat{j}+2\hat{k}\right)$$
$$= (6)(4)+(2)(-4)+(-8)(2)$$
$$= 24-8-16$$
$$= 0$$

So, $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ are perpendicular.

Scalar or Dot Product Ex 24.1 Q28

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\dfrac{\pi}{4}$ with $\hat{i}$, $\dfrac{\pi}{3}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$$\cos\frac{\pi}{4} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_1 \qquad \left[|\vec{a}| = 1\right]$$

$$\cos\frac{\pi}{3} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_2 \qquad \left[|\vec{a}| = 1\right]$$

Also, $\cos\theta = \dfrac{a_3}{|\vec{a}|}$.

$$\Rightarrow a_3 = \cos\theta$$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$$

Hence, $\theta = \dfrac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{2}, \dfrac{1}{2}\right)$.

Scalar or Dot Product Ex 24.1 Q29

$$\left(3\vec{a}-5\vec{b}\right)\cdot\left(2\vec{a}+7\vec{b}\right)$$

$$= 3\vec{a}\cdot2\vec{a}+3\vec{a}\cdot7\vec{b}-5\vec{b}\cdot2\vec{a}-5\vec{b}\cdot7\vec{b}$$

$$= 6\vec{a}\cdot\vec{a}+21\vec{a}\cdot\vec{b}-10\vec{a}\cdot\vec{b}-35\vec{b}\cdot\vec{b}$$

$$= 6\left|\vec{a}\right|^2+11\vec{a}\cdot\vec{b}-35\left|\vec{b}\right|^2$$

$$= 6*2^2+11*1-35*1^2$$

$$= 35-35$$

$$= 0$$

Scalar or Dot Product Ex 24.1 Q30(i)

We have,

$$\left(\vec{x}-\vec{a}\right)\cdot\left(\vec{x}+\vec{a}\right)=8$$

$$\Rightarrow \left|\vec{x}\right|^2-\left|\vec{a}\right|^2=8$$

$$\Rightarrow \left|\vec{x}\right|^2-1^2=8 \qquad sin\,ce\;\left|\vec{a}\right|=1$$

$$\Rightarrow \left|\vec{x}\right|^2=8+1$$

$$\Rightarrow \left|\vec{x}\right|^2=9$$

$$\Rightarrow \left|\vec{x}\right|=3$$

Scalar or Dot Product Ex 24.1 Q30(ii)

We have,

$$\left(\vec{x}-\vec{a}\right)\cdot\left(\vec{x}+\vec{a}\right)=12$$

$$\Rightarrow \left|\vec{x}\right|^2-\left|\vec{a}\right|^2=12$$

$$\Rightarrow \left|\vec{x}\right|^2-1^2=12 \qquad sin\,ce\;\left|\vec{a}\right|=1$$

$$\Rightarrow \left|\vec{x}\right|^2=12+1$$

$$\Rightarrow \left|\vec{x}\right|^2=13$$

$$\Rightarrow \left|\vec{x}\right|=\sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q31(i)

Here, $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 12$

$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 12$

$\left(2\left|\vec{b}\right|\right)^2 - \left|\vec{b}\right|^2 = 12$ $\left[\text{Using } \left|\vec{a}\right| = 2\left|\vec{b}\right|\right]$

$4\left|\vec{b}\right|^2 - \left|\vec{b}\right|^2 = 12$

$3\left|\vec{b}\right|^2 = 12$

$\left|\vec{b}\right|^2 = \dfrac{12}{3}$

$\left|\vec{b}\right|^2 = 4$

$\left|\vec{b}\right| = 2$

$\left|\vec{a}\right| = 2\left|\vec{b}\right| = 2(2)$

$\left|\vec{a}\right| = 4$

$\left|\vec{b}\right| = 2$

Scalar or Dot Product Ex 24.1 Q31(ii)

$\left(\vec{a}\cdot\vec{b}\right).\left(\vec{a}-\vec{b}\right) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} - \vec{a}.\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 8$

$\Rightarrow \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 8$

$\Rightarrow \left(8\left|\vec{b}\right|\right)^2 - \left|\vec{b}\right|^2 = 8$ $\left[\left|\vec{a}\right| = 8\left|\vec{b}\right|\right]$

$\Rightarrow 64\left|\vec{b}\right|^2 - \left|\vec{b}\right|^2 = 8$

$\Rightarrow 63\left|\vec{b}\right|^2 = 8$

$\Rightarrow \left|\vec{b}\right|^2 = \dfrac{8}{63}$

$\Rightarrow \left|\vec{b}\right| = \sqrt{\dfrac{8}{63}}$ [Magnitude of a vector is non-negative]

$\Rightarrow \left|\vec{b}\right| = \dfrac{2\sqrt{2}}{3\sqrt{7}}$

$\left|\vec{a}\right| = 8\left|\vec{b}\right| = \dfrac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \dfrac{16\sqrt{2}}{3\sqrt{7}}$

Scalar or Dot Product Ex 24.1 Q31(iii)

Here, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 3$

$$|\vec{a}|^2 - |\vec{b}|^2 = 3$$

$$(2|\vec{b}|)^2 - |\vec{b}|^2 = 3 \qquad\qquad [\text{Using } |\vec{a}| = 2|\vec{b}|]$$

$$4|\vec{b}|^2 - |\vec{b}|^2 = 3$$

$$3|\vec{b}|^2 = 3$$

$$|\vec{b}|^2 = \frac{3}{3}$$

$$|\vec{b}|^2 = 1$$

$$|\vec{b}| = 1$$

$$|\vec{a}| = 2|\vec{b}|$$

$$= 2(1)$$

$$|\vec{a}| = 2$$

$$|\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q32(i)

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$= (2)^2 + (5)^2 - 2(8)$$

$$= 4 + 25 - 16$$

$$|\vec{a}-\vec{b}|^2 = 13$$

$$|\vec{a}-\vec{b}| = \sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q32(ii)

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$= (3)^2 + (4)^2 - 2.(1)$$

$$= 9 + 16 - 2$$

$$|\vec{a}-\vec{b}|^2 = 23$$

$$|\vec{a}-\vec{b}| = \sqrt{23}$$

Scalar or Dot Product Ex 24.1 Q32(iii)

We have

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$$

$$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$$

$$= |\vec{a}|^2 - 2(\vec{a}.\vec{b}) + |\vec{b}|^2 = (2)^2 - 2(4) + (3)^2 = 5$$

$$\therefore \quad |\vec{a}-\vec{b}| = \sqrt{5}$$

Scalar or Dot Product Ex 24.1 Q33(i)

We have,

$\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$= \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$= \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{4}$$

Scalar or Dot Product Ex 24.1 Q33(ii)

Let the angle between $\vec{a}$ and $\vec{b}$ is θ, then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$= \frac{1}{3.3}$$

$$\cos\theta = \frac{1}{9}$$

$$\theta = \cos^{-1}\left(\frac{1}{9}\right)$$

Scalar or Dot Product Ex 24.1 Q34

Let $\vec{a} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ --- (i)

Such that $\vec{u}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$.

Now, $\vec{u}$ is parallel to $\vec{b}$

$\vec{u} = \lambda\vec{b}$

$= \lambda\left(3\hat{i} + \hat{k}\right)$

$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$ --- (ii)

Put value of $\vec{u}$ in equation (i),

$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(3\lambda\hat{i} + \lambda\hat{k}\right) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}$

$\vec{v}$ is perpendicular to $\vec{b}$

Then, $\vec{v}.\vec{b} = 0$

$$\left[(5-3\lambda)\hat{i} + (-2)\hat{j} + (5-\lambda)\hat{k}\right].\left(3\hat{i} + 0 \times \hat{j} + \hat{k}\right) = 0$$

$$(5-3\lambda)(3) + (-2)(0) + (5-\lambda)(1) = 0$$

$$15 - 9\lambda + 0 + 5 - \lambda = 0$$

$$20 - 10\lambda = 0$$

$$-10\lambda = -20$$

$$\lambda = \frac{-20}{-10}$$

$$\lambda = 2$$

Put λ in equation (ii)

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$$

$$= 3(2)\hat{i} + (2)\hat{k}$$

$$\vec{u} = 6\hat{i} + 2\hat{k}$$

Put the value of $\vec{u}$ in equation (i)

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \left(6\hat{i} + 2\hat{k}\right) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$$

$$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a} = \left(6\hat{i} + 2\hat{k}\right) + \left(-\hat{i} - 2\hat{j} + 3\hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q35

Vectors $\vec{a}$ and $\vec{b}$ have same magnitude, then
$$\left|\vec{a}\right| = \left|\vec{b}\right| = x \text{ (Say)}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then
$$\cos\theta = \frac{\vec{a}\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$\cos 30° = \frac{3}{x \cdot x}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{x^2}$$

$$\sqrt{3}x^2 = 6$$

$$x^2 = \frac{6}{\sqrt{3}}$$

Rationalizing the denominator,
$$x^2 = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$x^2 = \frac{6\sqrt{3}}{3}$$

$$x^2 = 2\sqrt{3}$$

$$x = \sqrt{2\sqrt{3}}$$

$$\left|\vec{a}\right| = \left|\vec{b}\right| = \sqrt{2\sqrt{3}}$$

Scalar or Dot Product Ex 24.1 Q36

Let $\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \vec{a} + \vec{b}$ --- (i)

Such that $\vec{a}$ is a vector parallel to vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$ and $\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$
$$\vec{a} = \lambda\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$$
$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k} \qquad --- (ii)$$

Put value of $\vec{a}$ in equation (i),
$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}\right) + \vec{b}$$
$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} - 2\lambda\hat{i} - 4\lambda\hat{j} + 2\lambda\hat{k}$$
$$\vec{b} = (2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right)$, then
$$\vec{b}.\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right) = 0$$
$$\left[(2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}\right]\left(2\hat{i} + 4\hat{j} - 2\hat{k}\right) = 0$$
$$(2 - 2\lambda)(2) + (-1 - 4\lambda)(4) + (3 + 2\lambda)(-2) = 0$$
$$4 - 4\lambda - 4 - 16\lambda - 6 - 4\lambda = 0$$
$$-6 - 24\lambda = 0$$
$$-24\lambda = 6$$

$$\lambda = -\frac{1}{4}$$

Put λ in equation (ii),

$$\vec{a} = 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}$$

$$= 2\left(-\frac{1}{4}\right)\hat{i} + 4\left(-\frac{1}{4}\right)\hat{j} - 2\left(-\frac{1}{4}\right)\hat{k}$$

$$\vec{a} = -\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}$$

Put the value of $\vec{a}$ in equation (i),

$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \vec{b}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} + \frac{1}{2}\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$$

$$= \frac{4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{2}$$

$$= \frac{5\hat{i} + 5\hat{k}}{2}$$

$$\vec{b} = \frac{5}{2}\left(\hat{i} + \hat{k}\right)$$

$$\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \frac{5}{2}\left(\hat{i} + \hat{k}\right)$$

Scalar or Dot Product Ex 24.1 Q37

Let $\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \vec{a} + \vec{b}$ --- (i)

Such that $\vec{a}$ is parallel to $\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{b}$ is perpendicular to $\left(\hat{i} + \hat{j} + \hat{k}\right)$.

Since, $\vec{a}$ is parallel to $\left(\hat{i} + \hat{j} + \hat{k}\right)$

$$\vec{a} = \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$$ --- (ii)

Put $\vec{a}$ in equation (i),

$$\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \left(\lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}\right) + \vec{b}$$

$$\vec{b} = 6\hat{i} - \lambda\hat{i} - 3\hat{j} - \lambda\hat{j} - 6\hat{k} - \lambda\hat{k}$$

$$\vec{b} = (6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}$$

$\vec{b}$ is a vector perpendicular to the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$, then

$$\vec{b}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$$\left[(6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}\right]\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$$(6 - \lambda)(1) + (-3 - \lambda)(1) + (-6 - \lambda)(1) = 0$$

$$6 - \lambda - 3 - \lambda - 6 - \lambda = 0$$

$$-3 - 3\lambda = 0$$

$$-3 = 3\lambda$$

$$\lambda = \frac{-3}{3}$$

$$\lambda = -1$$

Put value of λ in (ii),

$$\vec{a} = -1.\left(\hat{i} + \hat{j} + \hat{k}\right)$$
$$\vec{a} = -\hat{i} - \hat{j} - \hat{k}$$

Using $\vec{a}$ in equation (i),

$$\left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \vec{b}$$
$$\vec{b} = 6\hat{i} + \hat{i} - 3\hat{j} + \hat{j} - 6\hat{k} + \hat{k}$$
$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

Thus,

Vector $\vec{a} = -\hat{i} - \hat{j} - \hat{k}$ and
$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$
are required vectors.

Scalar or Dot Product Ex 24.1 Q38

Here, $\left(\vec{a} + \vec{b}\right)$ is orthogonal to $\left(\vec{a} - \vec{b}\right)$

Then, $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 0$

$$\left|\vec{a}\right|^2 = \left|\vec{b}\right|^2 = 0$$

$$\left\{\sqrt{(5)^2 + (-1)^2 + (7)^2}\right\}^2 - \left\{\sqrt{(1)^2 + (-1)^2 + (\lambda)^2}\right\}^2 = 0$$

$$(25 + 1 + 49) - \left(1 + 1 + \lambda^2\right) = 0$$

$$75 - \left(2 + \lambda^2\right) = 0$$

$$75 - 2 - \lambda^2 = 0$$

$$-\lambda^2 = -73$$

$$\lambda = \sqrt{73}$$

Scalar or Dot Product Ex 24.1 Q39

It is given that $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$.

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow \left|\vec{a}\right|^2 = 0 \Rightarrow \left|\vec{a}\right| = 0$$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector

Scalar or Dot Product Ex 24.1 Q40

Given that $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so,

$\vec{a}.\vec{c} = 0$ and $\vec{b}.\vec{c} = 0$

Now, $\vec{c}.\left(\vec{a}+\vec{b}\right)$

$= \vec{c}.\vec{a}+\vec{c}.\vec{b}$

$= 0+0$

$= 0$

$\therefore \vec{c}$ is perpendicular to $\left(\vec{a}+\vec{b}\right)$

$\vec{c}.\left(\vec{a}-\vec{b}\right)$

$= \vec{c}.\vec{a}-\vec{c}.\vec{b}$

$= 0-0$

$= 0$

$\therefore \vec{c}$ is perpendicular to $\left(\vec{a}-\vec{b}\right)$

Scalar or Dot Product Ex 24.1 Q41

Here $\left|\vec{a}\right| = a$, $\left|\vec{b}\right| = b$

LHS $= \left(\dfrac{\vec{a}}{a^2} - \dfrac{\vec{b}}{h^2}\right)^2$

$= \left(\dfrac{\vec{a}}{a^2}\right)^2 + \left(\dfrac{\vec{b}}{b^2}\right)^2 - 2\dfrac{\vec{a}}{a^2}.\dfrac{\vec{b}}{b^2}$

$= \dfrac{\left|\vec{a}\right|^2}{a^4} + \dfrac{\left|\vec{b}\right|^2}{b^4} - \dfrac{2\vec{a}\vec{b}}{a^2 b^2}$

$= \dfrac{a^2}{a^4} + \dfrac{b^2}{b^4} - \dfrac{2\vec{a}\vec{b}}{a^2 b^2}$ $\qquad \left[\text{Since } \left|\vec{a}\right| = a, \left|\vec{b}\right| = b\right]$

$= \dfrac{1}{a^2} + \dfrac{1}{b^2} - \dfrac{2\vec{a}\vec{b}}{a^2 b^2}$

$= \dfrac{b^2 + a^2 - 2\vec{a}\vec{b}}{a^2 b^2}$

$= \dfrac{\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\vec{b}}{a^2 b^2}$

$= \dfrac{\left(\vec{a}-\vec{b}\right)^2}{a^2 b^2}$

$= \left(\dfrac{\vec{a}-\vec{b}}{ab}\right)^2$

$= $ RHS

Hence proved

$\therefore \qquad \left(\dfrac{\vec{a}}{a^2} - \dfrac{\vec{b}}{b^2}\right)^2 = \left(\dfrac{\vec{a}-\vec{b}}{ab}\right)^2$

Scalar or Dot Product Ex 24.1 Q42

Given that

$\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that
$\vec{d}.\vec{a} = \vec{d}.\vec{b} = \vec{d}.\vec{c} = 0$

Given that

$\vec{d}.\vec{a} = 0$

$\Rightarrow$ $\vec{d}$ perpendicular to $\vec{a}$

or $\vec{d} = 0$ $---(i)$

$\vec{d}.\vec{b} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{b}$ or $\vec{d} = 0$ $---(ii)$

$\vec{d}.\vec{c} = 0$

$\Rightarrow$ $\vec{d}$ is perpendicular to $\vec{c}$ or $\vec{d} = 0$ $---(iii)$

From (i), (ii), (iii), we get
$\vec{d}$ is perpendicular to $\vec{a}, \vec{b}, \vec{c}$ or $\vec{d} = 0$, but $\vec{d}$ can not be perpendicular to $\vec{a}, \vec{b}$
and $\vec{c}$ because $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, so

$\vec{d} = 0$

Scalar or Dot Product Ex 24.1 Q43

Given that

$\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$

It means,

$\vec{a}.\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$ $---(i)$

Let $\vec{r}$ be some vector in the plane of $\vec{b}$ and $\vec{c}$
Then, $\vec{r}, \vec{b}, \vec{c}$ are coplanar

We know that,
Three vectors are coplanar if one of them is expressible as linear combination
of other two vectors.

Let $\vec{r} = x\vec{b} + y\vec{c}$
               where x and y are same scalar

      $\vec{r}.\vec{a} = \left(x\vec{b} + y\vec{c} \right).\vec{a}$ $\left[$ Taking dot product with $\vec{a}$ on both the side $\right]$
      $\vec{r}.\vec{a} = x\vec{b}.\vec{a} + y\vec{c}.\vec{a}$
          $= x.0 + y.0$ $\left[$ Using (i) $\right]$
      $\vec{r}.\vec{a} = 0 + 0$
      $\vec{r}.\vec{a} = 0$

So, $\vec{r}$ is perpendicular to $\vec{a}$

Thus,

$\vec{a}$ is perpendicular to every vector in the plane of $\vec{b}$ and $\vec{c}$

Scalar or Dot Product Ex 24.1 Q44

We have,

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\vec{b} + \vec{c} = -\vec{a}$$

Squaring both the sides.

$$\left(\vec{b} + \vec{c}\right)^2 = \left(-\vec{a}\right)^2$$

$$\left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{b}.\vec{c} = \left|\vec{a}\right|^2$$

$$2\vec{b}.\vec{c} = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2$$

$$2\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2 \qquad \left[\text{Since } \vec{b}\vec{c} = \left|\vec{b}\right|\left|\vec{c}\right|\cos\theta\right]$$

$$\cos\theta = \frac{\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - \left|\vec{c}\right|^2}{2\left|\vec{b}\right|\left|\vec{c}\right|}$$

Scalar or Dot Product Ex 24.1 Q45

Here, $\vec{u} + \vec{v} + \vec{w} = 0$

Squaring both the sides,

$$\left(\vec{u} + \vec{v} + \vec{w}\right)^2 = \left(0\right)^2$$

$$\left|\vec{u}\right|^2 + \left|\vec{v}\right|^2 + \left|\vec{w}\right|^2 + 2\vec{u}\vec{v} + 2\vec{v}\vec{w} + 2\vec{w}\vec{u} = 0$$

$$\left(3\right)^2 + \left(4\right)^2 + \left(5\right)^2 + 2\left(\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}\right) = 0$$

$$9 + 16 + 25 + 2\left(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}\right) = 0$$

$$2\left(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}\right) = -50$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = \frac{-50}{2}$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = -25$$

Scalar or Dot Product Ex 24.1 Q46

Given

$$\vec{a} = x^2\hat{i} + 2\hat{j} - 2\hat{k}$$
$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$
$$\vec{c} = x^2\hat{i} + 5\hat{j} - 4k$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{\left(x^2\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right)}{\sqrt{\left(x^2\right)^2 + (2)^2 + (-2)^2}\sqrt{(1)^2 + (-1)^2 + (1)^2}}$$

$$= \frac{\left(x^2\right)(1) + (2)(-1) + (-2)(1)}{\sqrt{x^4 + 4 + 4}\sqrt{1 + 1 + 1}}$$

$$= \frac{x^2 - 2 - 2}{\sqrt{8 + x^2}\sqrt{3}}$$

$$\cos\theta = \frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}}$$

Since θ is an acute angle, so

$$\cos\theta > 0$$

$$\frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}} > 0$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$\Rightarrow$ $x < -2$ or $x > 2$ $---$ (i)

Again, let ϕ be the angle between $\vec{b}$ and $\vec{c}$,

$$\cos\phi = \frac{\vec{b}\vec{c}}{|\vec{b}||\vec{c}|}$$

$$= \frac{\left(\hat{i} - \hat{j} + \hat{k}\right)\left(x^2\hat{i} + 5\hat{j} - 4k\right)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{\left(x^2\right)^2 + (5)^2 + (-4)^2}}$$

$$\cos\phi = \frac{(1)\left(x^2\right) + (-1)(5) + (1)(-4)}{\sqrt{3}\sqrt{x^4 + 25 + 16}}$$

$$\cos\phi = \frac{x^2 - 5 - 4}{\sqrt{3}\ \sqrt{x^2 + 41}}$$

$$\cos\phi = \frac{x^2 - 9}{\sqrt{3}\ \sqrt{x^2 + 41}}$$

Since ϕ is an obtuse angle, so

$$\cos\phi < 0$$

$$\frac{x^2 - 9}{\sqrt{3}\ \sqrt{x^2 + 41}} < 0$$

$$x^2 - 9 < 0$$

$$x^2 < 9$$

$\Rightarrow$ $\qquad$ $x > -3$ and $x < 3$ $\qquad\qquad\qquad\qquad\qquad$ – – – (ii)

From

$\qquad$ $-3 < x < -2$ and $2 < x < 3$

$\qquad$ $x \in (-3, -2) \cup (2, 3)$

Scalar or Dot Product Ex 24.1 Q47

Here, $\vec{a}$ and $\vec{b}$ are mutually perpenducular, then

$\vec{a}.\vec{b} = 0$

$\left(3\hat{i} + x\hat{j} - \hat{k}\right)\left(2\hat{i} + \hat{j} + y\hat{k}\right) = 0$

$(3)(2) + (x)(1) + (-1)(y) = 0$

$6 + x - y = 0$

$x - y = -6$ --- (i)

Also, $\vec{a}$ and $\vec{b}$ have equal magnitude,

$|\vec{a}| = |\vec{b}|$

$\sqrt{(3)^2 + (x)^2 + (-1)^2} = \sqrt{(2)^2 + (1)^2 + (y)^2}$

$9 + x^2 + 1 = 4 + 1 + y^2$

$x^2 + 10 = 5y^2$

$x^2 - y^2 = 5 - 10$

$x^2 - y^2 = -5$

$(x + y)(x - y) = -5$

$(x + y)(-6) = -5$ $\left[\text{Using (i)}\right]$

$-6x - 6y = -5$

$-(6x + 6y) = -5$

$6x + 6y = 5$ --- (ii)

Solving (i) and (ii),

$6x + 6y = 5$

$\underline{6x - 6y = -36}$ $\left[(i) \times 6\right]$

$12x \quad\quad = -31$

$x = \dfrac{-31}{12}$

Put value of x in equation (i),

$x - y = -6$

$\dfrac{-31}{12} - y = -6$

$-y = \dfrac{-6}{1} + \dfrac{31}{12}$

$-y = \dfrac{-72 + 31}{12}$

$y = \dfrac{41}{12}$

Scalar or Dot Product Ex 24.1 Q48

Given

$\vec{a}$ and $\vec{b}$ are unit vectors

Then, $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a} + \vec{b}| = \sqrt{3}$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = \left(\sqrt{3}\right)^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 3$$

$$1 + 1 + 2\vec{a}.\vec{b} = 3$$

$$2 + 2\vec{a}.\vec{b} = 3$$

$$2\vec{a}.\vec{b} = 3 - 2$$

$$2\vec{a}.\vec{b} = 1$$

$$\vec{a}.\vec{b} = \frac{1}{2}$$

Now, $\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right)$

$$= 2\vec{a}.3\vec{a} + 2\vec{a}.\vec{b} - 5\vec{b}.3\vec{a} - 5\vec{b}.\vec{b}$$

$$= 6\left(\vec{a}\right)^2 + 2\vec{a}.\vec{b} - 15\vec{a}.\vec{b} - 5\left(\vec{b}\right)^2$$

$$= 6|\vec{a}|^2 - 13\vec{a}.\vec{b} - 5|\vec{b}|^2$$

$$= 6\left(1\right)^2 - 13\left(\frac{1}{2}\right) - 5\left(1\right)^2$$

$$= \frac{6}{1} - \frac{13}{2} - \frac{5}{1}$$

$$= \frac{12 - 13 - 10}{2}$$

$$= \frac{12 - 23}{2}$$

$$= -\frac{11}{2}$$

$$\left(2\vec{a} - 5\vec{b}\right).\left(3\vec{a} + \vec{b}\right) = -\frac{11}{2}$$

Scalar or Dot Product Ex 24.1 Q49

$$|\vec{a} + \vec{b}|^2 = |\vec{b}|^2$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot (\vec{a} + 2\vec{b}) = 0$$

$$\therefore \vec{a} + 2\vec{b} \text{ is perpendicular to } \vec{a}.$$

Let $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

Let $|\vec{a}| = |\vec{b}|$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$\left(\vec{a}\right)^2 - \left(\vec{b}\right)^2 = 0$$

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$$

Thus,

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$