Very Short Answer Type Questions

[1 marks]

The graphs of y = p(x) for some polynomials (for questions 1 to 4) are given below. Find the number of zeros in each case.

Que 1.

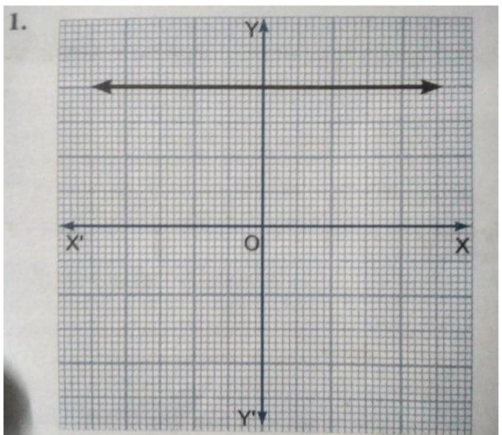

Fig. 2.3

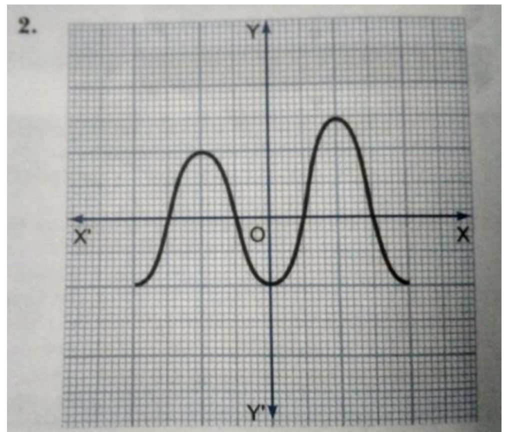

Fig. 2.4

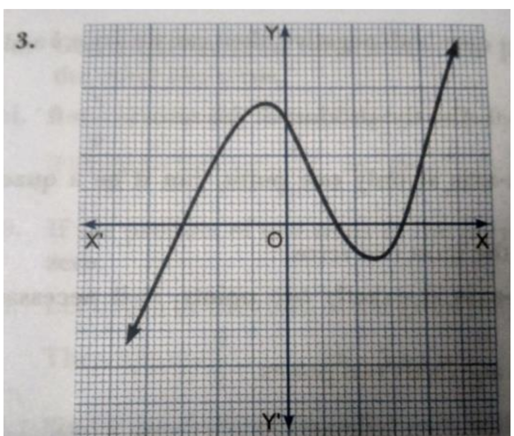

Fig. 2.5

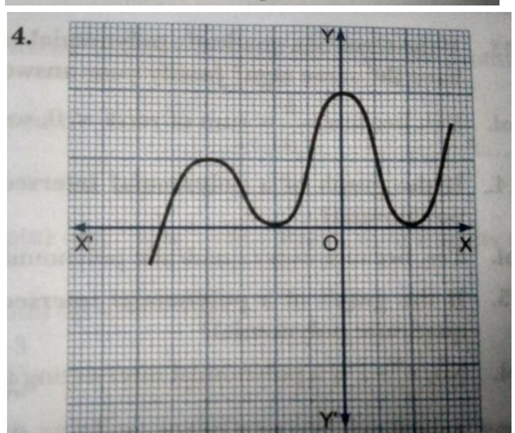

Fig. 2.6

Sol. 1. There is no zero as the graph does not intersect the x-axis.

2. The number of zeros is four as the graph intersects the x-axis at four points.

3. The number of zeros is three as the graph intersects the x-axis at three points.

4. The numbers of zeros is three as the graph intersects the x-axis at three points.

Answer the following questions and justify your answer

Que 5. What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + 5$, $p \neq 0$?

Sol. 0, $ax^2 + bx + c$.

Que 6. If on division of a polynomial p(x) by a polynomial g(x), the quotient is zero, what is the relation between the degrees of p(x) and g(x)?

Sol. Since the quotient is zero, therefore

deg p(x) < deg g(x)

Que 7. Can x – 2 be the remainder on division of a polynomial p(x) by x + 3?

Sol. No, as degree (x – 2) = degree (x + 3)

Que 8. Find the quadratic polynomial whose zeros are -3 and 4.

Sol. Sum of zeros = -3 + 4 = 1,

Product of zeros = $-3 \times 4 = -12$

∴ Required polynomial = $x^2 - x - 12$

Que 9. If one zero of the quadratic polynomial $x^2 - 5x - 6$ is 6 then find the other zero.

Sol. Let a, 6 be the roots of given polynomial.

Then a + 6 = 5 ⇒ a = -1

Que 10. If both the zeros of the quadratic polynomial $ax^2 + bx + c$ are equal and opposite in sign, then find the value of b.

Sol. Let a and -a be the roots of given polynomial.

Then a + (-a) = 0 ⇒ $-\frac{b}{a} = 0 \Rightarrow b = 0.$

Que 11. What number should be added to be the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the polynomial?

Sol. Let $f(x) = x^2 - 5x + 4$

Then $f(3) = 3^2 - 5 \times 3 + 4 = -2$

For f(3) = 0,2 must be added to f(x).

Que 12. Can a quadratic polynomial $x^2 + kx + k$ have equal zeros for some odd integer k > 1?

Sol. No, for equal zeros, k = 0,4 ⇒ k should be even.

Que 13. If the zeros of a quadratic polynomial $ax^2 + bx + c$ are both negative, then can we say a, b and c all have the same sign? Justify your answer.

Sol. Yes. Because $-\frac{b}{a}$ = sum of zeros < 0, so that $\frac{b}{a} > 0$. Also the product of the zeros =$\frac{c}{a} > 0$.

Que 14. If the graph of a polynomial intersects the x-axis at only one point, can it be a quadratic polynomial?

Sol. Yes, because every quadratic has at the most two zeros.

Que 15. If the graph of a polynomial intersects the x-axis at exactly two points, is it necessarily a quadratic polynomial?

Sol. No, $x^4 - 1$ is a polynomial intersecting the x-axis at exactly two points.

Short Answer Type Questions – I

[2 marks]

Que 1. If one of the zeros of the quadratic polynomial $f(x) = 4x^2 – 8kx – 9$ is equal in magnitude but opposite in sign of the other, find the value of k.

Sol. Let one root of the given polynomial be a.

Then the other root = -a

∴ Sum of the roots = (-a) + a = 0

$\Rightarrow \frac{-b}{a} = 0 \quad or \quad \frac{8k}{4} = 0 \quad or \quad k = 0$

Que 2. If one of the zeros of the quadratic polynomial $(k – 1)x^2 + kx + 1$ is -3 then find the value of k.

Sol. Since -3 is a zero of the given polynomial

∴ $(k – 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow 9k – 9 - 3k + 1 = 0 \Rightarrow k = \frac{4}{3}$

Que 3. If 1 is a zero of the polynomial $p(x) = ax^2 – 3(a- 1)x – 1$, then find the value of a.

Sol. Put x = 1 in p(x)

∴ $p(1) = a(1)^2 – 3(a – 1) \times 1 – 1 = 0$

$\Rightarrow a – 3a + 3 – 1 = 0 \quad \Rightarrow \quad -2a = -2 \quad \Rightarrow \quad a = 1$

Que 4. If a and b are zeros of polynomial $p(x) = x^2 – 5x + 6$, then find the value of a + b – 3ab.

Sol. Here, a + b = 5, ab = 6

∴ $a + b – 3ab = 5 – 3 \times 6 = -13$

Que 5. Find the zeros of the polynomial $p(x) = 4x^2 – 12x + 9$.

Sol. $P(x) = 4x^2 – 12x + 9 = (2x – 3)^2$

For zeros, p(x) = 0

$\Rightarrow (2x – 3)(2x – 3) = 0 \Rightarrow x = \frac{3}{2}, \frac{3}{2}$.

Que 6. If one root of the polynomial $p(y) = 5y^2 + 13y + m$ is reciprocal of other, then find the value of m.

Sol. Let the roots be a and $\frac{1}{a}$. Then $a\left(\frac{1}{a}\right) = \frac{m}{5} \quad or \quad 1 = \frac{m}{5} \quad or \quad m = 5$

Que 7. If a and b are zeros of p(x) = $x^2 + x - 1$, then find $\frac{1}{a} + \frac{1}{b}$.

Sol. Here, a + b = -1, ab = -1, so $\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = \frac{-1}{-1} = 1$

Que 8. Given that one of the zeros of the cubic polynomial ax3 + bx2 + d is zero, find the product of the other two zeros.

Sol. Let a, b, r, be the roots of the given polynomial and a = 0.

Then ab + br + ra = $\frac{c}{a}$ $\Rightarrow$ $br = \frac{c}{a}$

Que 9. If the product of two zeros of the polynomial p(x) = $2x^3 + 6x^2 - 4x + 9$ is 3, then find its third zero.

Sol. Let a, b, r be the roots of the given polynomial and ab = 3

Then abr = $-\frac{9}{2}$ $\Rightarrow 3 \times r = \frac{-9}{2}$ $or\ r = \frac{-3}{2}$

Que 10. Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $-\frac{1}{4}, \frac{1}{4}$ (*ii*) $\sqrt{2}, \frac{1}{3}$

Sol. Let a,b be the zeros of polynomial.

(i) we have, a + b = $-\frac{1}{4}$ and $ab = \frac{1}{4}$

Thus, polynomial is

P(x) = $x^2 - (a + b)x + ab$

$= x^2 - (-\frac{1}{4})x + \frac{1}{4} = x^2 + \frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(4x^2 + x + 1)$

∴ Quadratic polynomial = $4x^2 + x + 1$

(ii) We have, a + b = $\sqrt{2}$ $and\ ab = \frac{1}{3}$

Thus, polynomial is p(x) = $x^2 - (a + b)x + ab$

$= x^2 - \sqrt{2}x + \frac{1}{3} = \frac{1}{3}\left(3x^2 - 3\sqrt{2}x + 1\right)$

∴ Quadratic polynomial = 3x2 - $3x^2 - 3\sqrt{2}x + 1$

Short Answer Type Questions – II

[3 marks]

Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients (Q. 1 - 2)

Que 1. $6x^2 - 3 - 7x$

Sol. We have, $p(x) = 6x^2 - 3 - 7x$

$\Rightarrow P(x) = 6x^2 - 7x - 3$ (In general form)

$= 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

The zero of polynomial p (x) is given by

$P(x) = 0 \Rightarrow (2x - 3)(3x + 1) = 0 \Rightarrow x = \frac{3}{2}, -\frac{1}{3}$

Thus, the zeros of $6x^2 - 7x - 3$ are $a = \frac{3}{2}$ and $b = -\frac{1}{3}$

Now, sum of the zeros $= a + b = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6}$

And $\frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-7}{6} = \frac{7}{6}$

Therefore, sum of the zeros $= \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2}$

Again, product of zeros $= a.b = \frac{3}{2} \times (-\frac{1}{3}) = -\frac{1}{2}$

And $\frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{-3}{6} = -\frac{1}{2}$

Therefore, product of zeros $= \frac{Constant\ term}{Coefficient\ of\ x^2}$

Que 2. $4u^2 + 8u$

Sol. We have, $p(u) = 4u^2 + 8u \Rightarrow p(u) = 4u(u + 2)$

The zeros of polynomial p (u) is given by

$P(u) = 0 \Rightarrow 4u(u + 2) = 0$

$\therefore u = 0, -2$

Thus, the zeros of $4u^2 + 8u$ are a = 0 and b = -2

Now, sum of the zeros = a + b = 0 – 2 = -2

And $\frac{-(Coefficient\ of\ u)}{Coefficient\ of\ u^2} = \frac{-8}{4} = -2$

Therefore, sum of the zeros = $\frac{-(Coefficient\ of\ u)}{Coefficient\ of\ u^2}$

Again, product of the zeros = ab = 0 × (-2) = 0

And $\frac{Constant\ term}{Coefficient\ of\ u^2} = \frac{0}{4} = 0$

Therefore, product of zeros = $\frac{Constant\ term}{Coefficient\ of\ u^2}$

Que 3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) **$X^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$**

(ii) **t2 – 3, $2t^4 + 3t^3 - 2t^2 - 9t - 12$**

Sol. (i) we have,

$$3x^2 - 4x + 2$$

$$x^2 + 3x + 1 \overline{)\ 3x^4 + 5x^3 - 7x^2 + 2x + 2}$$

$$3x^4 + 9x^3 + 3x^2$$

$$- \quad - \quad -$$

$$-4x^3 - 10x^2 + 2x$$

$$-4x^3 - 12x^2 - 4x$$

$$+ \quad + \quad +$$

$$2x^2 + 6x + 2$$

$$2x^2 + 6x + 2$$

$$- \quad - \quad -$$

$$0$$

Clearly, remainder is zero, so $x^2 + 3x + 1$ is a factor of polynomial $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(ii) We have,

$$
\begin{array}{r}
2t^2 + 3t + 4 \\
t^2 - 3 \enclose{longdiv}{2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
2t^4 \qquad\quad - 6t^2 \qquad\qquad\quad \\
- \qquad\qquad\quad + \qquad\qquad\qquad \\
\hline
3t^3 + 4t^2 - 9t \qquad\; \\
3t^3 \qquad\;\; - 9t \qquad\; \\
- \qquad\qquad + \qquad\quad \\
\hline
4t^2 - 12 \\
4t^2 - 12 \\
- \qquad + \quad \\
\hline
0 \quad
\end{array}
$$

Clearly, remainder is zero, so $t^2 - 3$ is a factor of polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Que 4. If a and B are the zeros of the quadratic polynomial f(x) = $2x^2 - 5x + 7$, find a polynomial whose zeros are 2a + 3b and 3a + 2b.

Sol. Since a and b are the zeros of the quadratic polynomial f(x) = $2x^2 - 5x + 7$

∴ a + b = $= \frac{-(5)}{2} = \frac{5}{2}$ *and ab* $= \frac{7}{2}$

Let s and p denote respectively the sum and product of the zeros of the required polynomial.

Then, S = (2a + 3b) + (3a + 2b) = 5(a + b) = $5 \times \frac{5}{2} = \frac{25}{2}$

And p = (2a + 3b) (3a + 2b)

⇒ p = $6a^2 + 6b^2 + 13ab = 6a^2 + 6b^2 + 12ab + ab$

= $6(a^2 + b^2 + 2ab) + ab = 6(a + b)^2 + ab$

⇒ p = $6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$

Hence, the required polynomial g(x) is given by

g(x) = $k(x^2 - Sx + p)$

Or g(x) = k $\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

Que 5. What must be subtracted from p(x) = $8x^4 + 14x^3 – 2x^2 + 7x – 8$ so that the resulting polynomial is exactly divisible by g(x) = $4x^2 + 3x – 2$?

Sol. Let y be subtracted from p(x)

$\therefore$ $8x^4 + 14x^3 – 2x^2 + 7x – 8y$ is exactly divisible by g(x)

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\text{Now, } 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 - y \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + 7x - 8 - y \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 11x - 8 - y \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & 14x - 10 - y
\end{array}
$$

$\because$ Remainder should be 0.

$\therefore$ $14x – 10 – y = 0$

Or $14x – 10 = y$ or $y = 14x – 10$

$\therefore$ $(14x – 10)$ should be subtracted from p(x) so that it will be exactly divisible by g(x).

Que 6. What must be added to f(x) = $4x^4 + 2x^3 – 2x^2 + x – 1$ so that the resulting polynomial is divisible by g(x) = $x^2 + 2x – 3$?

Sol. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow f(x) – r(x) = g(x) \times q(x)$ $\Rightarrow f(x) + \{-r(x)\} = g(x) \times q(x)$

Clearly, RHS is divisible by g(x). Therefore, LHS is also divisible by g(x). Thus, if we add -r(x) to f(x), then the resulting polynomial is divisible by g(x). Let us now find the remainder when f(x) is divided by g(x).

$$
\begin{array}{r l}
x^2 + 2x - 3 \,\big)\, & 4x^4 + 2x^3 - 2x^2 + x - 1 \,\big(\, 4x^2 - 6x + 22 \\
 & 4x^4 + 8x^3 - 12x^2 \\
 & - \quad - \quad + \\
\hline
 & -6x^3 + 10x^2 + x - 1 \\
 & -6x^3 - 12x^2 + 18x \\
 & + \quad + \quad - \\
\hline
 & 22x^2 - 17x - 1 \\
 & 22x^2 + 44x - 66 \\
 & - \quad - \quad + \\
\hline
 & -61x + 65
\end{array}
$$

∴ r(x) = -61x + 65 or -r(x) = 61x – 65

Hence, we should add -r(x) = 61x – 65 to f(x) so that the resulting polynomial is divisible by g(x).

Que 7. Obtain the zeros of quadratic polynomial $\sqrt{3x^2}$ – 8x + 4$\sqrt{3}$ and verify the relation between its zeros and coefficients.

Sol. We have,

f(x) = $\sqrt{3}x^2 - 8x + 4\sqrt{3}$

= $\sqrt{3x^2} - 6x - 2x + 4\sqrt{3} = \sqrt{3x}\left(x - 2\sqrt{3}\right) - 2(x - 2\sqrt{3}$

$\Rightarrow \left(x - 2\sqrt{3}\right)\left(\sqrt{3x} - 2\right) = 0$

$x = 2\sqrt{3} \quad or \quad x = \frac{2}{\sqrt{3}}.$

So, the zeros of $f(x)$ are $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$.

Sum of zeros = $2\sqrt{3} + \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$

Product of the zeros = $2\sqrt{3} \times \frac{2}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

Hence verified.

Que 8. If a and b are the zeros of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeros are $\frac{1}{a}$ and $\frac{1}{b}$.

Sol. Let $p(y) = 6y^2 - 7y + 2$

$a + b = -\left(\frac{-7}{6}\right) = \frac{7}{6}; ab = \frac{2}{6} = \frac{1}{3}$

Now, $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{7}{6 \times \frac{1}{3}} = \frac{7}{2}$

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab} = \frac{1}{\frac{1}{3}} = 3$$

The required polynomial = $y^2 - \frac{7}{2}y + 3 = \frac{1}{2}(2y^2 - 7y + 6)$

Que 9. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Sol. Let a and b be the zeros of the polynomial. Then as per question b = 7a

Now sum of zeros = $a + b = a + 7a = -\left(\frac{-8}{3}\right)$

$$= 8a = \frac{8}{3} \; or = a = \frac{1}{3}$$

And $\quad a \times b = a \times 7a = \frac{2k+1}{3}$

$\Rightarrow \quad 7a^2 = \frac{2k+1}{3} \;\Rightarrow\; 7\left(\frac{1}{2}\right)^2 = \frac{2k+1}{3} \qquad \left(\because\; a = \frac{1}{3}\right)$

$\Rightarrow \quad \frac{7}{9} = \frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3} = 2k + 1$

$\Rightarrow \quad \frac{7}{3} - 1 = 2k \quad \Rightarrow \quad k = \frac{2}{3}$

Que 10. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and other zero.

Sol. Let $p(x) = 2x^2 + 3x + \boldsymbol{\lambda}$

Its one zero is $\frac{1}{2}$ $so\; p\left(\frac{1}{2}\right) = 0$

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} + \boldsymbol{\lambda} = \mathbf{0}$$

$\Rightarrow \quad \frac{1}{2} + \frac{-3}{2} + \lambda = 0 \quad \Rightarrow \quad \frac{4}{2} + \lambda = 0$

$\Rightarrow \quad 2 + \lambda = 0 \quad \Rightarrow \quad \lambda = -2$

Let the other zero be a.

Then $a + \frac{1}{2} = \frac{-3}{2} + \frac{1}{2} = -2$

Que 11. If one zero of polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.

Sol. Let one zero of the given polynomial be a.

Then, the other zero is $\frac{1}{a}$

$\therefore \quad Product\ of\ zeros = a \times \frac{1}{a} = 1$

But, as per the given polynomial product of zeros = $\frac{6a}{a^2+9}$

$\therefore \quad \frac{6a}{a^2+9} = 1 \quad \Rightarrow \quad a^2 + 9 = 6a$

$\Rightarrow \quad a^2 - 6a + 9 = 0 \quad \Rightarrow \quad (a-3)^2 = 0$

$\Rightarrow \quad a - 3 = 0 \quad \Rightarrow \quad a = 3$

Que 12. If the polynomial $(x^4 + 2x^3 + 8x^2 + 12x + 18)$ is divided by another polynomial $(x^2 + 5)$, the remainder comes out to be $(px + q)$. Find value of p and q.

Sol. Let $f(x) = (x^4 + 2x^3 + 8x^2 + 12x + 18)$ and $g(x) = (x^2 + 5)$.

On dividing f(x) by g(x), we get

$$
\begin{array}{l}
x^2+5 \overline{)\, x^4 + 2x^3 + 8x^2 + 12x + 18 \,(} \; x^2 + 2x + 3 \\
\qquad\quad x^4 \qquad\quad + 5x^2 \\
\qquad\quad - \qquad\quad\;\; - \\
\hline
\qquad\qquad\quad 2x^3 + 3x^2 + 12x \\
\qquad\qquad\quad 2x^3 \qquad\;\; + 10x \\
\qquad\qquad\quad - \qquad\qquad - \\
\hline
\qquad\qquad\qquad 3x^2 + 2x + 18 \\
\qquad\qquad\qquad 3x^2 \qquad + 15 \\
\qquad\qquad\qquad - \qquad\quad\; - \\
\hline
\qquad\qquad\qquad\qquad 2x + 3
\end{array}
$$

Now, $px + q = 2x + 3 \quad \Rightarrow \quad p = 2, q = 3$ (By comparing the coefficient of x and constant term).

Long Answer Type Questions

[4 marks]

Que 1. Verify that the numbers given alongside the cubic polynomial below are their zeros, Also verify the relationship between the zeros and the coefficients.

$\boldsymbol{x^3 - 4x^2 + 5x - 2;\ \ 2, 1, 1}$

Sol. Let $p(x) = x^3 - 4x^3 + 5x - 2$

On comparing with general polynomial $p(x) = ax^3 + bx^2 + cx + d$, we get a = 1, b = -4, c = 5 and d = -2.

Given zeros 2, 1, 1.

$\therefore \quad p(x) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

And $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0.$

Hence, 2, 1 and 1 are the zeros of the given cubic polynomial.

Again, consider a = 2, b = 1, y = 1

$\therefore \qquad \alpha + \beta + \gamma = 2 + 1 + 1 = 4$

$$and \quad \alpha + \beta + \gamma = \frac{-(coefficient\ of\ x^2)}{Coefficient\ of\ x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$$

$$and \quad \alpha\beta + \ \beta\gamma + \ \gamma\alpha = \frac{Coefficient\ x}{Coefficient\ of\ x^3} = \frac{c}{a} = \frac{5}{1} = 5$$

$$\alpha\beta\gamma = (2)(1)(1) = 2$$

$$and \quad \alpha\beta\gamma = \frac{-(Constant\ term)}{Coefficient\ of\ x^3} = \frac{-d}{a} = \frac{-(2)}{1} = 2.$$

Que 2. Find a cubic polynomial with the sum the zeros, sum of the products of its zeros taken two at a time, and the product of its zeros as 2, -7, -14 respectively.

Sol. Let the cubic polynomial be $p(x) = ax^3 + cx + d$. Then

Sum of zeros = $\frac{-b}{a} = 2$

Sum of the product of zeros taken two at a time = $\frac{c}{a} = -7$

And product of the zeros = $\frac{-d}{a} = -14$

$\Rightarrow \frac{b}{a} = -2, \frac{c}{a} = -7, -\frac{d}{a} = -14 \quad or \quad \frac{d}{a} = 14$

$\therefore \quad p(x) = ax^3 + bx^2 + cx + d \;\Rightarrow\; p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

$p(x) = a[x^3 + (-2)x^2 + (-7)x + 14] \qquad \Rightarrow \qquad p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

For real value of $a = 1, p(x) = x^3 - 2x^2 - 7x + 14$

Que 3. Find the zeros of the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$, if it is given that the product of its two zeros is 12.

Sol. Let α, β and γ be the zeros of polynomial $f(x)$ such that $ab = 12$.

We have, $\quad \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-2}{1} = -2 \; and \; \alpha\beta\gamma = \frac{-d}{a} = \frac{-24}{1} = -24$

Putting $\alpha\beta = 12 \; in \; \alpha\beta\gamma = -24, we \; get$

$12\gamma = -24 \qquad \Rightarrow \qquad \gamma = -\frac{24}{12} = -2$

Now, $\alpha + \beta + \gamma = 5 \qquad \Rightarrow \qquad \alpha + \beta - 2 = 5$

$\Rightarrow \alpha + \beta = 7 \qquad \Rightarrow \qquad \alpha = 7 - \beta$

$\therefore \quad \alpha\beta = 12$

$\Rightarrow (7 - \beta)\beta = 12 \qquad \Rightarrow \qquad 7\beta - \beta^2 = 12$

$\Rightarrow \beta^2 - 7\beta + 12 = 0 \qquad \Rightarrow \qquad \beta^2 - 3\beta - 4\beta + 12 = 0$

$\Rightarrow \beta = 4 \quad or \quad \beta = 3$

$\therefore \quad \alpha = 3 \quad or \quad \alpha = 4$

Que 4. If the remainder on division of $x^2 - kx^2 + 13x - 21$ by $2x - 1$ is -21, find the quotient and the value of k. Hence, find the zeros of the cubic polynomial $x^3 - kx^2 + 13x$.

Sol. Let $f(x) = x^3 - kx^2 + 13x - 21$

Then $f\left(\frac{1}{2}\right) = -21 \qquad \Rightarrow \qquad \left(\frac{1}{2}\right)^3 - k\left(\frac{1}{2}\right)^2 + 13\left(\frac{1}{2}\right) - 21 = -21$

Or $\quad \frac{1}{8} - \frac{1}{4}k + \frac{13}{2} - 21 + 21 = 0 \qquad or \qquad \frac{k}{4} = \frac{53}{8} \qquad \Rightarrow \qquad k = \frac{53}{2}$

$\therefore f(x) = x^3 - \frac{53}{2}x^2 + 13x - 21$

Now, $f(x) = q(x)(2x - 1) - 21$

$$\Rightarrow \qquad x^3 - \frac{53}{2}x^2 + 13x - 21 = q(x)(2x-1) - 21$$

$$\Rightarrow \qquad \left(x^3 - \frac{53}{2}x^2 + 13x\right) \div (2x-1) = q(x)$$

$$\begin{array}{r|l}
 & \frac{1}{2}x^2 - 13x \\
\hline
2x-1 & x^3 - \frac{53}{2}x^2 + 13x \\
 & x^3 - \frac{1}{2}x^2 \\
 & - \quad + \\
\hline
 & -26x^2 + 13x \\
 & -26x^2 + 13x \\
 & + \quad - \\
\hline
 & 0
\end{array}$$

$i.e.,\quad x^3 - \frac{53}{2}x^2 + 13x = (2x-1)\left(\frac{1}{2}x^2 - 13x\right) = \frac{1}{2}x(2x-1)(x-26)$

For zeros, $\quad x^3 - \frac{53}{2}x^2 + 13x = 0 \quad \Rightarrow \quad \frac{1}{2}x(2x-1)(x-26) = 0 \quad \Rightarrow \quad x = 0, \frac{1}{2}, 26$

Que 5. Obtain all other zeros of $3x^4 + 6x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since two zeros are $\sqrt{\frac{5}{3}}$ $and -\sqrt{\frac{5}{3}}, so$ $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of the given polynomial.

Now, we divide the given polynomial by $\left(x^2 - \frac{5}{3}\right)$ to obtain other zeros.

$$\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
\hline
x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad\quad - 5x^2 \\
 & - \qquad\qquad + \\
\hline
 & 6x^3 + 3x^2 - 10x \\
 & 6x^3 \qquad\quad - 10x \\
 & - \qquad\qquad + \\
\hline
 & 3x^2 - 5 \\
 & 3x^2 - 5 \\
 & - \quad + \\
\hline
 & 0
\end{array}$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

Now, $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x+1)^2 = 3(x+1)(x+1)$

So its zeros are -1, - 1.

Thus, all the zeros of given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1 \; and -1.$

Que 6. Given that $\sqrt{2}$ is a zero of the cubic polynomial space $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other zeros.

Sol. The given polynomial is $f(x) = \left(6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}\right)$. Since $\sqrt{2}$ is the zero of $f(x)$, it followa that $\left(x - \sqrt{2}\right)$ is a factor of $f(x)$.

On dividing $f(x)$ by $\left(x - \sqrt{2}\right)$, we get

$$
\begin{array}{r|l}
x-\sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \; (\; 6x^2 + 7\sqrt{2}x + 4 \\
 & 6x^3 - 6\sqrt{2}x^2 \\
 & - \quad\;\; + \\ \hline
 & \quad 7\sqrt{2}x^2 - 10x \\
 & \quad 7\sqrt{2}x^2 - 14x \\
 & \quad - \qquad\;\; + \\ \hline
 & \qquad\quad 4x - 4\sqrt{2} \\
 & \qquad\quad 4x - 4\sqrt{2} \\ \hline
 & \qquad\qquad 0
\end{array}
$$

$\therefore f(x) = 0 \Rightarrow \left(x - \sqrt{2}\right)\left(6x^2 + 7\sqrt{2x} + 4\right) = 0 \quad \Rightarrow \quad \left(x - \sqrt{2}\right)\left(3\sqrt{2x} + 4\right)\left(\sqrt{2x} + 1\right) = 0$

$x - \sqrt{2} = 0, 3\sqrt{2x} + 4 = 0, \sqrt{2x} + 1 = 0$

Hence, $x = \sqrt{2}, x = -\frac{2\sqrt{2}}{3}, x = \frac{-\sqrt{2}}{2}$ and all zeros of $f(x) are \; \sqrt{2}, \frac{-2\sqrt{2}}{3}, \frac{-\sqrt{2}}{2}$.

HOTS (Higher Order Thinking Skills)

Que 1. If α, β, γ be Zeros of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

Sol. P (x) = $6x^3 + 3x^2 - 5x + 1$ so a = 6, b = 3, c = – 5, d = 1

$\because$ α, β *and* γ are zeros of the polynomial p (x).

$\therefore$ $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-3}{6} = \frac{-1}{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{-5}{6}$ and $\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$

Now $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-5/6}{-1/6} = 5$

Que 2. Find the zeros of the polynomial $f\ (x) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeros are in A.P.

Sol. If α, β, γ are in A.P., then,

$\beta - a = \gamma - \beta \quad \Rightarrow \quad 2\beta = \alpha + \gamma$ ….(i)

$\alpha + \beta + y = \frac{-b}{a} = \frac{-(-12)}{1} = 12 \quad \Rightarrow \quad \alpha + \gamma = 12 - \beta$ ….(ii)

From (i) and (ii)

Putting the value of β in (i), we have

8 = $\alpha + \gamma$ ….(iii)

$\alpha\beta\gamma = -\frac{d}{\alpha} = \frac{-(-28)}{1} = 28$

$(\alpha\ \gamma)\ 4 = 28 \quad or \quad \alpha\ \gamma = 7 \quad or = \frac{7}{\alpha}$ ….(iv)

Putting the value of $\gamma = \frac{7}{\alpha} in\ (iii)$, we get

$8 = \alpha + \frac{7}{\alpha} \quad \Rightarrow \quad 8a = \alpha^2 + 7$

$\Rightarrow \quad \alpha^2 - 8a + 7 = 0 \quad \Rightarrow \quad \alpha^2 - 7\alpha - 1\alpha + 7 = 0$

$\Rightarrow \quad \alpha\ (\alpha - 7) - 1(\alpha - 7) = 0 \quad \Rightarrow \quad (\alpha - 1)(a - 7) = 0$

$\Rightarrow$ α = 1 or α = 7

Putting $\alpha = 1$ in (*iv*), we get	Putting $\alpha = 7$ in (*iv*), we get
$\gamma = \frac{7}{1}$	$\gamma = \frac{7}{7}$
or $\gamma = 7$	or $\gamma = 1$
and $\beta = 4$	and $\beta = 4$
$\therefore$ zeros are 1, 7, 4.	$\therefore$ zeros are 7, 4, 1.

Que 3. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be x +α. Find k andα.

Sol. By division algorithm, we have Dividend = Divisor × Quotient + Remainder

⇒ Dividend – Remainder = Divisor × Quotient

⇒ Dividend – Remainder is always divisible by the divisor.

When $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$ the remainder comes out to be x + a.

$\therefore\ f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a)$

$= x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$

is exactly divisible by $x^2 - 2x + k$.

Let us now divide $x^4 - 6x^2 + 16x^2 - 26x + 10 - a\ by\ \ x^2 - 2x + k.$

$$x^2 - 2x + k \overline{)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a \,(}\ x^2 - 4x + (8 - k)$$

$$\begin{array}{l}
x^4 - 2x^3 + kx^2 \\
- \quad + \quad - \\
\hline
-4x^3 + (16 - k)\,x^2 - 26x + 10 - a \\
-4x^3 + 8x^2 \qquad - 4kx \\
+ \quad - \qquad + \\
\hline
(8 - k)\,x^2 - (26 - 4k)\,x + 10 - a \\
(8 - k)\,x^2 - (16 - 2k)\,x + (8k - k^2) \\
- \qquad + \qquad - \\
\hline
(-10 + 2k)\,x + (10 - a - 8k + k^2)
\end{array}$$

For $f(x) - (c + \alpha) = x^4 - 6x^3 + 16x^2 - 26x + 10 - \alpha$ to be exactly divisible by $x^2 - 2x + k$, we must have

$(-10 + 2k)\,x + (10 - \alpha - 8k + k^2) = 0\ for\ all\ x$

⇒ $-10 + 2k = 0\ \ and\ \ 10 - a - 8k + k^2 = 0$

⇒ $k = 5\ \ and\ 10 - a - 40 + 25 = 0$

⇒ $k = 5\ \ and\ \ a = -5.$

Value Based Questions

Que 1. Some people collected money to be donated in some orphanages. A part of the donation is fixed and remaining depends on the number of children in the orphanage. If they donated ₹ 9,500 in the orphanage having 50 children and ₹ 13,250 with 75 children, find the fixed part of the donation and the amount donated for each child. What values do these people posses?

Sol. Let the fixed donation be ₹ x and amount donated for each child be ₹ y.

Then $x + 50y = 9500$ …(i)

$x + 75y = 13250$ …(ii)

Subtracting (i) from (ii), we get

$25y = 3750$ or $y = 150$

From (i), $x = 9500 - 50y = 9500 - 50 \times 150 = 2000$

∴ Fixed amount donated = ₹ 2,000.

Amount donated for each child = ₹ 150.

Helpfulness, cooperation, happiness, caring.

Que 2. The fraction of people in a society using CNG in their vehicles becomes $\frac{9}{11}$, if 2 is added to both its numerator and denominator. If 3 is added to both its numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction. What does this fraction show?

Sol. Let the fraction be $\frac{x}{y}$.

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \Rightarrow \quad 11x - 9y = -4$$

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \Rightarrow \quad 6x - 5y = -3$$

Solving for x and y gives x = 7 and y = 9

$$\therefore \quad Fraction = \frac{x}{y} = \frac{7}{9}$$

More people are becoming aware about scarcity of petrol, so they are switching on to alternative resources.

Que 3. Reading book in a library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days. While Bunty paid ₹ 21 for the book kept for five days.

(i) Find the fixed charge.

(ii) Find how much additional charge Shristi and Bunty paid.
(iii) Which mathematical concept is used in this problem?
(iv) Which value does it depict?

Sol. Let fix charge for reading book be ₹ x and additional charge for each day be ₹ y.

Then, $x + 4y = 27$ …(i)

On subtraction $\underline{\underline{x}+ 2y=21}$ ….(ii)

$2y = 6$

$Y = 3$

Putting value of y in equation (i) we get

$x + 4 \times 3 = 27 \quad \Rightarrow \quad x + 12 = 27 \quad \Rightarrow \quad x = 15$

(i) Fixed charge on the book is ₹ 15.
(ii) Bunty paid additional ₹ 6 and Shristi paid ₹ 12.
(iii) Pair of linear equation in two variables.
(iv) Reading is a good habit.

Que 4. An honest person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But if he had interchanged amount invested, he would have received ₹ 4 more as interest.

(i) How much amount did he invest at different rates?
(ii) Which mathematical concept is used in this problem?
(iii) Which value is being emphasized here?

Sol. (i) Let the person invest ₹ x at rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

Yearly interest = $\frac{12x}{100} + \frac{10y}{100}$

$\therefore \quad \frac{12x}{100} + \frac{10y}{100} = 130 \quad \Rightarrow \quad 12x + 10y = 13000$ …(i)

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$\frac{10x}{100} + \frac{12y}{100} = 134 \quad \Rightarrow \quad 10x + 12y = 13400$ …(ii)

Adding eqn. (i) and (ii) we get

$22x + 22y = 26400$

$x + y = 1200$ …(iii)

Subtracting (ii) from (i) we get

$2x - 2y = -400$

$x - y = -200$ …(iv)

Solving (iii) and (iv), we get

$x = 500, y = 700$

Thus, person invested ₹ 500 at 12% per annum and ₹ 700 at 10% per annum.

(ii) Pair of linear equation in two variables.
(iii) Honesty is the best policy.

Que 5. If the price of petrol is increased by ₹ 2 per litre, a person will have to buy 1 litre less petrol for ₹ 1740. Find the original price of petrol at that time.
(a) Why do you think the price of petrol is increasing day-by-day?
(b) What should we do to save petrol?

Sol. Let the original price of the petrol be ₹ x per litre.

Then, amount of petrol that can be purchased = $\frac{1740}{x}$

According to question

$$\frac{1740}{x} - \frac{1740}{x+2} = 1 \qquad \Rightarrow \quad 1740\,(x+2-x) = x\,(x+2)$$

$$\Rightarrow \quad x^2 + 2x - 3480 = 0 \qquad \Rightarrow \quad x^2 + 60x - 58x - 3480 = 0$$

$$\Rightarrow \quad (x+60)\,(x-58) = 0 \qquad \Rightarrow \quad x = 58, -60\ (rejected)$$

∴ Original cost of petrol was ₹ 58 per litre.

(a) Petrol is a natural resource which is depleting day-by-day. So, due to more demand and less supply, its price is increasing.

(b) We should use more of public, transport and substitute petrol with CNG or other renewable resource.

Que 6. One fourth of a group of people claim they are creative, twice the square root of the group claim to be caring and the remaining 15 claim they are optimistic. Find the total number of people in the group.

(a) How many persons in the group are creative?

(b) According to you, which one of the above three values is more important for development of a society?

Sol. Let x be the total

Then, number of creative persons = $\frac{x}{4}$

Number of caring persons = $2\sqrt{x}$

and number of optimistic persons = 15

Thus, total number of persons = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y,\quad then \quad x = y^2$

$$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \qquad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$$

$$\Rightarrow \quad 3y(y-6) + 10\,(y-6) = 0 \qquad \Rightarrow \quad (3y+10)\,(y-6) = 0$$

$$\Rightarrow \quad y = 6 \ \ or \ \ y = -\frac{10}{3}$$

Now, $y = -\frac{10}{3} \qquad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9} (\because\ x = y^2)$

But, the number of persons cannot be a fraction.

∴ y = 6 $\qquad \Rightarrow \quad x = 6^2 = 36$

Hence, the number of people in the group = 36

(a) 9 persons

(b) All of these values have their own importance. A person having these values will certainly contribute to the development of society. However, the level of importance given to each of them depends upon a person's own attitude. Hence, any value with justification is correct. (Do yourself)

Que 7. In the centre of a rectangular plot of land of dimensions 120 m × 100 m, a rectangular portion is to be covered with trees so that the area of the remaining part of the plot is 10500 m^2. Find the dimensions of the area to be planted. Which social act is being discussed here? Give its advantages.

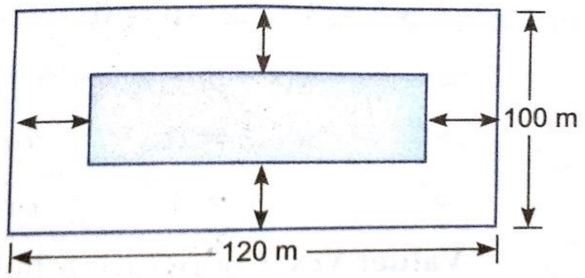

Fig. 1

Sol. Let the width of the unplanted area be x m

Then, dimension of area to be planted = (120 – 2x) and (100 – 2x)

$\therefore \quad (120 - 2x)\,(100 - 2x) = 120 \times 100 - 10500$

$\Rightarrow \quad 12000 - 440x + 4x^2 = 1500 \; or \; x^2 - 110x + 2625 = 0$

$\Rightarrow \quad (x - 75)\,(x - 35) = 0 \; or \; x = 75{,}35$

But x = 75 is not possible

$\therefore \quad x = 35$

Thus, dimension of area to be planted = (120 – 70) and (100 – 70) i.e., 50 m and 30 m.
Afforestation is being discussed here. Planting more trees helps in reducing air pollution and make the environment clean and green.

Que 8. Mr. Ahuja has to square plots of land which he utilises for two different purposes–one for providing free education to the children below the age of 14 years and the other to provide free medical services for the needy villagers. The sum of the areas of two square plots is 15425 m^2. If the difference of their perimeter is 60 m, find the sides of the two squares.
Which qualities of Mr. Ahuja are being depicted in the question?

Sol. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $\quad x^2 + y^2 = 15425 \quad$ …(i)

Let x be the length of the side of the bigger square.

$4x - 4y = 60$

$\Rightarrow \quad x - y = 15$ or $x = y + 15 \quad$ …(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 15)^2 + y^2 = 15425 \quad \Rightarrow \quad 2y^2 + 30y - 15200 = 0$

or $\quad y^2 + 15y - 7600 = 0 \quad$ or $\quad (y + 95)(y - 80) = 0 \quad \Rightarrow \quad y = -95{,}80$

But, sides cannot be negative, so y = 80

Therefore, $\quad x = 80 + 15 = 95$

Hence, sides of two squares are 80 m and 95 m.

Value: Caring, King, Social and generous.

Que 9. A takes 3 days longer than B to finish a work. But if they work together, then work is completed in 2 days. How long would each take to do it separately. Can you say cooperation helps to get more efficiency?

Sol. Let B finish a work in x days

then A finish a work in x + 3 days

According to question

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2} \qquad \Rightarrow \qquad \frac{(x+3)+x}{x(x+3)} = \frac{1}{2}$$

$\Rightarrow \quad 2x + 6 + 2x = x^2 + 3x \Rightarrow \quad x^2 + 3x - 2x - 6 - 2x = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

Solving the equation

$x^2 - (3 - 2)x - 6 = 0 \qquad \Rightarrow \quad x^2 - 3x + 2x - 6 = 0$

$\Rightarrow \quad x(x - 3) + 2(x - 3) = 0 \qquad \Rightarrow \quad (x - 3)(x + 2) = 0$

$\Rightarrow \quad x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow \quad x = 3$ or $x = -2$ (Days cannot be negative)

Value: Yes, Cooperation helps in improving work efficiency.

Que 10. In a class of 48 students, the number of regular students is more than the number of irregular students. Had two irregular students been regular, the product of the number of two types of students would be 380. Find the number of each type of students.

(a) Why is regularity essential in life?

(b) Write values other than regularity that a student must possess.

Sol. Let the number of regular students be x

Then, the number of irregular students = 48 – x

According to the question,

$(x + 2)(48 - x - 2) = 380$

$\Rightarrow \quad -x^2 + 44x + 92 - 380 = 0 \text{ or } x^2 - 44x + 288 = 0$

$\Rightarrow \quad (x - 36)(x - 8) = 0$ or $x = 36, 8$

But $x > 48 - x$ (given)

$\therefore \quad x = 36$

i.e., The number of regular students = 36

and the number of irregular students = 48 – 36 = 12

(a) Regularity in any sphere gives confidence which, in turn, leads to the development of an individual and the society as well.

(b) Honesty, Creativity, Confidence, Punctuality. (you may add more to this list)

Que 11. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?

Sol. Number of plants planted by class 1 = 2 (class 1 × 2 sections) = 2 × 1 × 2 = 4 trees

Similarly, number of plants planted by class 2 = 2 (class 2 x 2 sections)

= 2 × 2 × 2 = 8 trees

We now know, a = 4 and d = 4
Number of classes n = 12
Total number of plants planted = Sum of AP = S_{12}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 4 + (12-1)4]$$

$$S^{12} = 6[8 + 44] = 6 \times 52 = 312\ trees$$

Values shown by the students:
(i) Envrionmental friendly
(ii) Social awareness
(iii) Sense of responsibility towards the society.

Que 12. A sum of ₹ 3150 is to be used to give six cash prizes to students of a school for overall academic performance, punctuality, regularity, cleanliness, confidence and creativity. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes.
(a) Which value according to you should be awarded with the maximum amount? Justify your answer.
(b) Can you add more values to the above ones which should be awarded?

Sol. Let the six prizes (1st, 2nd, 3rd …..6th) be a, a – 50, a – 100, a – 150, a – 200, a – 250 respectively
Then a + (a – 50) + (a – 100) + (a – 150) + (a – 200) + (a – 250) = 3150

$$\Rightarrow\ 6a - 750 = 3150\ or\ a = \frac{3900}{6} = 650$$

∴ The value of the 1st, 2nd, 3rd …..6th Prizes are 650, 600, 550, 500, 450, 400 respectively.
(a) Any value with justification is correct.
(b) Many more can be added like, honesty, good habits, friendship, respectively elders, loving youngers,….. etc.

Que 13. A person donates money to a trust working for education of children and women in some village. If the person donates ₹ 5,000 in the first year and his donation increases by ₹ 250 every year, find the amount donated by him in the eighth year and the total amount donated in eight years.
(a) Which mathematical concept is being used here?
(b) Write any two values the person mentioned here possess.
(c) Why do you think education of women is necessary for the development of a society?

Sol. The amount donated by the person each year forms an AP.
Here, a = 5,000, d = 250
We have to find a_8 and S_8

$$a_8 = a + 7d = 5{,}000 + 7 \times 250 = ₹\,6{,}750$$

$$S_8 = \frac{8}{2}[2a + 7d] = 4(2 \times 5{,}000 + 7 \times 250) = 4 \times 11{,}750 = ₹\,47{,}000$$

(a) Arithmetic progression.
(b) Socially aware and responsible citizen.
(c) Educating a woman means educating the whole family and an educated family makes development in society.

Que 14. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

Sol. Let the usual speed of plane be x km/h.

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$\Rightarrow\ [1500(x+250) - 1500x]\,2 = x(x+250)$
$\Rightarrow\ x^2 + 250x - 750000 = 0$
$\Rightarrow (x+1000)(x-750) = 0 \ \Rightarrow\ x = 750\ or\ x = -1000$ (Which is neglected)
$\therefore$ Using speed of plane = 750 km/h
Values: Helping others

Que 15. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?
What value is reflected in this question?

Sol. Here a = ₹ 450, d = ₹ 20, n = 12

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20] = 6[1120] = 6720 > 6500$$

$\therefore$ Reshma will be able to send her daughter to school
Value: Encouraging efforts for girl education.