Very Short Answer Type Questions

[1 marks]

Que 1. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find value of k.

Sol. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \ i.e., k = \frac{15}{4}.$$

Que 2. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions.

Sol. The given system of equations will have infinitely many solutions if $\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$ which is not possible

∴ For no value of c, the given system of equations have infinitely many solutions.

Que 3. Do the equations $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent a pair of coincident lines?

Sol. Here, $\frac{4}{12} = \frac{3}{9} \neq \frac{6}{15}$ $\quad i.e. \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ Given equations do not not represent a pair of coincident lines.

Que 4. Find the co-ordinate where the line $x - y = 8$ will intersect y-axis.

Sol. The given line will intersect y-axis when x = 0.

$$\therefore 0 - y = 8 \qquad \Rightarrow \qquad y = -8$$

Required coordinate is (0, -8).

Que 5. Write the number of solutions of the following pair of linear equations:

$$\boldsymbol{x + 2y - 8 = 0, \qquad 2x + 4y = 16}$$

Sol. Here, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The given pair of linear equations has infinitely many solutions.

Short Answer Type Questions – I

[2 marks]

Que 1. Is the following pair of linear equations consistent? Justify your answer.

$$\boldsymbol{2ax + by = a, \quad 4ax + 2by - 2a = 0; \qquad a, b \neq 0}$$

Sol. Yes,

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

$\therefore \ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ The given system of equations is consistent.

Que 2. For all real value of c, the pair of equations

$$\boldsymbol{x - 2y = 8, \quad 5x + 10y = c}$$

Have a unique solution. Justify whether it is true or false.

Sol. Here, $\frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{+10} = \frac{-1}{5}, \frac{c_1}{c_2} = \frac{8}{c}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, for all real values of c, the given pair of equations have a unique solution.

$\therefore$ The given statement is true.

Que 3. Does the following pair of equations represent a pair of coincident lines? Justify your answer.

$$\boldsymbol{\frac{x}{2} + y + \frac{2}{5} = 0, \quad 4x + 8y + \frac{5}{16} = 0.}$$

Sol. Here, $a_1 = \frac{1}{2}, \ b_1 = 1, \ c_1 = \frac{2}{5}$ and $a_2 = 4, \ b_2 = 8, c_2 = \frac{5}{16}$

$$\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1}{8}, \qquad \frac{b_1}{b_2} = \frac{1}{8}, \qquad \frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{32}{25}$$

$\therefore \qquad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ The given system does not represent a pair of coincident lines.

Que 4. If x = a, y = b is the solution of the pair of equation x – y = 2 and x + y = 4 then find the value of a and b.

Sol. $x - y = 2$ …(i)

$x + y = 4$ …(ii)

On adding (i) and (ii), we get 2x = 6 or x = 3

From (i), 3 – y = 2 ⇒ y = 1

$\therefore \; a = 3, \; b = 1$

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, find out whether the following pair of linear equations is consistent or inconsistent. (5 to 6)

Que 5. $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

Q6. $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

Sol 5. We have, $\frac{3}{2}x + \frac{5}{3}y = 7$ **…(i)**

$9x - 10y = 14$ **…(ii)**

Here $a_1 = \frac{3}{2}, b_1 = \frac{3}{5}, c_1 = 7$

$a_2 = 9, b_2 = -10, c_2 = 14$

Thus. $\frac{a_1}{a_2} = \frac{3}{2 \times 9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{5}{3 \times (-10)} = -\frac{1}{6}$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. So, it has a unique solution and it is consistent.

Sol 6. We have, $\frac{4}{3}x + 2y = 8$ **…(i)**

$2x + 3y = 12$ **…(ii)**

Here, $a_1 = \frac{4}{3}, b_1 = 2, c_1 = 8$

And $a_2 = 2, b_2 = 3, c_2 = 12$

Thus, $\frac{a_1}{a_2} = \frac{4}{3 \times 2} = \frac{2}{3}$; $\frac{b_1}{b_2} = \frac{2}{3}$; $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so equations (i) and (ii) represent coincident lines.

Hence the pair of linear equations is consistent with infinitely many solutions.

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident: (7 to 9).

Que 7. $\mathbf{5x - 4y + 8 = 0}$

$\mathbf{7x + 6y - 9 = 0}$

Que 8. $\mathbf{9x + 3y + 12 = 0}$

$\mathbf{18x + 6y + 24 = 0}$

Que 9. $\mathbf{6x - 3y + 10 = 0}$

$\mathbf{2x - y + 9 = 0}$

Sol 7. We have, $5x - 4y + 8 = 0$ …(i)

$7x + 6y - 9 = 0$ …(ii)

Here, $a_1 = 5, b_1 = -4, c_1 = 8$

And, $a_2 = 7, b_2 = 6, c_2 = -9$

Here, $\frac{a_1}{a_2} = \frac{5}{7}$ *and* $\frac{b_1}{b_2} = -\frac{4}{6} = -\frac{2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. So, equations (i) and (ii) represent intersecting lines.

Sol 8. We have, $9x + 3y + 12 = 0$ …(i)

$18x + 6y + 24 = 0$ …(ii)

Here, $a_1 = 9, b_1 = 3, c_1 = 12$

And $a_2 = 18, b_2 = 6, c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$;

So equations (i) and (ii) represent coincident lines.

Sol 9. We have $6x - 3y + 10 = 0$ …(i)

$2x - y + 9 = 0$ …(ii)

Here, $a_1 = 6,\ b_1 = -3,\ c_1 = 10$

$a_2 = 2,\ b_2 = -1,\ c_2 = 9$

And $\frac{a_1}{a_2} = \frac{6}{2} = 3,$ $\frac{b_1}{b_2} = \frac{-3}{-1} = 3,$ $\frac{c_1}{c_2} = \frac{10}{9}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, equations (i) and (ii) represent parallel lines.

Short Answer Type Questions – II

[3 marks]

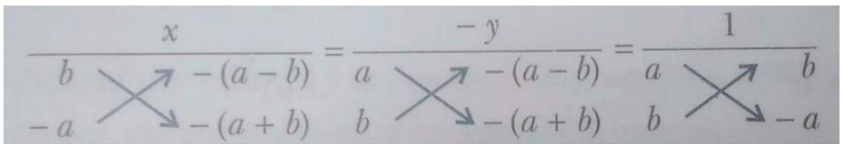
Que 1. Solve: $ax + by = a - b$ and $bx - ay = a + b$

Sol. The given system of equations may be written as

$$ax + by - (a - b) = 0$$

$$bx - ay - (a + b) = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a-b)} = \frac{1}{-a^2 - b^2}$$

$$\Rightarrow \quad \frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 + b^2} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad \frac{x}{-(a^2 + b^2)} = \frac{y}{(a^2 + b^2)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \quad x = -\frac{(a^2 + b^2)}{-(a^2 + b^2)} = 1 \text{ and } y = \frac{(a^2 + b^2)}{-(a^2 + b^2)} = -1$$

Hence, the solution of the given system of equations is $x = 1,\ y = -1$.

Que 2. Solve the following linear equations:

$$\mathbf{152x - 378y = -74 \text{ and } -378x + 152y = -604}$$

Sol. We have, $152x - 378y = -74$ …(i)

$-378x + 152y = -604$ …(ii)

Adding equation (i) and (ii), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ \hline -226x - 226y = -678 \end{array}$$

$\Rightarrow \quad -226(x + y) = -678$

$\Rightarrow \quad x + y = \frac{-678}{-226}$

$\Rightarrow \quad x + y = 3 \qquad \ldots(iii)$

Subtracting equation (ii) from (i), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ \hline 530x - 530y = 530 \end{array}$$

$\Rightarrow \quad x - y = 1 \qquad \ldots(iv)$

Adding equation (iii) and (iv), we get

$$\begin{array}{r} x + y = 3 \\ x - y = 1 \\ \hline 2x = 4 \end{array}$$

$\Rightarrow \quad x = 2$

Putting the value of x in (iii), we get

$$2 + y = 3 \quad \Rightarrow \quad y = 1$$

Hence, the solution of given system of equations is $x = 2, \; y = 1$

Que 3. Solve for x and y

$$\boldsymbol{\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; x + y = 2ab}$$

Sol. We have, $\quad \frac{b}{a}x + \frac{a}{b}y = a^2 + b^2 \qquad \ldots(i)$

$x + y = 2ab \qquad \ldots(ii)$

Multiplying (ii) by b/a, we get

$$\frac{b}{a}x + \frac{b}{a}y = 2b^2 \qquad \ldots(iii)$$

Subtracting (iii) from (i), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right) y = a^2 + b^2 - 2b^2 \quad \Rightarrow \quad \left(\frac{a^2 - b^2}{ab}\right) y = (a^2 - b^2)$$

$\Rightarrow \quad y = (a^2 - b^2) \times \frac{ab}{(a^2 - b^2)} \quad \Rightarrow \quad y = ab$

Putting the value of y in (ii), we get

$$x + ab = 2ab \quad \Rightarrow \quad x = 2ab - ab \quad \Rightarrow \quad x = ab$$

$\therefore \quad x = ab, y = ab$

Que 4. $(\boldsymbol{i})$ **For which values of $\boldsymbol{a}$ and $\boldsymbol{b}$ does the following pair of linear equations have and in finite number of solution?**

$$\mathbf{2x + 3y = 7}$$

$$\mathbf{(a - b)x + (a + b)y = 3a + b - 2}$$

$(\boldsymbol{ii})$ **For which value of $\boldsymbol{k}$ will the following pair of linear equations have no solution?**

$$\mathbf{3x + y = 1}$$

$$\mathbf{(2x - 1)x + (x - 1)y = 2k + 1}$$

Sol. (i) we have, $\quad 2x + 3y = 7 \quad \ldots(i)$

$$(a - b)x + (a + b)y = 3a + b - 2 \quad \ldots(ii)$$

Here, $\quad a_1 = 2, \quad b_1 = 3, \quad c_1 = 7$

And $\quad a_2 = a - b, \quad b_2 = a + b, \quad c_2 = 3a + b - 2$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b} = \frac{3}{a+b}$

$$\Rightarrow \quad 2a + 2b = 3a - 3b \quad \Rightarrow \quad 2a - 3a = -3b - 2b$$

$$\therefore \quad a = 5b$$

Again, we have

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \quad \Rightarrow \quad 9a + 3b - 6 = 7a + 7b$$

$$\Rightarrow \quad 9a - 7a + 3b - 7b - 6 = 0 \quad \Rightarrow \quad 2a - 4b - 6 = 0 \quad \Rightarrow \quad 2a - 4b = 6$$

$$\Rightarrow \quad a - 2b = 3$$

Putting $a = 5b$ in equation (iv), we get

$$5b - 2b = 3 \quad or \quad 3b = 3 \quad i.e., \quad b = \frac{3}{3} = 1$$

Putting the value of b in equation (iii), we get $a = 5$ (1) = 5

Hence, the given system of equations will have an infinite number of solutions for $a = 5$ and $b = 1$.

(ii) We have, $\quad 3x + y = 1 \quad \Rightarrow \quad 3x + y - 1 = 0$

$$(2k-1)x+(k-1)y=2k+1$$

$\Rightarrow$ $$(2k-1)x+(k+1)y-(2k+1)=0$$

Here, $a_1=3,\ \ b_1=1,\ \ c_1=-1$

$$a_2=2k-1, b_2=k-1, c_2=-(2k+1)$$

For no solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \quad\Rightarrow\quad \frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$$

Now, $\frac{3}{2k-1}=\frac{1}{k-1} \quad\Rightarrow\quad 3k-3=2k-1$

$\Rightarrow \quad 3k-2k=3-1 \quad\Rightarrow\quad k=2$

Hence, the given system of equations will have no solutions for $k=2$

Que 5. Find whether the following pair of linear equations has a unique solution. If yes, find the solution?

$$\boldsymbol{7x-4y=49 \quad and \quad 5x-6y=57}$$

Sol. We have, $7x-4y=49$ $\quad\ldots(i)$

And $5x-6y=57$ $\quad\ldots(ii)$

Here $a_1=7, b_1=-4, c_1=49$

$a_2=5, b_2=-6, c_2=57$

So, $\frac{a_1}{a_2}=\frac{7}{5}, \frac{b_1}{b_2}=\frac{-4}{-6}=\frac{2}{3}$

Since, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So, system has a unique solution.

Multiply equation (i) by 5 and equation (ii) by 7 and subtract

$$\begin{array}{r} 35x-\ \ 20y=\ \ 245 \\ 35x-\ \ 42y=\ \ 399 \\ \hline 22y=-154 \end{array} \Rightarrow \quad y=-7$$

Put $y=-7$ in equation (ii)

$$5x-6(-7)=57 \quad\Rightarrow\quad 5x=57-42 \quad\Rightarrow\quad x=3$$

Hence, $x=3$ and $y=-7$.

Que 6. Solve for x and y.

$$\frac{6}{x-1} - \frac{3}{y-2} = 1; \quad \frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \textbf{Where } x \neq 1, y \neq 2$$

Sol. Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

The given equations become

$$6p - 3q = 6 \qquad \ldots(i)$$

$$5p + q = 2 \qquad \ldots(ii)$$

Multiply equation (ii) by 3 and add in equation (i)

$$\begin{array}{rrr} 15p+ & 3q= & 6 \\ 6p- & 3q= & 1 \\ \hline 21p & = & 7 \end{array} \qquad \Rightarrow \quad p = \frac{7}{21} = \frac{1}{3}$$

Putting this value in equation (i) we get

$$6 \times \frac{1}{3} - 3q = 1 \quad \Rightarrow \quad 2 - 3q = 1 \quad \Rightarrow \quad 3q = 1, \quad \Rightarrow \quad q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3} \quad \Rightarrow \quad x - 1 = 3 \quad \Rightarrow \quad x = 4$

$\frac{1}{y-2} = q = \frac{1}{3} \quad \Rightarrow \quad y - 2 = 3 \quad \Rightarrow \quad y = 5$

Hence, $x = 4$ and $y = 5$

Que 7. Solve the following pair equations for x and y.

$$\frac{a^2}{x} - \frac{b^2}{y} = 0; \quad \frac{a^2 b}{y} = a + b, x \neq 0, y \neq 0.$$

Sol. $\frac{a^2}{x} - \frac{b^2}{y} = 0 \qquad \ldots(i)$

$$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b$$

Multiply equation (i) by a and adding to equation (ii)

$$\frac{a^2 a}{x} - \frac{b^2 a}{y} = 0 \qquad \Rightarrow \qquad \frac{a^2 b}{x} + \frac{b^2 a}{y} = (a + b)$$

$$\Rightarrow \frac{a^2}{x} - \frac{a^2 b}{x} = a + b \qquad \Rightarrow \qquad \frac{a^2}{x}(a + b) = a + b \qquad \Rightarrow \qquad x = \frac{a^2(a+b)}{a+b} = a^2$$

Putting the value of x in equation (i), we get

$$\frac{a^2}{a^2} - \frac{b^2}{y} = 0 \qquad \Rightarrow \quad 1 - \frac{b^2}{y} = 0 \quad \Rightarrow \quad \frac{b^2}{y} = 1 \quad \Rightarrow \quad y = b^2$$

Hence, $x = a^2, \; y = b^2$

Que 8. In $\Delta ABC, \angle A = x, \angle B = 3x, and\ \angle C = y$ if $3y - 5x = 30^o$ show that triangle is right angled.

Sol. $\angle A + \angle B + \angle C = 180^o$ (Sum of interior angles of ΔABC)

$x + 3x + y = 180^o$

$\Rightarrow$ $4x + y = 180^o$ $\ldots(i)$

And $3y - 5x = 30^0$ (Given) $\ldots(ii)$

Multiply equation (i) by 3 and subtracting from $eq.\ (ii)$, we get

$-17x = -510 \Rightarrow x = \frac{510}{17} = 30^o$

Then $\angle A = x = 30^o$ $and\ \angle B = 3x = 3 \times 30^o = 90^o$

$\angle C = y = 180^o - (\angle A + \angle B) = 180^o - 120^o = 60^o$

$\therefore$ $\angle A = 30^o,\ \angle B = 90^o, \angle C = 60^o$

Hence ΔABC is right tringle right angled at B.

Que 9. In fig. 3.1. $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. If the perimeter of $ABCDE$ is 21cm. Find the value of x and y.

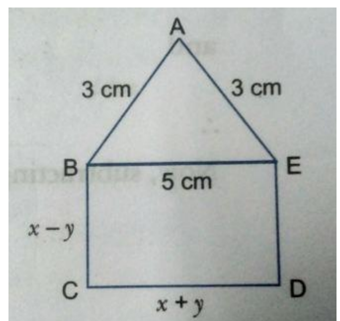


Fig. 3.1

Sol. Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp CD$.

$BCDE$ is a rectangle.

$\therefore$ Opposite sides are equal

$i.e.,$ $BE = CD$ $\therefore x + y = 5$ $\ldots(i)$

$DE = BC = x - y$

Since perimeter of $ABCDE$ is 21 cm.

$AB + BC + CD + DE + EA = 21$

$3 + x - y + x + y + x - y + 3 = 21$ $\Rightarrow$ $6 + 3x - y = 21$

$3x - y = 15$ $\ldots(ii)$

Adding (i) and (ii), we get

$4x = 20$ $\Rightarrow$ $x = 5$

On putting the value of x in (i), we get $y = 0$

Hence, $x = 5$ and $y = 0$.

Que 10. Five years ago, A was thrice as old as B and ten years later, A shall be twice old as B. What are the present ages of A and B?

Sol. Let the present ages of B and A be x years and y Year respectively. Then

$B's$ age 5 years ago = $(x-5)$ years

And $A's$ age 5 years ago = $(y-5)$

$\therefore \quad (y-5) = 3\,(x-5) \quad \Rightarrow \quad 3x - y = 10 \quad \ldots(i)$

$B's$ age 10 years hence = $(x+10)$ years

$A's$ age 10 years hence = $(y+10)$ years

$\therefore \quad y + 10 = 2\,(x+10) \quad \Rightarrow \quad 2x - y = -10 \quad \ldots(ii)$

On subtracting (ii) from (i) we get $x = 20$

Putting $x = 20$ and (i) we get

$(3 \times 20) - y = 10 \quad \Rightarrow \quad y = 50$

$\therefore \quad x = 20$ and $y = 50$

Hence, $B's$ present age = 20 years and $A's$ present age = 50 Years.

Que 11. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the numerator be x and denominator be y.

$\therefore \quad$ Fraction = $\frac{x}{y}$

Now, according to question,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - 3 = y$

$\therefore \quad 3x - y = 3 \quad \ldots(i)$

And $\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow \quad 4x = y + 8$

$\therefore \quad 4x - y = 8 \quad \ldots(ii)$

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{rrr} 3x- & y= & 3 \\ 4x- & y= & 8 \\ - & + & - \\ \hline & -x= & -5 \end{array}$$

$\therefore \quad x = 5$

Putting the value of x in equation (i), we have

$$3\ \times 5 - y = 3 \quad \Rightarrow \quad 15 - y = 3 \quad \Rightarrow \quad 15 - 3 = y$$

$\therefore \quad y = 12$

Hence, the required fraction is $\frac{5}{12}$.

Que 12. Solve the following pairs of equations by reducing them to a pair of linear equations:

$(\boldsymbol{i})$ $\frac{7x-2y}{xy} = \mathbf{5}$ $\qquad$ $(\boldsymbol{ii})$ $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{8x+7y}{xy} = \mathbf{15}$ $\qquad$ $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Sol. (i) We have

$$\frac{7x-2y}{xy} = 5 \quad \Rightarrow \quad \frac{7x}{xy} - \frac{2y}{xy} = 5 \quad \Rightarrow \quad \frac{7}{y} - \frac{2}{x} = 5$$

Let $\frac{1}{y} = u$ $\quad$ *and* $\quad$ $\frac{1}{x} = v$

$$7u - 2v = 5$$

$$8u + 7v = 15$$

Multiplying (i) by 7 and (ii) by 2 and adding, we have

$$\begin{aligned} 49u - 14v &= 35 \\ 16u + 14v &= 30 \\ \hline 65u &= 65 \end{aligned}$$

$\therefore \quad u = \frac{65}{65} = 1$

Putting the value of u in equation (i), we have

$$7\ \times 1 - 2u = 5 \quad \Rightarrow \quad -2v = 5 - 7 = \ -2$$

$\therefore \quad -2v = \ -2 \quad \Rightarrow \quad v = \frac{-2}{-2} = 1$

Here $\quad u = 1 \quad \Rightarrow \quad \frac{1}{y} = 1 \quad \Rightarrow \quad y = 1$

And $\quad v = 1 \quad \Rightarrow \quad \frac{1}{x} = 1 \quad \Rightarrow \quad x = 1$

Hence, the solution of given system of equations is $x = 1,\ y = 1$.

(ii) $\quad$ We have, $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$$

Let $\frac{1}{3x+y} = u$ and $\frac{1}{3x-y} = v$

We have, $u + v = \frac{3}{4}$ $\quad\ldots(i)$

$$\frac{u}{2} - \frac{u}{2} = -\frac{1}{8} \quad \Rightarrow \quad \frac{u-v}{2} = -\frac{1}{8}$$

$\Rightarrow$ $u - v = -\frac{2}{8} = -\frac{1}{4}$

$\therefore$ $u - v = -\frac{1}{4}$

Adding (i) and (ii), we have

$$u + v = \frac{3}{4}$$

$$u - v = -\frac{1}{4}$$

$$2u = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4}$$

$\Rightarrow$ $u = \frac{2}{4 \times 2} = \frac{1}{4}$ $\qquad \therefore \quad u = \frac{1}{4}$

Now putting the value of u in equation (i), we have

$$\frac{1}{4} + v = \frac{3}{4} \quad \Rightarrow \quad v = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \quad \Rightarrow \quad v = \frac{1}{2}$$

Here, $v = \frac{1}{4}$ $\quad \Rightarrow \quad \frac{1}{3x+y} = \frac{1}{4} \quad \Rightarrow \quad 3x + y = 4$ $\quad\ldots(iii)$

And $v = \frac{1}{2}$ $\quad \Rightarrow \quad \frac{1}{3x+y} = \frac{1}{2} \quad \Rightarrow \quad 3x - y = 2$

$\ldots(iv)$

Now, adding (iii) and (iv), we have

$$\begin{array}{l} 3x + y = 2 \\ 3x - y = 2 \\ \hline 6x = 6 \end{array}$$

$\therefore$ $x = \frac{6}{6} = 1$

Putting the value of x in equation (iii), we have

$$3 \times 1 + y = 4$$

$\Rightarrow$ $y = 4 - 3 = 1$

Hence, the solution of given system of equation is $x = 1,\ y = 1$

Long Answer Type Questions

[4 marks]

Que 1. From the pair of linear equations in this problem, and find its solution graphically: 10 students of class X took part in a Mathematics quiz. If the girls is 4 more than the number of boys, find the number of boy and girls who took part in the quiz.

Sol. Let x be the number of girls and y be the number of boys.

According to questions, we have

$$x = y + 4$$

$$\Rightarrow \quad x - y = 4$$

Again, total number of student = 10

Therefore, $x + y = 10$

Hence, we have following system of equations

$$x - y = 4$$

And $x + y = 10$

From equation (i), we have the following system of equations

x	0	4	7
y	-4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have

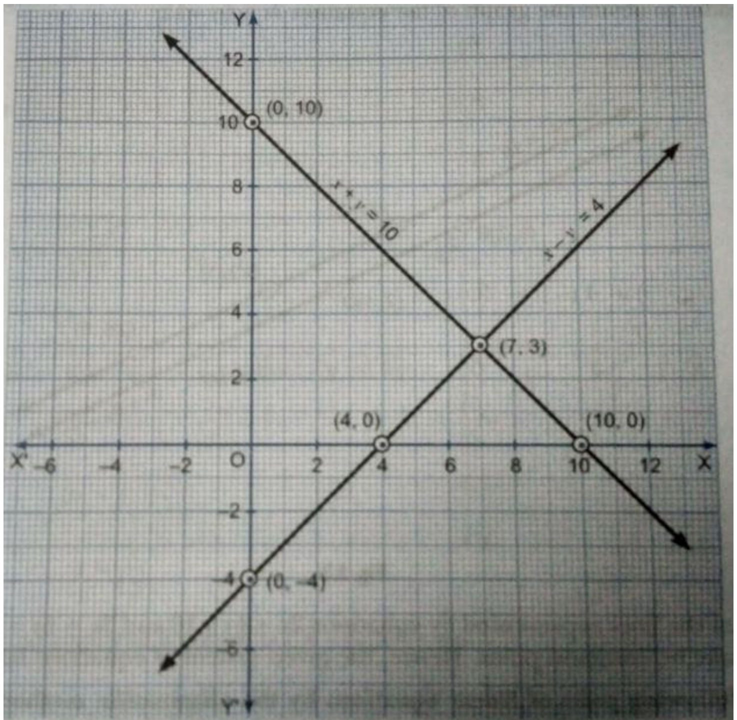


Fig. 3.2

Here, the two lines intersect at point (7,3) $i.e, x = 7, y = 3$.

So, the number of girls = 7

And number of boys = 3

Que 2. Show graphically the given system of equations

$\mathbf{2x + 4y = 10}$ **And** $\mathbf{3x + 6y = 12}$ **Has no solution.**

Sol. We have, $2x + 4y = 10$

$\Rightarrow \quad 4y = 10 - 2x \quad \Rightarrow \quad y = \frac{5-x}{2}$

Thus, we have he following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5, 0) on the graph paper. Join A, B and C and extend it on both sides to obtain the equation $2x + 4y = 10$.

We have, $3x + 6y = 12$

$\Rightarrow \quad 6y = 12 - 3x \quad \Rightarrow \quad y = \frac{5-x}{2}$

Thus, we have the following table:

x	2	0	4
y	1	2	0

Plot the points $D(2, 1)$, $E(0, 2)$ and $F(4, 0)$ on the same graph paper. Join D, E and F and extend on both sides to obtain the graph of the equation $3x + 6y = 12$.

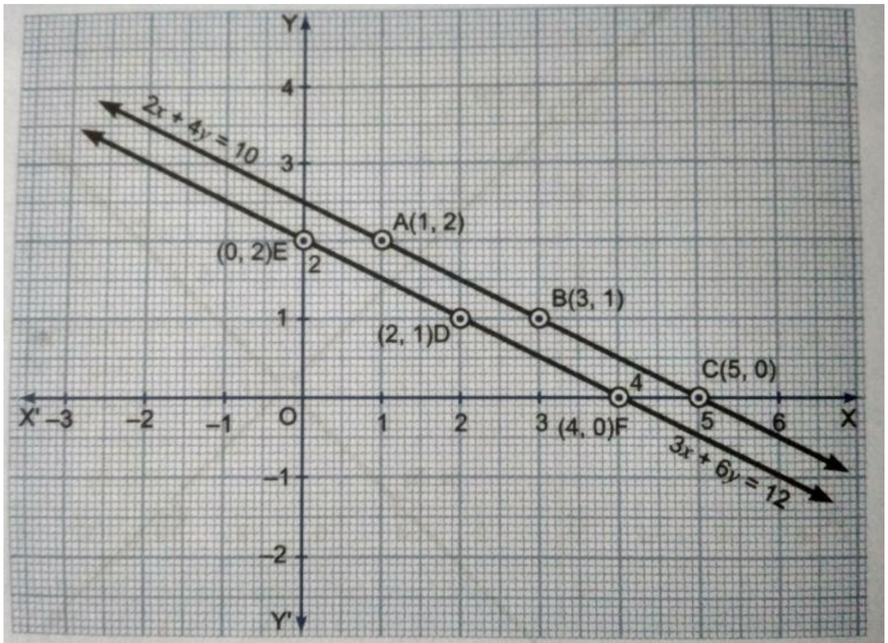


Fig. 3.3

We find that the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So the two lines have no common point. Hence, the given system of equations has no solution.

Que 3. Solve the following pairs of linear equations by the elimination method and the substitution method:

(i) $\mathbf{3x - 5y - 4 = 0}$ **And** $\mathbf{9x = 2y + 7}$

(ii) $\mathbf{\frac{x}{2} + \frac{2y}{3} = -1}$ **And** $\mathbf{x - \frac{y}{3} = 3}$

Sol. (i) we have, $3x - 5y - 4 = 0$

$\Rightarrow$ $3x - 5y = 4$ $\quad \ldots(i)$

Again, $9x = 2y + 7$

$\Rightarrow$ $9x - 2y = 7$ $\quad \ldots(ii)$

By Elimination Method:

Multiplying equation (i) by 3, we get

$$9x - 15y = 12 \quad \ldots(iii)$$

Subtracting (ii) from (iii), we get

$$\begin{array}{rcrcr} 9x & - & 15y & = & 12 \\ 9x & - & 2y & = & 7 \\ - & & + & & - \\ \hline & & -13y & = & 5 \end{array}$$

$\Rightarrow \qquad y = -\frac{5}{13}$

Putting the value of y in equation (ii), we have

$$9x - 2\left(-\frac{5}{13}\right) = 7 \quad \Rightarrow \quad 9x + \frac{10}{13} = 7 \quad \Rightarrow \quad 9x = 7 - \frac{10}{13}$$

$\Rightarrow \qquad 9x = \frac{91-10}{13} \quad \Rightarrow \quad 9x = \frac{81}{13} \quad \Rightarrow \quad x = \frac{9}{13}$

Hence, the required solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$x = \frac{4+5y}{3}$$

Substituting the value of x in equation (ii), we have

$$9 \times \left(\frac{4+5y}{3}\right) - 2y = 7$$

$\Rightarrow \qquad 3 \times (4+5y) - 2y = 7$

$\Rightarrow \qquad 12 + 15y - 2y = 7 \quad \Rightarrow \quad 13y = 7 - 12$

$\therefore \qquad y = -\frac{5}{13}$

Putting the value of y in equation (i), we have

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \quad \Rightarrow \quad 3x + \frac{25}{13} = 4$$

$\Rightarrow \qquad 3x = 4 - \frac{25}{13} \quad \Rightarrow \quad 3x = \frac{27}{13}$

$\therefore \qquad x = \frac{9}{13}$

Hence, the required solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

(ii) We have, $\frac{x}{2} + \frac{2y}{3} = -1 \quad \Rightarrow \quad \frac{3x+4y}{6} = -1$

$\therefore \qquad 3x + 4y = -6 \qquad \ldots(i)$

And $x - \frac{y}{3} = 3$ $\Rightarrow$ $\frac{3x-y}{3} = 3$

$\therefore$ $3x - y = 9$ $\ldots(ii)$

By Elimination Method:

Subtracting (ii) from (i), we have

$5y = -15$ or $y = -\frac{15}{5} = -3$

Putting the value of y in equation (i), we have

$3x + 4 \times (-3) = -6$ $\Rightarrow$ $3x - 12 = -6$

$\therefore$ $3x = -6 + 12$ $\Rightarrow$ $3x = 6$

Hence, Solution is $x = 2, y = -3$.

By Elimination Method:

Expressing x in terms of y from equation (i), we have

$x = \frac{-6-4y}{3}$

Substituting the value of x in from equation (i), we have

$3 \times \left(\frac{-6-4y}{3}\right) - y = 9$ $\Rightarrow$ $-6 - 4y - y = 9$ $\Rightarrow$ $-6 - 5y = 9$

$\therefore$ $-5y = 9 + 6 = 15$

$\therefore$ $y = \frac{15}{-5} = -3$

Putting the value of y in equation (i), we have

$3x + 4 \times (-3) = -6$ $\Rightarrow$ $3x - 12 = -6$

$\therefore$ $3x = 12 - 6 = 6$ $\therefore$ $x = \frac{6}{3} = 2$

Hence, the required solution is $x = 2, y = -3$.

Que 4. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the tringle formed by these lines and the x-axis, and shade the triangular region.

Sol. We have, $x - y + 1 = 0$ and $3x + 2y - 12 = 0$

Thus, $x - y = -1$ $\Rightarrow$ $x = y - 1$ $\ldots(i)$

$3x + 2y = 12$ $\Rightarrow$ $x = \frac{12-12y}{3}$ $\ldots(ii)$

From equation (i), we have

x	-1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have

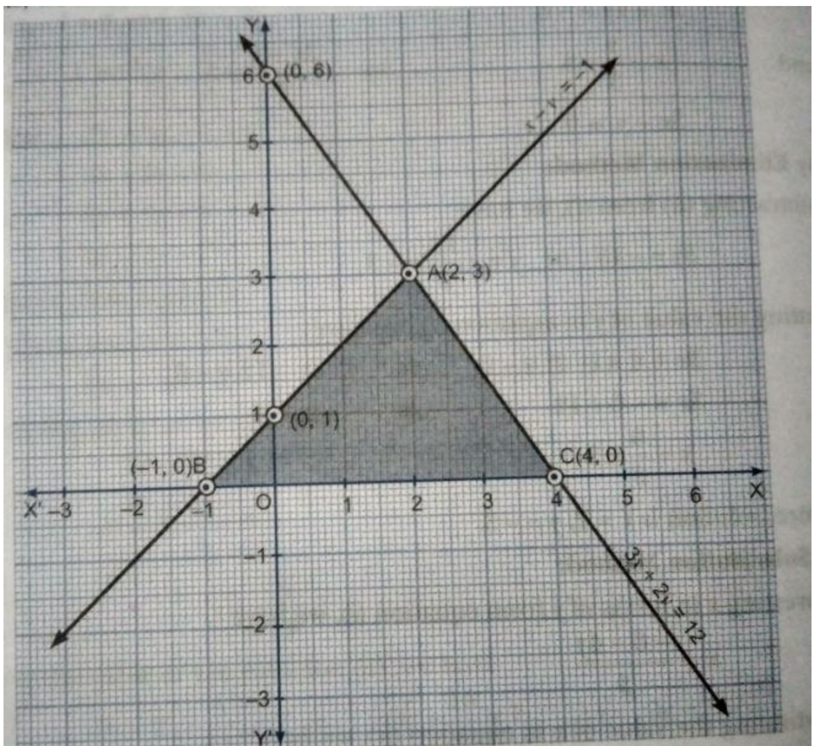


Fig. 3.4

ABC is the required (shaded) region and point of intersection is (2,3).

∴ The vertices of the tringle are (-1, 0), (4, 0), (2, 3).

From the pair of linear equations in the following problem and find their solutions (if they exist by any algebraic method (Q.5 to 8) :

Que 5. A Part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay 1000 as hostel charges whereas a student B, who takes food for 26 days, pays `1180 as hostel charges. Find the fixed charges and the cost of food per day.

Sol. Let the fixed charge be ₹ x and the cost of food per day be ₹ y.

Therefore, according to question,

$x + 20y = 1000$ …(i)

$x + 26y = 1180$ …(ii)

Now, subtracting equation (ii) from (i), we have

$x + 20y = 1000$

$$\underline{\ _{-}x + _{-}26y = _{-}1180}$$

$$-6y = -180$$

$$y = \frac{-180}{-6} = 30$$

Putting the value of y in equation (i), we have

$x + 20 \times 30 = 1000 \Rightarrow x + 600 = 1000 \Rightarrow x = 1000 - 600 = 400$

Hence, fixed charge is `400 and cost of food per day is ₹30.

Que 6. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deduced for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Sol. Let x be the number of questions of right answer and y be the number of questions of wrong answer.

∴ According to question,

$3x - y = 40$ …(i)

and $4x - 2y = 50$

or $2x - y = 25$ …(ii)

Subtracting (ii) from (i), we have

$3x - y = 40$

$$\underline{\ _{-}2x \underset{+}{-} y = _{-}25}$$

$$x = 15$$

Putting the value of x in equation (i), we have

3x 15 – y = 40 ⇒ 45 – y = 40

∴ y = 45 – 40 = 5

Hence, total number of question is x + y i. e., 5 + 15 = 20.

Que 7. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Sol. Let the speed of two cars be x km/h and y km/h respectively.

Case I: When two cars move in the same direction, they will meet each other at P after 5 hours.

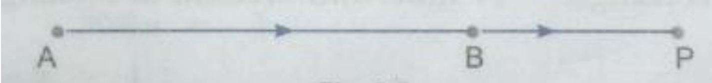


Fig. 3.5

The distance covered by car from A = 5x (Distance = Speed × Time)
and distance covered by the car from B = 5y

$\therefore \quad 5x - 5y = AB = 100 \quad \Rightarrow \quad x - y = \frac{100}{5}$

$\therefore \quad x - y = 20 \qquad \ldots (i)$

Case II: When two cars move in opposite direction, they will meet each other at Q after one hour.

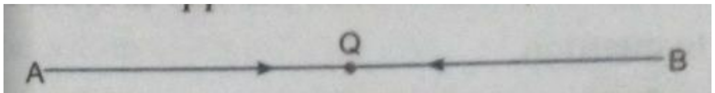


Fig. 3.6

The distance covered by the car from A = x
The distance covered by the car from B = y
$\therefore \quad x + y = AB = 100 \quad \Rightarrow \quad x + y = 100 \qquad \ldots (ii)$
Now, adding equations (i) and (ii), we have
$2x = 120 \quad \Rightarrow \quad x = \frac{120}{2} = 60$

Putting the value of x in equation (i), we get
$60 - y = 20 \quad \Rightarrow \quad -y = -40 \quad \therefore \quad y = 40$
Hence, the speeds of two cars are 60 km/h and 40 km/h respectively.

Que 8. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol. Let the length and breadth of a rectangle be x and y respectively.
Then area of the rectangle = xy
According to question, we have
$(x - 5)(y + 3) = xy - 9 \quad \Rightarrow \quad xy + 3x - 5y - 15 = xy - 9$

$\Rightarrow \quad 3x - 5y = 15 - 9 = 6 \quad \Rightarrow \quad 3x - 5y = 6 \qquad \ldots (i)$

Again, we have
$(x - 3)(y + 2) = xy + 67 \quad \Rightarrow \quad xy + 2x + 3y + 6 = xy + 67$

$\Rightarrow \quad 2x - 3y = 67 - 6 = 61 \quad \Rightarrow \quad 2x - 3y = 61 \qquad \ldots (ii)$

Now, from equation (i), we express the value of x in terms of y.

$$x = \frac{6+5y}{3}$$

Substituting the value of x in equation (ii), we have

$$2 \times \left(\frac{6+5y}{3}\right) + 3y = 61 \quad \Rightarrow \quad \frac{12+10y+9y}{3} = 61$$

$$\Rightarrow 19y = 183 - 12 = 171 \quad \Rightarrow \quad y = \frac{171}{19} = 9$$

Putting the value of y in equation (i), we have

$$3x - 5 \times 9 = 6 \quad \Rightarrow \quad 3x = 6 + 45 = 51$$

$$\therefore \quad x = \frac{51}{3} = 17$$

Hence, the length of rectangle = 17 units and breadth of rectangle = 9 units.

Que 9. Formulate the following problems as a pair of equations, and hence find their solutions:
(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find he speed of rowing in still water and the speed of the current.
(ii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by bus and the remaining by train. If she travels 100 km by bus and the remaining by train, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Sol. (i) Let her speed of rowing in still water be x km/h and the speed of the current be y km/h

Case I: When Ritu rows downstream

Her speed (downstream) = $(x + y)$ km/h

Now, We have speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow \quad (x + y) = \frac{20}{2} = 10$$

$$\therefore \quad x + y = 10 \qquad \dots (i)$$

Case II: When Ritu rows upstream

Her speed (upstream) = $(x - y)$ km/h

Again, Speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow \quad x - y = \frac{4}{2} = 2$$

$$\therefore \quad x - y = 2 \qquad \dots (ii)$$

Now, adding (i) and (ii), we have

$$2x = 12 \quad \Rightarrow \quad x = \frac{12}{2} = 6$$

Putting the value of x in equation (i), we have

$$6 + y = 10 \quad \Rightarrow \quad y = 10 - 6 = 4$$

Hence, speed of Ritu in still water = 6 km/h. and speed of current = 4 km/h.

(ii) Let the speed of the bus be x km/h and speed of the train be y km/h.

According to question, we have

$$\frac{60}{x} + \frac{240}{y} = 4$$

And $\frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60} = 4 + \frac{1}{6} = \frac{25}{6} \Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$

Now, let $\frac{1}{x} = u$ and $\frac{1}{y} = u$,

$\therefore \quad 60u + 240v = 4 \quad \ldots (i)$

$100u + 200v = \frac{25}{6} \quad \ldots (ii)$

Multiplying equation (i) by 5 and (ii) by 6 and subtracting, we have

$$\begin{array}{rcrcr} 300u & + & 1200v & = & 20 \\ \underset{-}{}600u & \overset{+}{-} & 1200v & = & \underset{-}{}25 \\ \hline & & -300u & = & -5 \end{array}$$

$\therefore \quad u = \frac{-5}{-300} = \frac{1}{60}$

Putting the value of u in equation (i), we have

$$60 \text{ x } \frac{1}{60} + 240v = 4 \quad \Rightarrow \quad 240v = 4 - 1 = 3$$

$\therefore \quad v = \frac{3}{240} = \frac{1}{80}$

Now, $u = \frac{1}{60} \quad \Rightarrow \quad \frac{1}{x} = \frac{1}{60} \quad \therefore \quad x = 60$

And $v = \frac{1}{80} \quad \Rightarrow \quad \frac{1}{y} = \frac{1}{80} \quad \therefore \quad y = 80$

Hence, speed of the bus is 60 km/h and speed of the train is 80 km/h.

Que 10. The sum of a two digit number and the number formed by interchanging its digits is 110.
If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

Sol. Let the digits at unit and tens places be x and y respectively.

Then, number = 10y + x … (i)

Number formed by interchanging the digits = 10x + y

According to the given condition, we have

(10y + x) + (10x+y) = 110 ⇒ 11x + 11y = 110 ⇒ x + y – 10 = 0

Again, according to question, we have

(1Oy + x) – 10 = 5 (x + y) + 4 ⇒ 10y + x – 10 = 5x + 5y + 4

$$\Rightarrow 10y + x - 5x - 5y = 4 + 10$$

$$5y - 4x = 14 \quad \text{Or} \quad 4x - 5y + 14 = 0$$

By using cross-multiplication, we have

$$\frac{x}{1 \text{ x } 14 - (-5) \text{ x } (-10)} = \frac{-y}{1 \text{ x } 14 - 4 \text{ x } (-10)} = \frac{1}{1 \text{ x } (-5) - 1 \text{ x } 4}$$

$$\Rightarrow \frac{x}{14-50} = \frac{-y}{14+40} = \frac{1}{-5-4} \Rightarrow \frac{x}{-36} = \frac{-y}{54} = \frac{1}{-9}$$

$$\Rightarrow x = \frac{-36}{-9} \text{ and } y = \frac{-54}{-9} \Rightarrow x = 4 \text{ and } y = 6$$

Putting the values of x and y in equation (i), we get Number = 10 x 6 + 4 = 64.

Que 11. Jamila sold a table and a chair for ₹1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹1065. Find cost price of each.

Sol. Let cost price of table be ₹x and the cost price of the chair be ₹y.

The selling price of the table, when it is sold at profit of 10% = ₹ $\left(x + \frac{10x}{100}\right) = \frac{110x}{100}$

The selling price of the chair when it is sold at a profit of 25% = ₹ $\left(y + \frac{25y}{100}\right) = \frac{125y}{100}$

So, $\frac{110x}{100} + \frac{125y}{100} = 1050$ … (i)

When the table is sold at a profit of 25%

Its selling price = ₹$\left(x + \frac{25}{100}x\right)$ = ₹ $\frac{125}{100}x$

When the chair is sold at a profit of 10%

its selling price = ₹ $\left(y + \frac{10y}{100}\right)$ = ₹$\frac{110y}{100}$

So, $\frac{125}{100}x + \frac{110y}{100} = 1065$ … (ii)

Form equation (i) and (ii) we get

$110x + 125y = 105000$

and $125x + 110y = 106500$

On adding and subtracting these equations we get

$235x + 235y = 211500$

and $15x - 15y = 1500$

$i.e.,$ $x + y = 900$ … (iii)

$x - y = 100$ … (iv)

Solving equation (iii) and (iv) we get

$x = 500,\ y = 400$

So, the cost price of the table is ₹ 500 and the cost price of the chair is ₹ 400.

HOTS (Higher Order Thinking Skills)

Que 1. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Sol. Let one men alone can finish the work in x days and one boy alone can finish the work in y days. Then,

One day work of one man = $\frac{1}{x}$, One day work of one boy = $\frac{1}{y}$

∴ One day work of 8 men = $\frac{8}{x}$, One day work of 12 boys = $\frac{12}{y}$,

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x}+\frac{12}{y}\right)=1 \quad \Rightarrow \quad \frac{80}{x}+\frac{120}{y}=1 \qquad \text{....(i)}$$

Again, 6 men and 8 boys can finish the work in 14 days

$$\therefore \quad 14\left(\frac{6}{x}+\frac{8}{y}\right)=1 \quad \Rightarrow \quad \frac{84}{x}+\frac{112}{y}=1 \qquad \text{.....(ii)}$$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equation (i) and (ii), we get

80u + 120v – 1 = 0 and 84u + 112v – 1 = 0

By using cross-multiplication, we have

$$\frac{u}{120\times-1-112\times-1}=\frac{-v}{80\times-1-84\times-1}=\frac{1}{80\times112-84\times120}$$

$$\Rightarrow \quad \frac{u}{-120+112}=\frac{-v}{-80+84}=\frac{1}{80\times112-84\times120}$$

$$\Rightarrow \quad \frac{u}{-8}=\frac{-v}{4}=\frac{1}{-1120}$$

$$\Rightarrow \quad u=\frac{-8}{-1120}=\frac{1}{140} \quad \text{and} \quad v=\frac{-4}{-1120}=\frac{1}{280}$$

We have, $u=\frac{1}{140} \Rightarrow \frac{1}{x}=\frac{1}{140} \Rightarrow x=140$

and $v=\frac{1}{280} \Rightarrow \frac{1}{y}=\frac{1}{280} \Rightarrow y=280.$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

Que 2. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Sol. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream = (x – y) km/h

Speed downstream = (x + y) km/h

Now, time taken to cover 25 km upstream = $\frac{25}{x-y} hours$

Time taken to cover 44 km downstream = $\frac{44}{x+y} hours$

The total time of journey is 9 hours

∴ $$\frac{25}{x-y} + \frac{44}{x+y} = 9 \qquad \text{....(i)}$$

Time taken to cover 15 km upstream = $\frac{15}{x-y}$

Time taken to cover 22 km downstream = $\frac{22}{x+y}$

In this case, total time of journey is 5 hours.

∴ $$\frac{15}{x-y} + \frac{22}{x+y} = 5 \qquad \text{....(ii)}$$

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

25u + 44v = 9 ⇒ 25u + 44v – 9 = 0 ....(iii)

15u + 22v = 5 ⇒ 15u + 22v – 5 = 0 ....(iv)

By cross-multiplication, we have

$$\frac{u}{44\times(-5)-22\times(-9)} = \frac{-v}{25\times(-5)-15\times(-9)} = \frac{1}{25\times22-15\times44}$$

$$\Rightarrow \quad \frac{u}{-220+198} = \frac{-v}{-125+135} = \frac{1}{550-660}$$

$$\Rightarrow \quad \frac{u}{-22} = \frac{-v}{10} = \frac{1}{-110} \quad \Rightarrow \quad \frac{u}{22} = \frac{v}{10} = \frac{1}{110}$$

$$\Rightarrow \quad \frac{u}{22} = \frac{1}{110} \quad \text{and} \quad \frac{v}{10} = \frac{1}{110}$$

$$\Rightarrow \quad u = \frac{22}{110} = \frac{1}{5} \quad \text{and} \quad v = \frac{1}{11}$$

We have, $u = \frac{1}{5}$ ⇒ $\frac{1}{x-y} = \frac{1}{5}$ ⇒ $x - y = 5$ ....(v)

and $v = \frac{1}{11}$ ⇒ $\frac{1}{x+y} = \frac{1}{11}$ ⇒ $x + y = 11$ ....(vi)

Solving equations (v) and (vi), we get x = 8 and y = 3.

Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Que 3. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Sol. Let total number of roes be y

and total number of students in each row be x.

∴ Total number of students = xy

Case I: If one student is extra in a row, there would be two rows less.

Now, number of rows = (y – 2)

Number of students in each row = (x + 1)

Total number of students = Number of rows × Number of students in each row

$xy = (y - 2)(x + 1) \Rightarrow xy = xy + y - 2x - 2$

$\Rightarrow xy - xy - y + 2x = -2 \Rightarrow 2x - y = -2$ …(i)

Case II: If one student is less in a row, there would be 3 rows more.

Now, number of rows = (y + 3)

and number of students in each row = (x – 1)

Total number of students = Number of rows × Number of students in each row

$\therefore xy = (y + 3)(x - 1) \Rightarrow xy = xy - y + 3x - 3$

$xy - xy + y - 3x = -3 \Rightarrow -3x + y = -3$ ….(ii)

On adding equation (i) and (ii), we have

$$\begin{array}{r} 2x - y = -2 \\ -3x + y = -3 \\ \hline -x = -5 \end{array}$$

Or $x = 5$

putting the value of x in equation (i), we get

$2(5) - y = -2 \Rightarrow 10 - y = -2$

$-y = -2 - 10 \Rightarrow -y = -12$

or $y = 12$

∴ Total number of students in the class = 5 × 12 = 60.

Que 4. Draw the graph of 2x + y = 6 and 2x – y + 2 = 0. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.

Sol. We have, $2x + y = 6 \Rightarrow y = 6 - 2x$

when x = 0, we have $y = 6 - 2 \times 0 = 6$

when x = 3, we have $y = 6 - 2 \times 3 = 0$

when x = 2, we have $y = 6 - 2 \times 2 = 2$

Thus, we get the following table:

x	0	3	2
y	6	0	2

Now, we plot the points A (0, 6), B (3, 0) and C (2, 2) on the graph paper. We join A, B and C and extend it on both sides to obtain the graph of the equation 2x + y = 6.

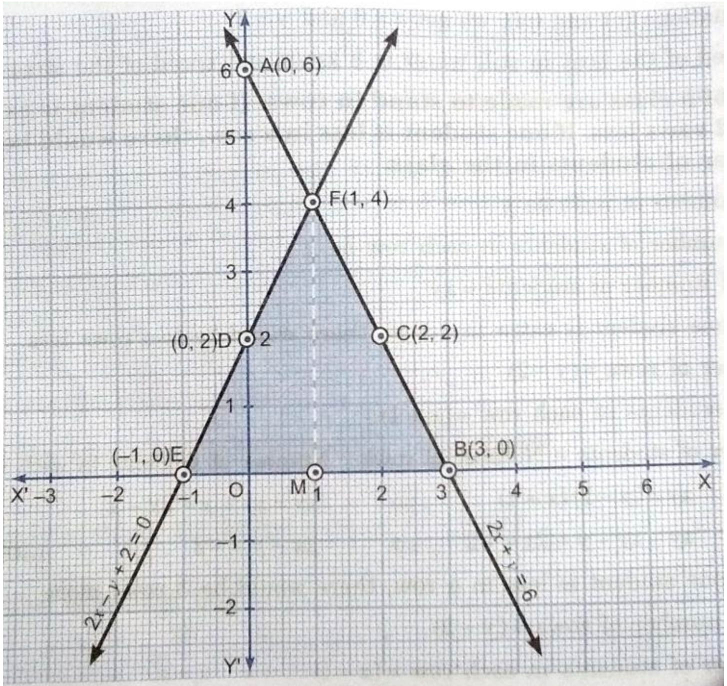


Fig. 3.7

We have, 2x – y + 2 = 0 ⇒ $y = 2x + 2$

When x = 0, we have y = 2 × 0 + 2 = 2

When x = – 1, we have y = 2 × (– 1) + 2 = 0

When x = 1, we have y = 2 × 1 + 2 = 4

Thus, we have the following table:

x	0	– 1	1
y	2	0	4

Now, we plot the points D (0, 2), E (– 1, 0) and F (1, 4) on the same graph paper. We join D, E and F and extend it on both sides to obtain the graph of the equation 2x – y + 2 = 0. It is evident from the graph that the two lines intersect at point F (1, 4). The area enclosed by the given lines and x-axis is shown in Fig. 3.7.

Thus, x = 1, y = 4 is the solution of the given system of equations. Draw FM perpendicular from F on x-axis.

Clearly, we have

FM = y-coordinates of point F (1, 4) = 4 and BE = 4

∴ Area of the shaded region = Area of ΔFBE

⇒ Area of the shaded region = $\frac{1}{2}(Base \times Height) = \frac{1}{2}(BE \times FM)$

$$= \left(\frac{1}{2} \times 4 \times 4\right) sq.units = 8\ sq.units.$$

Que 5. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol. Let the ages of Ani and Biju be x and y years respectively. Then x – y = ± 3

Age of Dharam = 2x years

Age of Cathy = $\frac{y}{2} years$

Clearly, Dharam is older than Cathy.

$\therefore \quad 2x - \frac{y}{20} = 30 \quad \Rightarrow \quad \frac{4x-y}{2} = 30 \quad \Rightarrow \quad 4x - y = 60$

Thus, we have following two systems of linear equations

$$x - y = 3 \qquad \text{....(i)}$$
$$4x - y = 60 \qquad \text{....(ii)}$$

and

$$x - y = -3 \qquad \text{....(iii)}$$
$$4x - y = 60 \qquad \text{....(iv)}$$

Subtracting equation (i) from (ii), we get

$$\begin{array}{r} 4x - y = 60 \\ \underset{-}{x} \underset{+}{-} y = \underset{-}{3} \\ \hline 3x = 57 \end{array} \quad \Rightarrow \quad x = 19$$

Putting $x = 19$ in equation (i), we get

$$19 - y = 3 \quad \Rightarrow \quad y = 16$$

Now, subtracting equation (iii) from (iv)

$$\begin{array}{r} 4x - y = 60 \\ \underset{-}{x} \underset{+}{-} y = \underset{+}{-} 3 \\ \hline 3x = 63 \end{array} \quad \Rightarrow \quad x = 21$$

Putting x = 21 in equation (ii), we get

$$21 - y = -3 \qquad y = 24$$

Hence, age of Ani = 19 years and age of Biju = 16 years or age of Ani = 21 years and age of Biju = 24 years

Value Based Questions

Que 1. Some people collected money to be donated in some orphanages. A part of the donation is fixed and remaining depends on the number of children in the orphanage. If they donated ₹ 9,500 in the orphanage having 50 children and ₹ 13,250 with 75 children, find the fixed part of the donation and the amount donated for each child. What values do these people posses?

Sol. Let the fixed donation be ₹ x and amount donated for each child be ₹ y.

Then $x + 50y = 9500$ …(i)

$x + 75y = 13250$ …(ii)

Subtracting (i) from (ii), we get

$25y = 3750$ or $y = 150$

From (i), $x = 9500 - 50y = 9500 - 50 \times 150 = 2000$

∴ Fixed amount donated = ₹ 2,000.

Amount donated for each child = ₹ 150.

Helpfulness, cooperation, happiness, caring.

Que 2. The fraction of people in a society using CNG in their vehicles becomes $\frac{9}{11}$, if 2 is added to both its numerator and denominator. If 3 is added to both its numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction. What does this fraction show?

Sol. Let the fraction be $\frac{x}{y}$.

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \Rightarrow \quad 11x - 9y = -4$$

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \Rightarrow \quad 6x - 5y = -3$$

Solving for x and y gives x = 7 and y = 9

$$\therefore \quad Fraction = \frac{x}{y} = \frac{7}{9}$$

More people are becoming aware about scarcity of petrol, so they are switching on to alternative resources.

Que 3. Reading book in a library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days. While Bunty paid ₹ 21 for the book kept for five days.

(i) Find the fixed charge.
(ii) Find how much additional charge Shristi and Bunty paid.
(iii) Which mathematical concept is used in this problem?
(iv) Which value does it depict?

Sol. Let fix charge for reading book be ₹ x and additional charge for each day be ₹ y.

Then, $x + 4y = 27$ …(i)

On subtraction $\frac{\underline{\underline{x +}} 2y = 21}{2y = 6}$ ….(ii)

Y = 3

Putting value of y in equation (i) we get

$x + 4 \times 3 = 27 \quad \Rightarrow \quad x + 12 = 27 \quad \Rightarrow \quad x = 15$

(i) Fixed charge on the book is ₹ 15.

(ii) Bunty paid additional ₹ 6 and Shristi paid ₹ 12.

(iii) Pair of linear equation in two variables.

(iv) Reading is a good habit.

Que 4. An honest person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But if he had interchanged amount invested, he would have received ₹ 4 more as interest.

(i) How much amount did he invest at different rates?

(ii) Which mathematical concept is used in this problem?

(iii) Which value is being emphasized here?

Sol. (i) Let the person invest ₹ x at rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

Yearly interest = $\frac{12x}{100} + \frac{10y}{100}$

$$\therefore \quad \frac{12x}{100} + \frac{10y}{100} = 130 \quad \Rightarrow \quad 12x + 10y = 13000 \qquad \ldots(i)$$

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$$\frac{10x}{100} + \frac{12y}{100} = 134 \quad \Rightarrow \quad 10x + 12y = 13400 \qquad \ldots(ii)$$

Adding eqn. (i) and (ii) we get

$22x + 22y = 26400$

$x + y = 1200$ …(iii)

Subtracting (ii) from (i) we get

$2x – 2y = – 400$

$x – y = – 200$ …(iv)

Solving (iii) and (iv), we get

$x = 500, y = 700$

Thus, person invested ₹ 500 at 12% per annum and ₹ 700 at 10% per annum.

(ii) Pair of linear equation in two variables.

(iii) Honesty is the best policy.

Que 5. If the price of petrol is increased by ₹ 2 per litre, a person will have to buy 1 litre less petrol for ₹ 1740. Find the original price of petrol at that time.

(a) Why do you think the price of petrol is increasing day-by-day?

(b) What should we do to save petrol?

Sol. Let the original price of the petrol be ₹ x per litre.

Then, amount of petrol that can be purchased = $\frac{1740}{x}$

According to question

$\frac{1740}{x} - \frac{1740}{x+2} = 1 \qquad \Rightarrow \quad 1740\,(x + 2 - x) = x\,(x + 2)$

$\Rightarrow \quad x^2 + 2x - 3480 = 0 \qquad \Rightarrow \quad x^2 + 60x - 58x - 3480 = 0$

$\Rightarrow \quad (x + 60)\,(x - 58) = 0 \qquad \Rightarrow \quad x = 58, -60\ (rejected)$

∴ Original cost of petrol was ₹ 58 per litre.

(a) Petrol is a natural resource which is depleting day-by-day. So, due to more demand and less supply, its price is increasing.

(b) We should use more of public, transport and substitute petrol with CNG or other renewable resource.

Que 6. One fourth of a group of people claim they are creative, twice the square root of the group claim to be caring and the remaining 15 claim they are optimistic. Find the total number of people in the group.

(a) How many persons in the group are creative?

(b) According to you, which one of the above three values is more important for development of a society?

Sol. Let x be the total

Then, number of creative persons = $\frac{x}{4}$

Number of caring persons = $2\sqrt{x}$

and number of optimistic persons = 15

Thus, total number of persons = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$

Let $\sqrt{x} = y, \quad then \quad x = y^2$

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \qquad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y - 6) + 10\,(y - 6) = 0 \qquad \Rightarrow \quad (3y + 10)\,(y - 6) = 0$

$\Rightarrow \quad y = 6 \ or \ y = -\frac{10}{3}$

Now, $\quad y = -\frac{10}{3} \qquad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9} (\because\ x = y^2)$

But, the number of persons cannot be a fraction.

∴ y = 6 $\qquad \Rightarrow \quad x = 6^2 = 36$

Hence, the number of people in the group = 36

(a) 9 persons

(b) All of these values have their own importance. A person having these values will certainly contribute to the development of society. However, the level of importance given to each of them depends upon a person's own attitude. Hence, any value with justification is correct. (Do yourself)

Que 7. In the centre of a rectangular plot of land of dimensions 120 m × 100 m, a rectangular portion is to be covered with trees so that the area of the remaining part of the plot is 10500 m^2. Find the dimensions of the area to be planted. Which social act is being discussed here? Give its advantages.

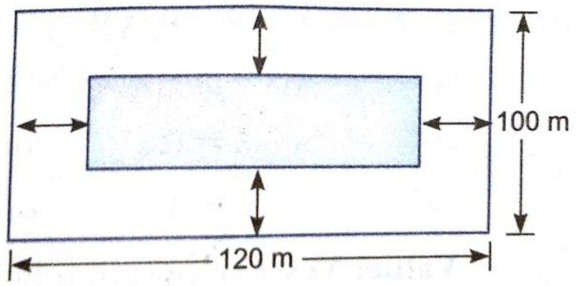


Fig. 1

Sol. Let the width of the unplanted area be x m

Then, dimension of area to be planted = (120 – 2x) and (100 – 2x)

∴ $(120 - 2x)\,(100 - 2x) = 120 \times 100 - 10500$

⇒ $12000 - 440x + 4x^2 = 1500 \; or \; x^2 - 110x + 2625 = 0$

⇒ $(x - 75)\,(x - 35) = 0 \; or \; x = 75, 35$

But x = 75 is not possible

∴ $x = 35$

Thus, dimension of area to be planted = (120 – 70) and (100 – 70) i.e., 50 m and 30 m.
Afforestation is being discussed here. Planting more trees helps in reducing air pollution and make the environment clean and green.

Que 8. Mr. Ahuja has to square plots of land which he utilises for two different purposes–one for providing free education to the children below the age of 14 years and the other to provide free medical services for the needy villagers. The sum of the areas of two square plots is 15425 m^2. If the difference of their perimeter is 60 m, find the sides of the two squares.
Which qualities of Mr. Ahuja are being depicted in the question?

Sol. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 15425$ …(i)

Let x be the length of the side of the bigger square.

$4x - 4y = 60$

⇒ $x - y = 15$ or $x = y + 15$ …(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 15)^2 + y^2 = 15425 \quad \Rightarrow \quad 2y^2 + 30y - 15200 = 0$

or $y^2 + 15y - 7600 = 0$ or $(y + 95)(y - 80) = 0 \quad \Rightarrow \quad y = -95, 80$

But, sides cannot be negative, so y = 80

Therefore, $x = 80 + 15 = 95$

Hence, sides of two squares are 80 m and 95 m.

Value: Caring, King, Social and generous.

Que 9. A takes 3 days longer than B to finish a work. But if they work together, then work is completed in 2 days. How long would each take to do it separately. Can you say cooperation helps to get more efficiency?

Sol. Let B finish a work in x days

then A finish a work in x + 3 days

According to question

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2} \quad \Rightarrow \quad \frac{(x+3)+x}{x(x+3)} = \frac{1}{2}$$

⇒ $2x + 6 + 2x = x^2 + 3x$ ⇒ $x^2 + 3x - 2x - 6 - 2x = 0$

⇒ $x^2 - x - 6 = 0$

Solving the equation

$x^2 - (3 - 2)x - 6 = 0$ ⇒ $x^2 - 3x + 2x - 6 = 0$

⇒ $x(x - 3) + 2(x - 3) = 0$ ⇒ $(x - 3)(x + 2) = 0$

⇒ $x - 3 = 0$ or $x + 2 = 0$

⇒ $x = 3$ or $x = -2$ (Days cannot be negative)

Value: Yes, Cooperation helps in improving work efficiency.

Que 10. In a class of 48 students, the number of regular students is more than the number of irregular students. Had two irregular students been regular, the product of the number of two types of students would be 380. Find the number of each type of students.

(a) Why is regularity essential in life?

(b) Write values other than regularity that a student must possess.

Sol. Let the number of regular students be x

Then, the number of irregular students = 48 – x

According to the question,

$(x + 2)(48 - x - 2) = 380$

⇒ $-x^2 + 44x + 92 - 380 = 0 \ or \ x^2 - 44x + 288 = 0$

⇒ $(x - 36)(x - 8) = 0$ or $x = 36, 8$

But $x > 48 - x$ (given)

∴ $x = 36$

i.e., The number of regular students = 36

and the number of irregular students = 48 – 36 = 12

(a) Regularity in any sphere gives confidence which, in turn, leads to the development of an individual and the society as well.

(b) Honesty, Creativity, Confidence, Punctuality. (You may add more to this list)

Que 11. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?

Sol. Number of plants planted by class 1 = 2 (class 1 × 2 sections) = 2 × 1 × 2 = 4 trees

similarly, number of plants planted by class 2 = 2 (class 2 x 2 sections)

= 2 × 2 × 2 = 8 trees

We now know, a = 4 and d = 4
Number of classes n = 12
Total number of plants planted = Sum of AP = S_{12}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 4 + (12-1)4]$$

$$S^{12} = 6[8 + 44] = 6 \times 52 = 312\ trees$$

Values shown by the students:
(i) Envrionmental friendly
(ii) Social awareness
(iii) Sense of responsibility towards the society.

Que 12. A sum of ₹ 3150 is to be used to give six cash prizes to students of a school for overall academic performance, punctuality, regularity, cleanliness, confidence and creativity. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes.
(a) Which value according to you should be awarded with the maximum amount? Justify your answer.
(b) Can you add more values to the above ones which should be awarded?

Sol. Let the six prizes (1st, 2nd, 3rd …..6th) be a, a – 50, a – 100, a – 150, a – 200, a – 250 respectively
Then a + (a – 50) + (a – 100) + (a – 150) + (a – 200) + (a – 250) = 3150

$$\Rightarrow\ 6a - 750 = 3150\ or\ a = \frac{3900}{6} = 650$$

∴ The value of the 1st, 2nd, 3rd …..6th Prizes are 650, 600, 550, 500, 450, 400 respectively.
(a) Any value with justification is correct.
(b) Many more can be added like, honesty, good habits, friendship, respectively elders, loving youngers,….. etc.

Que 13. A person donates money to a trust working for education of children and women in some village. If the person donates ₹ 5,000 in the first year and his donation increases by ₹ 250 every year, find the amount donated by him in the eighth year and the total amount donated in eight years.
(a) Which mathematical concept is being used here?
(b) Write any two values the person mentioned here possess.
(c) Why do you think education of women is necessary for the development of a society?

Sol. The amount donated by the person each year forms an AP.
Here, a = 5,000, d = 250
We have to find a_8 and S_8

$$a_8 = a + 7d = 5{,}000 + 7 \times 250 = ₹\ 6{,}750$$

$$S_8 = \frac{8}{2}[2a + 7d] = 4(2 \times 5{,}000 + 7 \times 250) = 4 \times 11{,}750 = ₹\ 47{,}000$$

(a) Arithmetic progression.
(b) Socially aware and responsible citizen.
(c) Educating a woman means educating the whole family and an educated family makes development in society.

Que 14. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

Sol. Let the usual speed of plane be x km/h.

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$\Rightarrow \quad [1500(x+250) - 1500x]\,2 = x(x+250)$
$\Rightarrow \quad x^2 + 250x - 750000 = 0$
$\Rightarrow \quad (x+1000)(x-750) = 0 \;\Rightarrow\; x = 750 \text{ or } x = -1000$ (Which is neglected)
∴ Using speed of plane = 750 km/h
Values: Helping others

Que 15. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?
What value is reflected in this question?

Sol. Here a = ₹ 450, d = ₹ 20, n = 12

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20] = 6[1120] = 6720 > 6500$$

∴ Reshma will be able to send her daughter to school
Value: Encouraging efforts for girl education.