Very Short Answer Type Questions

[1 marks]

Que 1. What is the HCF of the smallest composite number and the smallest prime number?

Sol. Smallest composite number = 4

Smallest prime number = 2

So, HCF (4,2) = 2

Que 2. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Sol. $\frac{6}{1250} = \frac{3}{625} = \frac{3}{5^4} \times \frac{2^4}{2^4} = \frac{48}{(5 \times 2)^4} = \frac{48}{10^4} = 0.0048$

This representation will terminate after 4 decimal places.

Que 3. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?

Sol. Product of two numbers = Product of their LCM and HCF

$\Rightarrow$ 1800 = 12 X LCM

$\Rightarrow$ LCM = $\frac{1800}{12}$ = 150.

Que 4. What is the HCF of 3^3 x 5 and $3^2 \times 5^2$?

Sol. HCF of 3^3 x 5 and 3^2 x 5^2 - 3^2 x 5 = 45

Que 5. If a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of 3a and 2b?

Sol. 6p

Que 6. If P is prime number then, what is the LCM of P, P^2, P^3?

Sol. P^3

Que 7. Two positive integers p and q can be expressed as p = ab^2 and q = a^2b, a and b are prime numbers. What is the LCM of p and q?

Sol. a^2b^2

Que 8. A number N when divided by 14 gives the remainder 5. What is the remainder when same number is divided by 7?

Sol. 5, because 14 is multiple of 7.

Therefore, remainder in both cases are same.

Que 9. Examine whether $\frac{17}{30}$ is a terminating decimal or not.

Sol. $\frac{17}{30} = \frac{17}{2 \times 3 \times 5}$, since the denominator has 3 as its factor.

$\frac{17}{30}$ is a non-terminating decimal.

Que 10. What are the possible values of remainder r, when a positive integer a is divided by 3?

Sol. According to Euclid's division lemma a 3q+ r, where $0 \leq r < 3$ and r is an integer.

Therefore, the values of r can be 0, 1 or 2.

Que 11. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form $\frac{p}{q}$? Give reason.

Sol. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^m$, where m,n are natural numbers.

Que 12. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating repeating decimal expansion. Give reason for your answer.

Sol. $\frac{987}{10500} = \frac{47}{500}$ and $500 = 2^2 \times 5^3$, so it has terminating decimal expansion.

Value Based Questions

Que 1. In order to celebrate Van Mahotsav, the students of a school planned to plant two types of trees in the nearby park. They decided to plant 144 trees of type A and 84 trees of type B. If the two types of plants are to be in the same number of columns, find the maximum number of columns in which they can be planted.
What values do these students possess?

Sol. Maximum number of columns in which the two types of plants can be planted = HCF of 144 and 84 Since 144 > 84
So, by division lemma, $144 = 84 \times 1 + 60$
Again, applying division lemma (since remainder $\neq 0$), we get

$$84 = 60 \times 1 + 24$$

Continuing the same way,

$$60 = 24 \times 2 + 12 \quad \text{and} \quad 24 = 12 \times 2 + 0$$

$\because$ Remainder at this stage = 0
$\therefore$ HCF (144, 84) = 12
Environmental protection, sincerity, social work, cooperation.

Que 2. In a seminar on the topic 'Liberty and Equality', the number of participants in Hindi, Social Science and English are 60, 84 and 108 respectively.
(i) Find the minimum number of rooms required if in each room the same number of participants
are to be seated and all of them being from the same subject.
(ii) Which mathematical concept has been used in this problem?
(iii) Which values are discussed in the above problem?

Sol. (i) The number of rooms will be minimum if each room accommodates maximum number of participants. Since in each room the same number of participants are to be seated and all of them must be on the same subject, therefore, the number of participants in each room must be the HCF of 60, 84, 108.
The prime factorisation of 60, 84 and 108 are as under.

$$60 = 2^2 \times 3 \times 5 \quad \text{and} \quad 84 = 2^2 \times 3 \times 7$$
$$108 = 2^2 \times 3^3$$
$$\text{HCF} = 2^2 \times 3 = 12$$

$\therefore$ Number of rooms required = $\frac{Total\ Number\ of\ Participants}{12}$

$$= \frac{60+84+108}{12} = \frac{252}{12} = 21$$

(ii) Mathematical concept used in the above problem is highest common factor, i.e., H.C.F.
(iii) Liberty and equality are the pay marks of democracy.

Que 3. Three sets of English, Hindi and Sociology books dealing with cleanliness have to be stacked in such a way that all the boobs are stored topic wise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Sociology books is 336.

(i) Assuming that the books are of the same thickness, determine the number of stacks of each subject.
(ii) Which mathematical concept is used in this problem?
(iii) Which habits are discussed in this problem?

Sol. (i) In order to arrange the books as required we have to find the largest number that divides 96, 240 and 336 exactly. Clearly, such a number is their HCF.

$$96 = 2^5 \times 3$$
$$240 = 2^4 \times 3 \times 5$$
$$336 = 2^4 \times 3 \times 7$$

∴ HCF of 96, 240 and 336 is $2^4 \times 3 = 48$

So, there must be 48 books in each stack.

∴ Number of stacks of English books = $\frac{96}{48}$ = 2

Number of stacks of Hindi books = $\frac{240}{48}$ = 5

Number of stacks of sociology books = $\frac{336}{48}$ = 7

(ii) HCF of number.
(iii) Cleanliness and orderliness have been discussed in this question. Cleanliness leads to good health and orderliness makes a person better organised in life.

Que 4. There is a circular path around a sports field. Priya takes 12 minutes to drive one round of the field. While Ravish takes 10 minutes for the same. Suppose they both start from the same point and at the same time and go in the same direction.
(i) After how many minutes will they meet again at the starting point?
(ii) Which mathematical concept is used in this problem?
(iii) What is the value discussed in this problem?

Sol. (i) Required number of minutes is
LCM of (12, 10)
$12 = 2 \times 2 \times 3$
$10 = 2 \times 5$
LCM = $2 \times 2 \times 3 \times 5 = 60$
Hence, they meet at starting point after 60 minutes.
(ii) LCM of number.
(iii) Health competition is necessary for person development and progress.

HOTS (Higher Order Thinking Skills)

Que 1. Show that there is no positive integer n for which $\sqrt{\mathbf{n-1}}+\sqrt{\mathbf{n+1}}$ is rational.

Sol. Let there be a positive integer n for which $\sqrt{\mathrm{n}-1}+\sqrt{\mathrm{n}+1}$ be rational number.

$\sqrt{n-1}+\sqrt{n+1}=\frac{p}{q}$; Where p, q are integer and q ≠ 0 ……(i)

$\Rightarrow \frac{1}{\sqrt{n-1}+\sqrt{n+1}}=\frac{q}{p}$ $\Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})\times(\sqrt{n-1}-\sqrt{n+1})}\frac{q}{p}$

$\Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}=\frac{q}{p}$ $\Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{n-1-n-1}=\frac{q}{p}$

$\Rightarrow \frac{\sqrt{n+1}-\sqrt{n-1}}{2}=\frac{q}{p}$ $\Rightarrow \sqrt{n+1}-\sqrt{n-1}=\frac{2q}{p}$ …..(ii)

Adding (i) and (ii), we get

$$\sqrt{n-1}+\sqrt{n+1}+\sqrt{n+1}-\sqrt{n-1}=\frac{p}{q}+\frac{2q}{p}$$

$\Rightarrow 2\sqrt{n+1}=\frac{p^2+2q^2}{pq}$ $\Rightarrow \sqrt{n+1}=\frac{p^2+2q^2}{2pq}$

⇒ $\sqrt{n+1}$ is rational number as $\frac{p^2+2q^2}{2pq}$ is rational.

⇒ n + 1 is perfect square of positive integer …(A)

Again subtracting (ii) from (i), we get

$\sqrt{n-1}+\sqrt{n+1}-\sqrt{n+1}+\sqrt{n-1}=\frac{p}{q}-\frac{2q}{p}$ ⇒ $2\sqrt{n-1}=\frac{p^2-2q^2}{pq}$

⇒ $\sqrt{n-1}$ is rational number as $\frac{p^2-2q^2}{2pq}$ is rational.

⇒ $\sqrt{n-1}$ is also perfect square of positive integer. ….(B)

From (A) and (B)

$\sqrt{n+1}\ and\ \sqrt{n-1}$ are perfect squares of positive integer. It contradict the fact that two perfect differ at least by 3.

Hence, there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

Que 2. Let a, b, c, k be rational numbers such that k is not a perfect cube. If a + bk$^{1/3}$ + ck$^{2/3}$ = 0, then prove that a = b = c = 0.

Sol. Given, $a+bk^{1/3}+ck^{2/3}=0$ …(i)

Multiplying both sides by k$^{1/3}$, we have

$ak^{1/3}+bk^{2/3}+ck=0$

Multiplying (i) by b and (ii) by c and then subtracting, we have

$$(ab+b^2k^{1/3}+bck^{2/3})-(ack^{1/3}+bck^{2/3}+c^2k)=0$$

$\Rightarrow \quad (b^2 - ac)k^{1/3} + ab - c^2k = 0$
$\Rightarrow \quad b^2 - ac = 0 \quad and \quad ab - c^2 = 0$ [Since $k^{1/3}$ is irrational]
$\Rightarrow \quad b^2 = ac \quad and \quad ab = c^2k$
$\Rightarrow \quad b^2 = ac \quad and \quad a^2b^2 = c^4k^2$
$\Rightarrow \quad a^2(ac) = c^4k^2$ [By putting b^2 = ac in $a^2 b^2 = c^4 k^2$]
$\Rightarrow \quad a^3c - k^2c^4 = 0 \quad \Rightarrow \quad (a^3 - k^2c^3)\, c = 0$
$\Rightarrow \quad a^3 - k^2c^3 = 0, or \;\; c = 0$

Now, $a^3 - k^2c^3 = 0 \quad \Rightarrow \quad k^2 = \frac{a^3}{c^3}$

$$\Rightarrow \quad (k^2)^{1/3} = \left(\frac{a^3}{c^3}\right)^{1/3} \quad \Rightarrow \quad k^{2/3} = \frac{a}{c}$$

This is impossible as $k^{2/3}$ is irrational and $\frac{a}{c}$ is rational.

$\therefore \quad a^3 - k^2c^3 \neq 0$
Hence, c = 0
Subtracting c = 0 in b^2 – ac = 0, we get b = 0
Substituting b = 0 and c = 0 in a + $bk^{1/3}$ + $ck^{2/3}$ = 0, we get a = 0
Hence, a = b = c = 0.

Que 3. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Sol. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required number. In other words, required number is a factor of 391.
Similarly required positive integer is a factor of 436 – 11 = 425 and 542 – 15 = 527.
Clearly, required number is the HCF of 391, 425 and 527.
Using the factor tree we get the prime factorisations of 391, 425 and 425 and 527 as follows: 391 = 17 × 23, 425 = 5^2 × 17 and 527 = 17 × 31
∴ HCF of 391, 425 and 527 is 17.
Hence, required number = 17

Long Answer Type Questions

[4 marks]

Que 1. Use Euclid's division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol. Let a be an arbitrary positive integer.

Then by Euclid's Division algorithm, corresponding to the positive integers a and 3 there exist non-negative integers q and r such that

$a = 3q + r$ where $0 < r < 3$

$\Rightarrow$ $a^2 = 9q^2 + 6qr + r^2$ …. (i) $0 \leq r < 3$

Case-**I:** When r = 0 putting on (i)

$a^2 = 9q^2 = 3(3q^2) = 3m$ where $m = 3q^2$

Case-**II:** r = 1

$A^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$ where $m = 3q^2 + 2q$

Case-**III:** r = 2

A2 = 9q2 + 12q + 4 = 3(3q2 + 4q + 1) + 1 = 3m + 1 where m = (3q2 + 4q + 1)

Hence, square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Que 2. Show that one and only one out of n, n+2, n+4 is divisible by 3, where n is any positive integer.

Sol. Let q be the quotient and r be the remainder when n is divided by 3.

Therefore, $n = 3q + r$ where r = 0, 1,2

$\Rightarrow$ $n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$

Case (i) if n = 3q, then n is divisible by 3, n + 2 and n + 4 are not divisible by 3.

Case (ii) if n = 3q + 1 then n + 2 = 3q + 3(q + 1), which is not divisible by 3 and n + 4 = 3q + 5, which is not divisible by 3.

So, only (n + 4) is divisible by 3.

Hence one and only one out of n, (n+2), (n+4) is divisible by 3.

Que 3. Use Euclid's division algorithm to find the HCF of:

(i) **960 and 432**
(ii) **4052 and 12576**

Sol. (i) since 960 > 432, we apply the division Lemma to 960 and 432.

We have, $960 = 432 \times 2 + 96$

Since the remainder $96 \neq 0$, so we apply the division lemma to 432 and 96.

We have, $960 = 432 \times 4 + 48$

Again remainder $48 \neq 0$, so we again apply division Lemma to 96 and 48.

We have, $96 = 48 \times 2 + 0$

The remainder has now become zero. So our procedure stops.

Since the divisor at this stage is 48.

Hence, HCF of 960 and 432 is 48

i.e., HCF (960, 432) = 48

(ii) Since 12576 > 4052, we apply the division lemma to 12576 and 4052 to get

$$12576 = 4052 \times 3 + 420$$

Since the remainder $420 \neq 0$, we apply the division lemma to 4052 and 420, to get

$$4052 = 420 \times 9 + 272$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$420 = 272 \times 1 + 148$$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$$148 = 124 \times 1 + 24$$

We consider the new divisor 124 and the new remainder 4, and apply the division lemma to get

$$24 = 4 \times 6 + 0$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 4, the HCF of 12576 and 4052 is 4.

Que 4. Using prime factorization method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.

Sol. Given numbers = 30, 72, 432

$$30 = 2 \times 3 \times 5; \qquad 72 = 2^3 \times 3^2 \quad \text{and} \quad 432 = 2^4 \times 3^3$$

Here, 21 and 31 are the smallest powers of the common factors 2 and 3 respectively.

So, HCF (30, 72, 432) = $2^1 \times 3^1 = 2 \times 3 = 6$

Again, 2^4, 3^3 and 5^1 are the greatest powers of the prime factors 2,3 and 5 respectively.

So, LCM (30, 72, 432) = $2^4 \times 3^3 \times 5^1 = 2160$

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120

Therefore, HCF × LCM ≠ Product of the numbers.

Que 5. Prove that $\sqrt{7}$ is an irrational number.

Sol. Let us assume, to the contrary, that $\sqrt{7}$ is a rational number.

Then, there exist co-prome positive integers a and b such that

$$\sqrt{7} = \frac{a}{b}, \qquad b \neq 0$$

So, $\quad a = \sqrt{7}b$

Squaring both sides, we have

$$a^2 = 7b^2$$

⇒ $\quad$ 7 divides a^2 $\quad$ ⇒ 7 divides a

So, we can write

$$a = 7c, \qquad \text{(where c is any integer)}$$

Putting the value of a = 7c in (i), we have

$$49c^2 = 7b^2 \quad \Rightarrow \quad 7c^2 = b^2$$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that $\sqrt{7}$ is rational is wrong.

Hence, we conclude that $\sqrt{7}$ is an irrational number.

Que 6. Show that 5-$\sqrt{3}$ is an irrational number.

Sol. Let us assume that 5-$\sqrt{3}$ is rational.

So, 5-$\sqrt{3}$ may be written as

5-$\sqrt{3}$ = $\frac{p}{q}$, where p and q are integers, having no common factor except 1 and q ≠ 0.

$$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \qquad \Rightarrow \qquad \sqrt{3} = \frac{5q - p}{q}$$

Since $\frac{5q-p}{q}$ is a rational number which is a contradiction.

∴ $\sqrt{3}$ is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, 5 - $\sqrt{3}$ is an irrational number.

Que 7. Using Euclid's division algorithm, find whether the pair of numbers 847, 2160 are co-prime or not.

Sol. Since 2160 > 847 we apply the division lemma to 2160 and 847

We have, $2160 = 847 \times 2 + 466$

Since remainder 466 ≠ 0. So we apply the division lemma to 847 and 466

$$847 = 466 \times 1 + 381$$

Again remainder 381 ≠ 0. So we again apply the division lemma to 466 and 381.

$$466 = 381 \times 1 + 85$$

Again remainder 85 ≠ 0. So we again apply the division lemma to 381 and 85.

$$381 = 85 \times 4 + 41$$

Again remainder 41 ≠ 0. So we again apply the division lemma to 85 and 41.

$$85 = 41 \times 2 + 3$$

Again remainder $3 \neq 0$. So we again apply the division lemma to 41 and 3.

$$41 = 3 \times 13 + 2$$

Again remainder $2 \neq 0$. So we again apply the division lemma to 3 and 2.

$$3 = 2 \times 1 + 1$$

Again remainder $1 \neq 0$. So we again apply the division lemma to 2 and 1.

$$2 = 1 \times 4 + 0$$

The remainder now becomes 0. So, our procedure stops.

Since the divisor at this stage is 1.

Hence, HCF of 847 and 2160 is 1 and numbers are co-prime.

Que 8. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. But $6^n = (2 \times 3)^n = 2^n \times 3^n$ So the primes in factorisation of 6^n are2 and 3. So the uniqueness of the fundamental theorem of arithmetic guarantees that there are no other primes except 2 and 3 in the factrosation of 6^n. So there is no natural number n for which 6^n ends with digit zero.

Short Answer Type Questions – II

[3 marks]

Que 1. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since 616 > 32, we apply division lemma to 616 and 32.

We have, 616 32 x 19 + 8

Here, remainder 8 ≠ 0 So, we again apply division lemma to 32 and 8.

We have, 32 = 8 x 4 + 0

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns is 8.

Que 2. Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.

Sol. The prime factors of 12, 15 and 21 are

$12 = 2^2 \times 3$, $15 = 3 \times 5$ and $21 = 3 \times 7$

Therefore, the HCF of these integers is 3.

$2^2, 3^1, 5^1$ and 7^1 are the greatest powers involved in the prime factors of 12, 15 and 21.

So, LCM (12, 15, 21) = $2^2 \times 3^1 \times 5^1 \times 7^1 = 420$.

Que 3. Find the LCM and HCF of the following pairs of integers and verify that LCM x HCF = product of the two numbers.

(i) 26 and 91 **(ii) 198 and 144**

Sol.
(i) We have, 26 = 2 x 13 and 91 = 7 x 13

Thus, LCM (26, 91) = 2 x 7 x 13 = 182

HCF (26, 91) = 13
Now, LCM x HCF = Product of two numbers = 26 x 91 = 2366
Hence, LCM x HCF = Product of two numbers.

(ii) $144 = 2^4 \times 3^2$ and $198 = 2 \times 3^2 \times 11$

$\therefore$ LCM (198, 144) = $2^4 \times 3^2 \times 11 = 1584$

HCF (198, 144) = $2 \times 3^2 = 18$

Now, LCM (198,144) x HCF (198, 144) = 1584 x 18 = 28512

And product of 198 and 144 = 28512

Thus, product of LCM (198, 144)

And HCF (198, 144) = Product of 198 and 144.

Que 4. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. To find the time after which they meet again at the starting point,

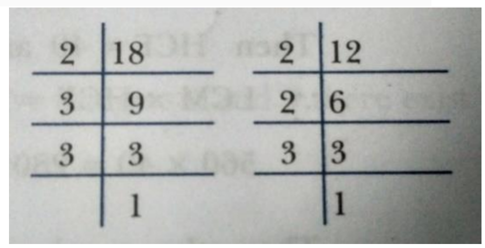
we have to find LCM of 18 and 12 minutes. We have

$18 = 2 \times 3^2$

And $12 = 2^2 \times 3$

Therefore, LCM of 18 and 12 - $2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.



Que 5. Write down the decimal expansions of the following numbers:

(i) $\frac{35}{50}$ **(ii)** $\frac{15}{1600}$

(i) we have, $\frac{35}{50} = \frac{35}{5^2 \times 2} = \frac{35 \times 2}{52 \times 2 \times 2} = \frac{70}{5^2 \times 2^2}$

$= \frac{70}{10^2} = \frac{70}{100} = 0.70$

(ii) We have, $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^4 \times 2^2 \times 5^2 \times 5^4} = \frac{15 \times 625}{2^6 \times 5^6}$

$= \frac{9375}{10^6} = \frac{9375}{1000000} = 0.009375$

Que 6. Express the number 0.3178 in the form of rational number $\frac{a}{b}$.

Sol. Let x $0.3\overline{178}$

Then x = 0.3178178178 …(i)

10x = 3.178178178 …(ii)

10000x = 3178.178178 …(iii)

On subtracting (ii) from (iii)

$$9990x = 3175 \Rightarrow x = \frac{3175}{9990} = \frac{635}{1998}$$

$\therefore 0.3\overline{178} = \frac{635}{1998}$

Que 7. If n is an odd positive integer, show that (n^2 - 1) divisible by 8.

Sol. We know that an odd positive integer n is of the form (4q+1) or (4q+3) for some integer q.

Case-I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q = 8q\,(2q + 1)$

Which is clearly divisible by 8.

Case-II When n = (4q +3)

In this case, we have

$n2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

Which is clearly divisible by 8.

Hence (n^2 – 1) is divisible by 8.

Que 8. The LCM of two numbers is 14 times their HCF. The sum of LMC and HCF is 600. If one number is 280, then find the other number.

Sol. Let HCF of the numbers br x then according to question LCM of the number will be 14x

And $x + 14x = 600 \Rightarrow x = 40$

Then HCF = 40 and LCM = 14 × 40 = 560

∵ LCM × HCF = Product of the numbers

560 × 40 = 280 × Second number ⇒ Second numbers = $\frac{560 \times 40}{280} = 80$

Then other number is 80.

Que 9. Find the value of x, y and z, in the given factor tree. Can the value of 'x' be found without finding the value of 'y' and 'z'? If yes, explain.

Sol. Yes, value of x can be found without finding value of y or z as x = 2 × 2 × 2 × 17 which are prime factors of x.

Que 10. Show that any positive odd integer is of the form 6q +1, or 6q + 3, or 6q +5, where q is some integer.

Sol. Let a be any positive odd integer and b = 6. Then, by Euclid's algorithm, a = 6q + r, for some
integer q ≥ 0 and 0 ≤ r < 6.

i.e, the possible remainders are 0, 1, 2, 3, 4, 5.

Thus, a can be of the form 6q, or 6q + 1, or 6q + 2, or 6q + 4 (since they are even).

Thus, a is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Hence, any odd positive integer is of the form 6q+ 1 or 6q + 3 or 6q + 5, where q is some integer.

Que 11. The decimal expansions of some real numbers are given below. In each case, decide whether they are rational or not. If they are rational, write it in the form $\frac{p}{q}$. What can you say about the prime factors of q?

(i) 0.140140014000140000… **(ii) $0.\overline{16}$**

Sol. (i) we have, 0.140140014000140000… a non-terminating and non-repeating decimal expansion. So it is irrational. It cannot be written in the form of $\frac{p}{q}$.

(ii) We have, $0.\overline{16}$ a non-terminating but repeating decimal expansion. So, it is rational.

Let x = $0.\overline{16}$

Then x = 0.1616… …(i)

⇒ 100x = 16.1616… …(ii)

On subtracting (i) from (ii), we get

100x – x = 16.1616 – 0.1616

⇒ 99x = 16 ⇒ x = $\frac{16}{99} = \frac{p}{q}$

The denominator (q) has factors other than 2 or 5.

Short Answer Type Questions – I

[2 marks]

Que 1. Can the number 4", n being a natural number, end with the digit 0? Give reason.

Sol. If 4^n ends with 0, then it must have 5 as a factor. But, $(4)^n = (2^2)^n$ i.e., the only prime factor of 4^n is 2. Also, we know from the fundamental theorem of arithmetic that the prime factorization of each number is unique.

$\therefore$ 4^n can never end with 0.

Que 2. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Sol. No, because any positive integer can be written as 3q, 3q + 1, 3q + 2, therefore, square will $9q^2 = 3m$, $9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$,

$9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3m + 1$.

Que 3. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.

Sol. No, because here HCF (18) does not divide LCM (380).

Que 4. Write a rational number between $\sqrt{3}$ and $\sqrt{5}$.

Sol. A rational number between $\sqrt{3}$ and $\sqrt{5}$ is $\sqrt{324} = 1.8 = \frac{18}{10} = \frac{9}{5}$

Que 5. The product of two consecutive integers is divisible by 2. Is this statement true or false? Give reason.

Sol. True, because n (n + l) will always be even, as one out of the n or (n+ l) must be even.

Que 6. Explain why 3 x 5 x 7 + 7 is a composite number.

Sol. $3 \times 5 \times 7 + 7 = 7(3 \times 5 + 1) = 7 \times 16$, which has more than two factors.

Que 7. What is the least number that is divisible by all the numbers from 1 to 10?

Sol. Required number = LCM of 1, 2, 3,… 10 = 2520

Que 8. Find the sum $\mathbf{0.\overline{68} + 0.\overline{73}}$.

Sol. $0.\overline{68} + 0.\overline{73} = \frac{68}{99} + \frac{73}{99} = \frac{141}{99} = 1.\overline{42}$

Que 9. "the product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Sol. True, because n(n + 1)(n + 2) will always be divisible by 6, as at least one of the factors will be divisible by 2 and at least one of the factors will be divisible by 3.