Very Short Answer Type Questions

[1 mark]

Que 1. What will be the nature of roots of quadratic equation $2x^2 + 4x – 7 = 0$?

Sol. D = b^2 4ac = 4^2 – 4 x 2 x (-7) = 16 + 56 = 72 > 0

Hence, root of quadratic equation are real and unequal.

Que 2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then find the value of k.

Sol. ∴ $\frac{1}{2}$ is a root of quadratic equation.

∴ It must satisfy the quadratic equation.

$x^2 + kx - \frac{5}{4} = 0$

$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \quad \Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$

$\frac{1+2k-5}{4} = 0 \quad \Rightarrow \quad 2k - 4 = 0$

$\Rightarrow \quad k = 2$

Que 3. If $ax^2 + bx + c = 0$ has equal roots, find the value of c.

Sol. For equal roots D = 0

i.e., $b^2 - 4ac = 0 \quad \Rightarrow \quad b^2 = 4\,ac \quad \Rightarrow \quad c = \frac{b2}{4a}$

Que 4. If a and b are the roots of the equation $x^2 + ax – b = 0$, then find a and b.

Sol. Sum of the roots = $a + b = -\frac{B}{A} = -a$

Product of the roots = $ab = \frac{C}{A} = -b$

$\Rightarrow a + b = -a$ and $ab = -b$

$\Rightarrow 2a = -b$ and $a = -1 \quad \Rightarrow \quad$ b = 2 and a = -1

Que 5. Show that x = -2 is a solution of $3x^2 + 13x + 14 = 0$.

Sol. Put the value of x in the quadratic equation,

LHS = $3x^2 + 13x + 14 = 3(-2)^2 + 13(-2) + 14$

= 12 – 26 + 14 = 0 = RHS

Hence, x = -2 is a solution.

Que 6. Find the discriminant of the quadratic equation $4\sqrt{2}x^2 + 8x + 2\sqrt{2} = 0$.

Sol. $D = b^2 - 4ac = (8)^2 - 4(4\sqrt{2})(2\sqrt{2}) = 64 - 64 = 0$

Short Answer Type Questions – I

[2 marks]

Que 1. State whether the equation (x + 1) (x – 2) + x = 0 has two distinct real roots or not. Justify your answer.

Sol. $(x + 1)(x - 2) + x = 0 \Rightarrow x^2 - x - 2 + x = 0 \Rightarrow x^2 - 2 = 0$

$D = b^2 - 4ac = 0 - 4(1)(-2) = 8 > 0$

∴ Given equation has two distinct real roots.

Que 2. Is 0.3 a root of the equation x^2 -0.9 = 0? Justify.

Sol. ∴ 0.3 is a root of the equation $x^2 - 0.9 = 0$

∴ $x^2 - 0.9 = (0.3)^2 - 0.9 = 0.09 - 0.9 \neq 0$

Hence, 0.3 is not a root of given equation.

Que 3. For what value of k, is 3 a root of the equation 2x^2 + x + k = 0?

Sol. 3 is a root of $2x^2 + x + k = 0$, when

$2(3)^2 + 3 + k = 0$

$\Rightarrow 18 + 3 + k = 0 \Rightarrow k = -21$

Que 4. Find the values of k for which the quadratic equation 9x^2 - 3kx + k = 0 has equal roots.

Sol. For equal roots:

$D = 0 \Rightarrow b^2 - 4ac = 0$

$(-3k)^2 - 4 \times 9 \times k = 0 \Rightarrow 9k^2 = 36k \Rightarrow k = 4$

Que 5. If – 5 is a root of the quadratic equation 2x^2 + px – 15 = 0 and the quadratic equation p(x^2 + x) + k = 0 has equal roots, then find the value of k.

Sol. Since – 5 is a root of the equation $2x^2 + px - 15 = 0$

∴ $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow 50 - 5p - 15 = 0$ or $5p = 35$ or $p = 7$

Again $p(x^2 + x) + k = 0$ or $7x^2 + 7x + k = 0$ has equal roots

∴ $D = 0$

i.e., $b^2 - 4ac = 0$ or $49 - 4 \times 7k = 0 \Rightarrow k = \frac{49}{28} = \frac{7}{4}$

Que 6. Does there exist a quadratic equation whose co-efficients are rational but both of its roots are irrational? Justify your answer.

Sol. Yes, $x^2 - 4x + 1 = 0$ is a quadratic equation with rational co-efficients.

Its roots are $\frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$, which are irrational.

Que 7. Write the set of values of k for which the quadratic equation $2x^2 + kx + 8 = 0$ has real roots.

Sol. For real roots, D ≥ 0

⇒ $b^2 - 4ac \geq 0$ ⇒ $k^2 - 4(2)(8) \geq 0$

⇒ $k^2 - 64 \geq 0$ ⇒ $k^2 \geq 64$ ⇒ $k \geq -8$ and $k \geq 8$

Que 8. Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for x.

Sol. $2x^2 + ax - a^2 = 0$

Here, a = 2, b = a and c = $-a^2$.

Using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ we get,}$$

$$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times -a^2}}{2 \times 2} = \frac{-a \pm \sqrt{9a^2}}{4} = \frac{-a \pm 3a}{4}$$

$$\Rightarrow \quad x = \frac{-a+3a}{4} = \frac{a}{2}, x = \frac{-a+3a}{4} = -a \quad \Rightarrow \quad x = \frac{a}{2}, -a$$

Que 9. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.

Sol. For equal roots; D = 0

$b^2 - 4ac = 0$

i.e., $p^2 - 4 \times 4 \times 3 = 0$ ⇒ $p^2 - 48 = 0$

$p^2 = 48$ ⇒ $p = \pm\sqrt{48}$

$p = 4\sqrt{3}$ or $-4\sqrt{3}$

Que 10. Solve for x: $\sqrt{3}x^2 - 2\sqrt{2x} - 2\sqrt{3} = 0$

Sol. $\sqrt{3}x^2 - 2\sqrt{2x} - 2\sqrt{3} = 0$ ⇒ $\sqrt{3}x^2 - 3\sqrt{2x} + \sqrt{2x} - 2\sqrt{3} = 0$

$\sqrt{3x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6})} = 0$ ⇒ $(\sqrt{3x} + \sqrt{2)}(x - \sqrt{6}) = 0$

⇒ $\sqrt{3x} + \sqrt{2} = 0$ or $x - \sqrt{6} = 0$

⇒ $x = \frac{-\sqrt{2}}{\sqrt{3}}$ or $x = \sqrt{6}$

Que 11. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol. Let us assume the quadratic equation be Ax^2 + Bx + C = 0.

Sum of the roots = - $\frac{B}{A}$

⇒ $\frac{-7}{a} = \frac{2}{3} - 3$ ⇒ $a = 3$

Product of the roots = $\frac{C}{A}$

⇒ $\frac{b}{a} = \frac{2}{3}$ x (-3) ⇒ b = - 6

Que 12. A two digit number is four times the sum of the digit. It is also equal to 3 times the product of digit. Find the number.

Sol. Let the ten's digit be x and unit's digit = y

Number = 10x + y

∴ $10x + y = 4(x + y)$ ⇒ $6x = 3y$ ⇒ $2x = y$

Again $10x + y = 3xy$

$10x + 2x = 3x\,(2x)$ ⇒ $12x = 6x^2$ ⇒ $x = 2$ (rejecting $x = 0$)

$2x = y$ ⇒ $y = 4$

∴ The required number is 24

Short Answer Type Questions – II

[3 marks]

Que 1. Find the roots of the following quadratic equation by factorisation:

(i) $\sqrt{2x}^2 + 7x + 5\sqrt{2} = 0$ (ii) $2x^2 - x + \frac{1}{8} = 0$

Sol. (i) We have, $\sqrt{2x}^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow$ $\sqrt{2x}^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow$ $x(\sqrt{2x} + 5) + \sqrt{2}(\sqrt{2x} + 5) = 0$ $\Rightarrow$ $(\sqrt{2x} + 5)(x + \sqrt{2}) = 0$

$\therefore$ Either $\sqrt{2x} + 5 = 0$ or $x + \sqrt{2} = 0$

$\therefore$ $x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(ii) We have, $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow$ $\frac{16x^2 - 8x + 1}{8} = 0$ $\Rightarrow$ $16x^2 - 8x + 1 = 0$

$\Rightarrow$ $16x^2 - 4x - 4x + 1 = 0$ $\Rightarrow$ $4x(4x-1) - 1(4x-1) = 0$

$\Rightarrow$ $(4x-1)(4x-1) = 0$

So, either $4x - 1 = 0$ or $4x - 1 = 0$

$\therefore$ $x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots of given equation are $\frac{1}{4}$ and $\frac{1}{4}$.

Que 2. Find the roots of the following quadratic equation, if they exist, by the method of completing the square:

(i) $2x^2 + x - 4 = 0$ (ii) $4x^2 + 4\sqrt{3x} + 3 = 0$

Sol. (i) We have, $2x^2 + x - 4 = 0$

On dividing both sides by 2, we have

$x^2 + \frac{x}{2} - 2 = 0$

$\Rightarrow$ $x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$ [b = $\frac{1}{2}$ (coefficient of x) $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$]

$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0 \quad \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2 = \frac{1+32}{16} = \frac{33}{16} > 0$

$\Rightarrow$ Roots exist.

$\therefore \quad x + \frac{1}{4} = \pm\sqrt{\frac{33}{16}} \Rightarrow \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x + \frac{1}{4} = \frac{\sqrt{33}}{4}$ or $x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\therefore \quad x - \frac{1}{4} = +\frac{\sqrt{33}}{4}$ or $x = -\frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{\sqrt{33}-1}{4}$ or $x = \frac{-(\sqrt{33}+1)}{4}$

Hence, roots of given equation are $\frac{\sqrt{33}-1}{4}$ and $\frac{-(\sqrt{33}+1)}{4}$.

(ii) We have, $4x^2 + 4\sqrt{3x} + 3 = 0$

On dividing both sides by 4, we have

$x^2 + \sqrt{3x} + \frac{3}{4} = 0 \quad \Rightarrow x^2 + \sqrt{3x} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0 \quad \Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \quad \ldots (i)$

$\Rightarrow$ Roots exist. $\quad \therefore (i) \quad \Rightarrow x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$.

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

Que 3. Find the roots of the following quadratic equation by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$ **(ii) $4x^2 + 4\sqrt{3x} + 3 = 0$**

Sol. (i) We have, $2x^2 - 7x + 3 = 0$

Here, a = 2, b = - 7 and c = 3

Therefore, D = $b^2 - 4ac$

$\Rightarrow \quad D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$

$\therefore$ D > 0, $\therefore$ roots exist.

Thus, $\quad x = \frac{-b \pm\sqrt{D}}{2a} = \frac{-(-7) \pm \sqrt{25}}{2 \times 2} = \frac{7 \pm 5}{4}$

$= \frac{7+5}{4}$ or $\frac{7-5}{4}$

$= 3$ or $\frac{1}{2}$

So, the roots of given equation are 3 and $\frac{1}{2}$.

(ii) We have, $4x^2 + 4\sqrt{3x} + 3 = 0$

Here, a = 4, b = $4\sqrt{3}$ and c = 3

Therefore, $D = b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

$\therefore$ D = 0, roots exist and are equal.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-4\sqrt{3} \pm 0}{2 \times 4} = \frac{-\sqrt{3}}{2}$

Hence, the roots of given equation are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

Que 4. Using quadratic formula solve the following quadratic equation:
$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Sol. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

$\therefore$ $D = b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$

$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0$

So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2q^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

Que 5. Find the roots of the following equation:
$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}$; $x \neq -3, 6$

Sol. Given, $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}$; $x \neq -3, 6$

$\Rightarrow$ $\frac{(x-6)-(x+3)}{(x+3)(x-6)} = \frac{9}{20}$ $\Rightarrow$ $\frac{-9}{(x+3)(x-6)} = \frac{9}{20}$

$\Rightarrow$ $(x + 3)(x - 6) = -20$ or $x^2 - 3x + 2 = 0$

$\Rightarrow$ $x^2 - 2x - x + 2 = 0$ $\Rightarrow$ $x(x - 2) - 1(x - 2) = 0$

$\Rightarrow$ $(x - 1)(x - 2) = 0$ $\Rightarrow$ $x = 1$ or $x = 2$

Both x = 1 and x = 2 are satisfying the given equation. Hence, x = 1, 2 are the solution of
the equation.

Que 6. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them:

(i) $3x^2 - 4\sqrt{3x} + 4 = 0$ **(ii) $2x^2 - 6x + 3 = 0$**

Sol. (i) We have, (i) $3x^2 - 4\sqrt{3x} + 4 = 0$

Here, a = 3, b = - 4 $\sqrt{3x}$ and c = 4

Therefore, D = b^2 – 4ac = (- 4 $\sqrt{3x}$)^2 – 4 x 3 x 4 = 48 - 48 = 0

Hence, the given quadratic equation has real and equal roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\ \sqrt{3} \pm \sqrt{0}}{2 \times 3} = \frac{2\sqrt{3}}{3}$.

Hence, equal roots of given equation are $\frac{2\sqrt{3}}{3}$, $\frac{2\sqrt{3}}{3}$.

(ii) We have, $2x^2 - 6x + 3 = 0$

Here a = 2, b = - 6, c = 3

Therefore, D = b^2 – 4ac

= $(-6)^2$ – 4 x 2 x 3 = 36 – 24 = 12 > 0

Hence, given quadratic equation has real and distinct roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence, roots of given equation are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

Que 7. Find the value of k for each of the following quadratic equation, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$ **(ii) $kx(x-2) + 6 = 0$**

Sol. (i) We have, $2x^2 + kx + 3 = 0$

Here, a = 2, b = k, c = 3

∴ D = b^2 – 4ac = k^2 – 4 x 2 x 3 = k^2 – 24

For equal roots

D = 0 *i.e.,* k^2 – 24 = 0

⇒ k^2 = 24

⇒ k = $\pm$ $\sqrt{24}$ ⇒ k = $\pm$ 2 $\sqrt{6}$

(ii) We have, $kx(x-2) + 6 = 0$ ⇒ $kx^2 - 2kx + 6 = 0$

Here, a = k, b = - 2k, c = 6

For equal roots, we have

D = 0

i.e., $b^2 - 4ac = 0$ $\Rightarrow$ $(-2k)^2 - 4 \times k \times 6 = 0$
$\Rightarrow$ $4k^2 - 24k = 0$ $\Rightarrow$ $4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$ $\Rightarrow$ $k = 0$ or $k = 6$
But $k \neq 0$ (because if k = 0 then given equation will not be a quadratic equation).
So, k =6.

Que 8. If the roots of the equation (a – b) x^2 + (b – c) x + (c – a) = 0 are equal, prove that 2a = b + c.

Sol. Since the equation (a – b) x^2 + (b – c) x + (c – a) = 0 has equal roots, therefore discriminant
$D = (b - c)^2 - 4(a - b)(c - a) = 0$
$\Rightarrow b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$ $\Rightarrow b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$
$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$
$\Rightarrow (2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)2a = 0$
$\Rightarrow (2a - b - c)^2 = 0$ $\Rightarrow 2a - b - c = 0$ $\Rightarrow$ $2a = b + c$. Hence proved

Que 9. If the equation (1 + m²) x^2 + $2mcx$ + c² – a² = 0 has equal root, show that c² = a² (1 + m²).

Sol. The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$
Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$
Since the given equation has equal roots, therefore D = 0 $\Rightarrow B^2 - 4AC = 0$.
$\Rightarrow$ $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$
$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$
$\Rightarrow$ $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$ [Dividing throughout by 4]
$\Rightarrow$ $-c^2 + a^2(1 + m^2) = 0$ $\Rightarrow$ $c^2 = a^2(1 + m^2)$ Hence proved

Que 10. If sin θ and cos θ are roots of the equation ax^2 + bx + c = 0, prove that a² – b² + 2ac = 0.

Sol. Sum of the roots = $\frac{-B}{A}$ $\Rightarrow$ $\sin\theta + \cos\theta = \frac{-b}{a}$ … (i)

Product of the roots = $\frac{C}{A}$ $\Rightarrow$ $\sin\theta \, . \cos\theta = \frac{c}{a}$ … (ii)

Now, we have, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow$ $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$ $\Rightarrow$ $\left(\frac{-b}{a}\right)^2 - 2.\frac{c}{a} = 1$

$\Rightarrow$ $\frac{\boldsymbol{b^2}}{\boldsymbol{a^2}} - \frac{2c}{a} = 1$ or $b^2 - 2ac = a^2$

$\Rightarrow \qquad a^2 - b^2 + 2ac = 0$

Que 11. Determine the condition for one root of the quadratic equation ax^2 + bx + c = 0 to be thrice the other.

Sol. Let the roots of the given equation be α and 3 α.

Then sum of the roots = $\alpha + 3\alpha = 4\alpha = \frac{-b}{a}$ … (i)

Product of the roots = $(\alpha)(3\alpha) = 3\alpha^2 = \frac{c}{a}$ … (ii)

From (i), $\alpha = \frac{-b}{4a}$

$(ii) \Rightarrow 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a} \quad \Rightarrow \quad \frac{3b^2}{16a^2} = \frac{c}{a}$

$\Rightarrow$ $3b^2 = 16ac$, which is the required condition.

Que 12. Solve for x: $\mathbf{2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5; \ x \neq -3, \frac{1}{2}}$

Sol. $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5 \quad \Rightarrow \quad \left(\frac{4x-2}{x+3}\right) - \left(\frac{3x+9}{2x-1}\right) = 5$

$(4x - 2)(2x - 1) - (3x + 9)(x + 3) = 5(x + 3)(2x - 1)$

$(8x^2 - 4x - 4x + 2) - (3x^2 + 9x + 9x + 27) = 5(2x^2 - x + 6x - 3)$

$8x^2 - 8x + 2 - 3x^2 - 18x - 27 = 10x^2 + 25x - 15$

$5x^2 - 26x - 25 = 10x^2 + 25x - 15$

$5x^2 + 51x + 10 = 0$

$5x^2 + 50x + x + 10 = 0$

$5x(x + 10) + 1(x + 10) = 0$

$(5x + 1)(x + 10) = 0$

$5x + 1 = 0$ or $x + 10 = 0$

$x = \frac{-1}{5}$ or $x = -10$

Que 13. Solve the equation $\mathbf{\frac{4}{x} - 3 = \frac{5}{2x+3}; \ x \neq 0, -\frac{3}{2}}$, for x,

Sol. $\frac{4}{x} - 3 = \frac{5}{2x+3} \quad \Rightarrow \quad \frac{4-3x}{x} = \frac{5}{2x+3}$

$(4 - 3x)(2x + 3) = 5x \quad \Rightarrow \quad 8x - 6x^2 + 12 - 9x = 5x$

$6x^2 + 6x - 12 = 0 \quad \Rightarrow \quad x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0 \quad \Rightarrow \quad x(x+2) - 1(x+2) = 0$

$(x - 1)(x + 2) = 0$

$\Rightarrow \quad x - 1 = 0 \quad$ or $\quad x + 2 = 0$

$x = 1 \quad$ or $\quad x = -2$

Que 14. Solve for x: $\frac{16}{x} - 1 = \frac{15}{x+1}$; $x \neq 0, -1$

Sol. $\frac{16}{x} - 1 = \frac{15}{x+1} \Rightarrow \frac{16-x}{x} = \frac{15}{x+1}$

$(16 - x)(x + 1) = 15x \quad \Rightarrow \quad 16x - x^2 + 16 - x = 15x$

$x^2 + 15x - 15x - 16 = 0 \quad \Rightarrow \quad x^2 = 16$

$x = \pm 4$

Que 15. Solve for x. $x^2 + 5x - (a^2 + a - 6) = 0$

Sol. $x^2 + 5x - (a^2 + a - 6) = 0$

$x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$

$x^2 + 5x - (a - 2)(a + 3) = 0$

$\therefore \quad x^2 + (a + 3)x - (a - 2)x - (a - 2)(a + 3) = 0$

$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$[\{x + (a + 3)\}\{x - (a - 2)\}] = 0$

$\therefore \quad x = -(a + 3) \; x = (a - 2)$

$\Rightarrow \quad x = -(a + 3), (a - 2)$

Alternative method

$x^2 + 5x - (a^2 + a - 6) = 0$

$\therefore \quad x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times [-(a^2 + a - 6)]}}{2 \times 1}$

$= \frac{-5 \pm \sqrt{25 - 4a^2 + 4a - 24}}{2} = \frac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$

$= \frac{-5 \pm \sqrt{(2a)^2 + 2.(2a).1 + 1^2}}{2} = \frac{-5 \pm \sqrt{(2a+1)^2}}{2}$

$= \frac{-5 \pm 2a+1}{2} = \frac{-5+2a+1}{2}, \frac{-5-2a-1}{2}$

$= \frac{2a-4}{2}$, $-\frac{2a-6}{2}$ = (a – 2), - (a + 3)

Que 16. Solve for x: $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$

Sol. $\frac{2x(2x+3)+(x-3)+(3x+9)}{(x-3)(2x+3)} = 0$

$\Rightarrow 2x(2x + 3) + (x - 3) + (3x + 9) = 0$

$\Rightarrow 4x^2 + 10x + 6 = 0 \quad \Rightarrow \quad 2x^2 + 5x + 3 = 0$

$\Rightarrow (x + 1)(2x + 3) = 0 \quad \Rightarrow \quad x = -1, x = -\frac{3}{2}$

But $x \neq -\frac{3}{2}$ $\quad \therefore \quad x = -1$

Que 17. Solve for x: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1,2,3.$

Sol. $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3} \Rightarrow \frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\Rightarrow 3(x-3+x-1) = 2(x-1)(x-2)(x-3) \quad \Rightarrow \quad 3(2x-4) = 2(x-1)(x-2)(x-3)$

$\Rightarrow 3 \times 2(x - 2) = 2(x - 1)(x - 2)(x - 3) \quad \Rightarrow \quad 3 = (x - 1)(x - 3)$ *i.e.,* $x^2 - 4x = 0$

$\Rightarrow x(x - 4) = 0 \quad \therefore \quad x = 0, x = 4$

Long Answer Type Questions

[4 marks]

Que 1. Using quadratic formula, solve the following equation for x:

abx^2 + (b^2 – ac) $x - bc$ = 0

Sol. We have, abx^2 + (b^2 - ac) $x - bc$ = 0

Here, A = ab, B = b^2 - ac, C = -bc

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4\,AC}}{2A}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2-ac)^2 - 4\,(ab)\,(-bc)}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2-ac)^2 - 4ab^2c)}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{b^4 - 2ab^2c + a^2\,c^2 + 4ab^2c}}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) \pm \sqrt{(b^2-ac)^2}}{2ab} \qquad \Rightarrow x = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \quad \text{or} \quad x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$\Rightarrow x = \frac{2ac}{2ab} \quad \text{or} \quad x = \frac{-2b^2}{2ab} \qquad \Rightarrow x = \frac{c}{b} \quad \text{or} \quad x = \frac{-c}{b}$$

Que 2. Find the value of p for which the quadratic equation
(2p + 1) x^2 - (7p + 2)x + (7p - 3) = 0 has equal roots. Also find these roots.

Sol. Since the quadratic equation has equal roots, D = 0

$i.e.,$ b^2 - 4ac = 0

In (2p + 1) x^2 - (7p + 2) x + (7p - 3) = 0

Here, a = (2p +1), b = -(7p + 2) , c = (7p - 3)

∴ $(7p + 2)^2$ – 4 (2p + 1) (7p – 3) = 0

⇒ $49p^2$ + 4 + 28p – (8p + 4) (7p – 3) = 0

⇒ $49p^2$ + 4 + 28p – $56p^2$ + 24p – 28p + 12 = 0

⇒ $-7p^2$ + 24p + 16 = 0 ⇒ $7p^2$ – 24p – 16 = 0

⇒ $7p^2$ – 28p + 4p – 16 = 0 ⇒ 7p (p - 4) + 4 (p – 4) = 0

⇒ (7p + 4) (p – 4) = 0 ⇒ p = $-\frac{4}{7}$ or p = 4

For p = $\frac{-4}{7}$

$\left(2 \times \frac{-4}{7} + 1\right) x^2$ - $\left(7 \times \frac{-4}{7} + 2\right) x$ + $\left(7 \times \frac{-4}{7} - 3\right)$ = 0

⇒ $\frac{-1}{7} x^2 + 2x - 7 = 0$ ⇒ $x^2 - 14x + 49 = 0$

⇒ $x^2 - 7x - 7x + 49 = 0$ ⇒ $x(x - 7) - 7(x - 7) = 0$

⇒ $(x - 7)^2 = 0$ ⇒ $x = 7, 7$

For p = 4,

$(2 \text{ x } 4 + 1)\, x^2 - (7 \text{ x } 4 + 2)\, x + (7 \text{ x } 4 - 3) = 0$

⇒ $9x^2 - 30x + 25 = 0$ ⇒ $9x^2 - 15x - 15x + 25 = 0$

⇒ $3x(3x - 5) - 5(3x - 5) = 0$ ⇒ $(3x - 5)(3x - 5) = 0$

⇒ $x = \frac{5}{3}, \frac{5}{3}$

Que 3. Solve for x: $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; x \neq 5,7$

Sol. $\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}$ ⇒ $\frac{(x-4)(x-7)+(x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$

⇒ $\frac{x^2-7x-4x+28+x^2-5x-6x+30}{x^2-7x-5x+35} = \frac{10}{3}$

⇒ $\frac{2x^2-22x+58}{x^2-12x+35} = \frac{10}{3}$ ⇒ $\frac{x^2-11x+29}{x^2-12x+35} = \frac{5}{3}$

⇒ $3x^2 - 33x + 87 = 5x^2 - 60x + 175$ ⇒ $2x^2 - 27x + 88 = 0$

⇒ $2x^2 - 16x - 11x + 88 = 0$ ⇒ $2x(x - 8) - 11(x - 8) = 0$

⇒ $(2x - 11)(x - 8) = 0$ ⇒ $2x - 11 = 0$ or $x - 8 = 0$

⇒ $x = \frac{11}{2}$ or $x = 8$

Que 4. The sum of the reciprocal of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age = $(x - 3)$ years

And 5 years from now, Rehman's age = $(x - 5)$ years

Now' according to question, we have

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

⇒ $\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$ ⇒ $\frac{2x + 2}{(x - 3)(x + 5)} = \frac{1}{3}$

$\Rightarrow$ $6x + 6 = (x - 3)(x + 5)$ $\Rightarrow$ $6x + 6 = x^2 + 5x - 3x - 15$

$\Rightarrow$ $x^2 + 2x - 15 - 6x - 6 = 0$ $\Rightarrow$ $x^2 - 4x - 21 = 0$

$\Rightarrow$ $x^2 - 7x + 3x - 21 = 0$ $\Rightarrow$ $x(x - 7) + 3(x - 7) = 0$

$\Rightarrow$ $(x - 7)(x + 3) = 0$ $\Rightarrow$ $x = 7$ or $x = -3$

But $x \neq -3$ (age cannot be negative)
Therefore, present age of Rehman = 7 years.

Que 5. The different of two nature numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the number.

Sol. Let the Two nature numbers be x and y such that $x > y$.

According to the question
Difference of numbers, $x - y = 5$ $\Rightarrow$ $x = 5 + y$ … (i)
Difference of the reciprocals,

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \qquad \ldots (ii)$$

Putting the value of (i) in (ii)

$$\frac{1}{y} - \frac{1}{5+y} = \frac{1}{10} \Rightarrow \frac{5+y-y}{y(5+y)} = \frac{1}{10}$$

$\Rightarrow$ $50 = 5y + y^2$ $\Rightarrow$ $y^2 + 5y - 50 = 0$

$\Rightarrow$ $y^2 + 10y - 5y - 50 = 0$ $\Rightarrow$ $y(y + 10) - 5(y + 10) = 0$

$\Rightarrow$ $(y - 5)(y + 10) = 0$

$\therefore$ $y = 5$ or $y = -10$

$\therefore$ y is a nature number. $\therefore$ $y = 5$

Putting the value of y in (i), we have

$x = 5 + 5$ $\Rightarrow$ $x = 10$

The required numbers are 10 and 5.

Que 6. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Sol. Let the two consecutive odd numbers be x and $x + 2$.

$\Rightarrow$ $x^2 + (x + 2)^2 = 394$ $\Rightarrow$ $x^2 + x^2 + 4 + 4x = 394$

$\Rightarrow$ $2x^2 + 4x + 4 = 394$ $\Rightarrow$ $2x^2 + 4x - 390 = 0$

$\Rightarrow$ $x^2 + 2x - 195 = 0$ $\Rightarrow$ $x^2 + 15x - 13x - 195 = 0$

$\Rightarrow$ $x(x + 15) - 13(x + 15) = 0$ $\Rightarrow$ $(x - 13)(x + 15) = 0$

$\Rightarrow$ $x - 13 = 0$ or $x + 15 = 0$ $\Rightarrow$ $x = 13$ or $x = -15$

Que 7. The sum of two number is 15 and the sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Sol. Let the numbers be x and $15 - x$.

According to given condition,

$$\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10} \Rightarrow \frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$$

$\Rightarrow$ $150 = 3x(15 - x)$ $\Rightarrow$ $50 = 15x - x^2$

$\Rightarrow$ $x^2 - 15x + 50 = 0$ $\Rightarrow$ $x^2 - 5x - 10x + 50 = 0$

$\Rightarrow$ $x(x - 5) - 10(x - 5) = 0$ $\Rightarrow$ $(x - 5)(x - 10) = 0$

$\Rightarrow$ $x = 5$ or 10.

When $x = 5$, then $15 - x = 15 - 5 = 10$

When $x = 10$, then $15 - x = 15 - 10 = 5$

Hence, the two numbers are 5 and 10.

Que 8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

Sol. Let Shefali's marks in Mathematics be x.

Therefore, Shefali's marks in English is $(30 - x)$.

Now, according to question,

$(x + 2)(30 - x - 3) = 210$ $\Rightarrow$ $(x + 2)(27 - x) = 210$

$\Rightarrow$ $27x - x^2 + 54 - 2x = 210$ $\Rightarrow$ $25x - x^2 + 54 - 210 = 0$

$\Rightarrow$ $25x - x^2 - 156 = 0$ $\Rightarrow$ $-(x^2 - 25x + 156) = 0$

$\Rightarrow$ $x^2 - 25x + 156 = 0$ $\Rightarrow$ $x^2 - 13x - 12x + 156 = 0$

$\Rightarrow$ $x(x - 13) - 12(x - 13) = 0$ $\Rightarrow$ $(x - 13)(x - 12) = 0$

Either $x - 13 = 0$ or $x - 12 = 0$

$\therefore$ $x = 13$ or $x = 12$

Therefore, Shefali's marks in Mathematics = 13

Marks in English = 30 - 13 = 17

or Shefali's marks in Mathematics = 12, marks in English = 30 – 12 = 18.

Que 9. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. Let the uniform speed of the train be x km/h.

Then, time taken to cover 360 km = $\frac{360}{x}$ h

Now, new increased speed = $(x + 5)$ km/h

So, time taken to cover 360 km = $\frac{360}{x+5}$ h

According to question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1 \qquad \Rightarrow \frac{360\,(x+5-x)}{x\,(x+5)} = 1$

$\Rightarrow \frac{360 \times 5}{x\,(x+5)} = 1 \qquad \Rightarrow 1800 = x^2 + 5x$

$\therefore x^2 + 5x - 1800 = 0 \qquad \Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow x\,(x + 45) - 40\,(x + 45) = 0 \qquad \Rightarrow (x + 45)\,(x - 40) = 0$

Either $x + 45 = 0$ or $x - 40 = 0$

$\therefore x = -45$ or $x = 40$

But x cannot be negative, so $x \neq -45$

Therefore, $x = 40$

Hence, the uniform speed of train is 40 km/h.

Que 10. The sum of the areas of two squares is 468 m2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol. Let x be the length of the side of First Square and y be the length of side of the second square.

Then, $x^2 + y^2 = 468 \qquad \ldots (i)$

Let x be the length of the side of the bigger square.

$4x - 4y = 24$

$\Rightarrow x - y = 6$ or $x = y + 6 \qquad \ldots (ii)$

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow y^2 + 12y + 36 + y^2 = 468$ or $2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0 \qquad \Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0 \qquad \Rightarrow (y + 18)(y - 12) = 0$

Either $y + 18 = 0$ or $y - 12 = 0$

$\Rightarrow y = -18$ or $y = 12$

But, sides cannot be negative, so y = 12

Therefore, $x = 12 + 6 = 18$

Hence, sides of two squares are 18 m and 12 m.

Que 11. Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.

Sol. Seven years ago, let Swati's age be x years. Then, seven years ago Varun's age was $5x^2$ years.

∴ Swat's present age = (x + 7) years

Varun's present age = ($5x^2$ + 7) years

Three years hence,

Swati's age = (x + 7 + 3) years = (x +10) years

Varun's age = ($5x^2$ + 7 + 3) years = ($5x^2$ + 10) years

According to the questions,

$x + 10 = \frac{2}{5}(5x^2 + 10)$ ⇒ $x + 10 = \frac{2}{5} \times 5\,(x^2 + 2)$

⇒ $x + 10 = 2x^2 + 4$ ⇒ $2x^2 - x - 6 = 0$

⇒ $2x^2 - 4x + 3x - 6 = 0$ ⇒ $2x\,(x - 2) + 3\,(x - 2) = 0$

⇒ $(2x + 3)(x - 2) = 0$

⇒ $x - 2 = 0$ or $2x + 3 = 0$

⇒ $x = 2$ [∴ $2x + 3 \neq 0$ as $x > 0$]

Hence, Swati's present age = (2+7) years = 9 years

and Varun's present age = (5×2^2 +7) years = 27 years

Que 12. A train takes 2 hours less for a journey of 300 km, if its speed is increased by 5 km/h from its usual speed. Find the usual speed of the train.

Sol. Let the usual speed of the train = x km/h.

Therefore, time taken to cover 300 km = $\frac{300}{x}$ hours … (i)

When its speed is increased by 5 km/h, then time taken by the train to cover the distance of 300 km = $\frac{300}{x+5}$ hour … (ii)

According to the question, $\left(\frac{300}{x} - \frac{300}{x+5}\right)$ hours = 2 hours

⇒ $\frac{300\,(x+5) - 300\,x}{x\,(x+5)} = 2$ ⇒ $\frac{300\,\{(x+5) - x\}}{x\,(x+5)} = 2$

⇒ $2x\,(x + 5) = 300 \times 5$ ⇒ $2x^2 + 10x - 1500 = 0$

⇒ $x^2 + 5x - 750 = 0$ ⇒ $x^2 + 30x - 25x - 750 = 0$

⇒ $x\,(x + 30) - 25\,(x + 30) = 0$ ⇒ $(x - 25)(x + 30) = 0$

$\Rightarrow$ x = 25 or x = - 30

$\Rightarrow$ x = 25 ($\because$ speed cannot be negative)

Therefore, the usual speed of the train 25 km/h.

Que 13. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Sol. Let the digit at tens place be x.

Then, digit at unit place = $\frac{18}{x}$

$\therefore$ Number = $10x + \frac{18}{x}$

And number obtained by interchanging the digits = $10 \times \frac{18}{x} + x$

According to the question,

$$\left(10x + \frac{18}{x}\right) - 63 = 10 \times \frac{18}{x} + x \quad \Rightarrow \quad \left(10x + \frac{18}{x}\right) - \left(10 \times \frac{18}{x} + x\right) = 63$$

$\Rightarrow$ $10x + \frac{18}{x} - \frac{180}{x} - x = 63$ $\Rightarrow$ $9x - \frac{162}{x} - 63 = 0$

$\Rightarrow$ $9x^2 - 63x - 162 = 0$ $\Rightarrow$ $x^2 - 7x - 18 = 0$

$\Rightarrow$ $x^2 - 9x + 2x - 18 = 0$ $\Rightarrow$ $x(x - 9) + 2(x - 9) = 0$

$\Rightarrow$ $(x - 9)(x + 2) = 0$ $\Rightarrow$ $x = 9$ or $x = -2$

$\Rightarrow$ $x = 9$ [$\because$ a digit can never be negative]

Hence, the required number = $10 \times 9 + \frac{18}{9} = 92$.

Que 14. If twice the area of a smaller square is subtracted from the area of a larger square; the result is 14 cm². However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm². Determine the sides of the two squares.

Sol. Let the sides of the larger and smaller squares be x and y respectively. Then

$x^2 - 2y^2 = 14$ … (i)

and $2x^2 + 3y^2 = 203$ … (ii)

Operating $(ii) - 2 \times (i)$, we get

$2x^2 + 3y^2 - (2x^2 - 4y^2) = 203 - 2 \times 14$

$\Rightarrow$ $2x^2 + 3y^2 - 2x^2 - 4y^2 = 203 - 28$

$\Rightarrow$ $7y^2 = 175$ $\Rightarrow$ $y^2 = 25$ $\Rightarrow$ $y = \pm 5$

$\Rightarrow$ $y = 5$ [$\because$ side cannot be negative]

By putting the value of y in equation (i), we get

$x^2 - 2 \times 5^2 = 14 \Rightarrow x^2 - 50 = 14$ or $x^2 = 64$

∴ $x = \pm 8$ or $x = 8$

∴ x side of the two squares are 8 cm and 5 cm.

Que 15. If Zeba was younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Sol. Let the present age of Zeba be x years

Age before 5 years = $(x - 5)$ years

According to given condition,

$\Rightarrow (x - 5)^2 = 5x + 11 \Rightarrow x^2 + 25 - 10x = 5x + 11$

$\Rightarrow x^2 - 10x - 5x + 25 - 11 = 0 \Rightarrow x^2 - 15x + 14 = 0$

$\Rightarrow x^2 - 14x - x + 14 = 0 \Rightarrow x(x - 14) - 1(x - 14) = 0$

$\Rightarrow (x - 1)(x - 14) = 0 \Rightarrow x - 1 = 0$ or $x - 14 = 0$

$x = 1$ or $x = 14$

But present age cannot be 1 year.

∴ Present age of Zeba is 14 years.

Que 16. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be x km/h.

Speed of motor boat in still water = 18 km/h

Speed of motor boat in upstream = $(18 - x)$ km/h

Speed of motor boat in downstream = $(18 + x)$ km/h

Distance travelled = 24 km.

Time taken by motor boat to travel upstream = $\frac{24}{18 - x}$

Time taken by motor boat to travel downstream = $\frac{24}{18 + x}$

$\frac{24}{18 - x} = \frac{24}{18 + x} + 1 \Rightarrow 24(18 + x) = (18 - x)(24 + 18 + x)$

$\Rightarrow 432 + 24x = (18 - x)(42 + x) \Rightarrow 432 + 24x = 756 + 18x - 42x - x^2$

$\Rightarrow x^2 + 48x - 324 = 0 \Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x + 54) - 6(x + 54) = 0 \Rightarrow (x - 6)(x + 54) = 0$

$x = 6$ or $x = -54$

speed of motorboat = 6 km/h.

Que 17. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

Sol. Let the natural number be x

According to the question,

$$x + 12 = \frac{160}{x}$$

$\Rightarrow$ $x^2 + 12x - 160 = 0$ $\Rightarrow$ $x^2 + 20x - 8x – 160 = 0$

$\Rightarrow$ $x(x + 20) – 8(x + 20) = 0$ $\Rightarrow$ $(x + 20)(x – 8) = 0$

$\Rightarrow$ $x = -20$ (Not possible) or $x = 8$

Hence, the required natural number is 8.

Que 18. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

Sol. Let the two consecutive multiples of 7 be x and $x + 7$.

$x^2 + (x + 7)^2 = 637$

$\Rightarrow$ $x^2 + x^2 + 49 + 14x = 637$ $\Rightarrow$ $2x^2 + 14x – 588 = 0$

$\Rightarrow$ $x^2 + 7x – 294 = 0$ $\Rightarrow$ $x^2 + 21x - 14x – 294 = 0$

$\Rightarrow$ $x(x + 21) – 14(x + 21) = 0$ $\Rightarrow$ $(x – 14)(x + 21) = 0$

$x = 14$ or $x = -21$

The multiples are 14 and 21.

Que 19. Solve for x: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq -1, -2, -4$

Sol. $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$\Rightarrow$ $\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$ $\Rightarrow$ $(x + 4)(x + 2 + 2x + 2) = 4(x + 1)(x + 2)$

$\Rightarrow$ $(x + 4)(3x + 4) = 4(x^2 + 3x + 2)$ $\Rightarrow$ $x^2 - 4x – 8 = 0$

$\Rightarrow$ $x = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

Que 20. Find the positive value(s) of k for which both quadratic equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots.

Sol. (i) For $x^2 + kx + 64 = 0$ to have real roots

$K^2 – 4(I)(64) \geq 0$ $i.e.$, $k^2 – 256 \geq 0$ $\Rightarrow$ $k \geq \pm 16$

(ii) For $x^2 - 8x + k = 0$ to have real roots

$(-8)^2 – 4(k) \geq 0$ $i.e.$, $64 – 4k \geq 0$ $\Rightarrow$ $k \leq \pm 16$

For (i) and (ii) to hold simultaneously k = 16

HOTS (Higher Order Thinking Skills)

Que 1. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Sol. Let x be the total number of camels.

Then, number of camels in the forest = $\frac{x}{4}$

Number of camels on mountains = $2\sqrt{x}$

And number of camels on the bank or river = 15

Thus, total number of camels = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y$, then x = y^2

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \qquad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y-6) + 10(y-6) = 0 \quad \Rightarrow \quad (3y+10)(y-6) = 0$

$\Rightarrow$ y = 6 or $y = -\frac{10}{3}$

Now, $y = -\frac{10}{3} \qquad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{3} \qquad (\because \; x = y^2)$

But the number of camels cannot be a fraction.

$\therefore$ y = 6 $\quad \Rightarrow \quad x = 6^2 = 36$

Hence, the number of camels = 36

Que 2. Solve the following quadratic equation:

$$\mathbf{9x^2 - 9\,(a+b)\,x + [2a^2 + 5ab + 2b^2] = 0.}$$

Sol. Consider the equation $9x^2 - 9\,(a+b)\,x + [2a^2 + 5ab + 2b^2] = 0.$

Now comparing with $Ax^2 + Bx + C = 0$, we get

$$A = 9, B = -9\,(a+b) \; and \; C = [2a^2 + 5ab + 2b^2]$$

Now discriminant

$D = B^2 - 4AC$

$= \{-9(a+b)\}^2 - 4 \times 9(2a^2 + 5ab + 2b^2) = 9^2(a+b)^2 - 4 \times 9\,(2a^2 + 5ab + 2b^2)$

$= 9\{9(a+b)^2 - 4(2a^2 + 5ab + 2b^2)\} = 9\{9a^2 + 9b^2 + 18ab - 8a^2 - 20ab - 8b^2\}$

$= 9\{a^2 + b^2 - 2ab\} = 9(a-b)^2$

Now using the quadratic formula,

$x = \frac{-B \pm \sqrt{D}}{2A}$, we get $\quad x = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2\times 9}$

$\Rightarrow \quad x = \frac{9(a+b) \pm 3\,(a-b)}{2\times 9} \qquad \Rightarrow \quad x = \frac{3(a+b) \pm (a-b)}{6}$

$\Rightarrow \quad x = \frac{(3a+3b) + (a-b)}{6}$ and $x = \frac{(3a+3b)-(a-b)}{6}$

$\Rightarrow \quad x = \frac{(4a+2b)}{6}$ and $x = \frac{(2a+4b)}{6}$

$\Rightarrow \quad x = \frac{2a+b}{3}$ and $x = \frac{a+2b}{3}$ are required solutions.

Que 3. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the order to fil it, find the time in which each pipe would fill the cistern.

Sol. Let, time taken by faster pipe to fill cistern be x minutes

Therefore, time taken by slower pipe to fill the cistern = (x + 3) minutes

Since the faster pipe takes x minutes to fill the cistern.

∴ Portion of the cistern filled by the faster pipe in one minute = $\frac{1}{x}$

Portion of the cistern filled by the slower pipe in one minute = $\frac{1}{x+3}$

Portion of the cistern filled by the two pipes together in one minute = $\frac{1}{\frac{40}{13}} = \frac{13}{40}$

According to question,

$\Rightarrow \quad \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40} \qquad \Rightarrow \quad \frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \quad 40(2x+3) = 13x(x+3) \qquad \Rightarrow \quad 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0 \qquad \Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0 \qquad \Rightarrow \quad (x-5)(13x+24) = 0$

Either x – 5 = 0 or 13x + 24 = 0

$\Rightarrow \quad x = 5 \quad or \quad x = \frac{-24}{13}$

$\Rightarrow \quad x = 5 \qquad [\because \ x \text{ cannot be negative}]$

Hence, time taken by faster pipe to fill the cistern = x = 5 minutes

and time taken by slower pipe = x + 3 = 5 + 3 = 8 minutes.

Que 4. In the centre of a rectangular lawn of dimension 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

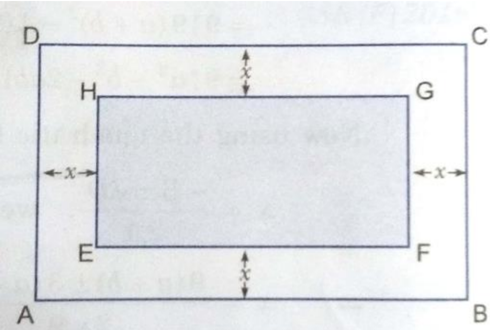

Fig. 4.1

Sol. Let ABCD be rectangular lawn and EFGH be rectangular pound. Let x m be the width of

grass area, which is same around the pound.

Given, Length of lawn = 50 m

Width of lawn = 40 m

⇒ Length of pond = (50 – 2x) m

Breadth of pond = (40 – 2x) m

Also given,

Area of grass surrounding the pond = 1184 m2

⇒ Area of rectangular lawn – Area of pond = 1184 m^2

⇒ 50 × 40 – {(50 – 2x) × (40 – 2x) = 1184

⇒ $2000 - (2000 - 80x - 100x + 4x^2) = 1184$

⇒ $4x^2 - 180x + 1184 = 0$ ⇒ $x^2 - 45x + 296 = 0$

⇒ $x^2 - 37x - 8x + 296 = 0$ ⇒ $x(x - 37) - 8(x - 37) = 0$

⇒ $(x - 37)(x - 8) = 0$ ⇒ $x - 37 = 0\ or\ x - 8 = 0$

⇒ $x = 37\ or\ x = 8$

x = 37 is not possible as in this case length of pond becomes 50 – 2 × 37 = – 24 (not possible)

Hence, x = 8 is acceptable

∴ Length of pond = 50 – 2 × 8 = 34 m

Breadth of pond = 40 – 2 × 8 = 24 m

Value Based Questions

Que 1. Some people collected money to be donated in some orphanages. A part of the donation is fixed and remaining depends on the number of children in the orphanage. If they donated ₹ 9,500 in the orphanage having 50 children and ₹ 13,250 with 75 children, find the fixed part of the donation and the amount donated for each child.
What values do these people posses?

Sol. Let the fixed donation be ₹ x and amount donated for each child be ₹ y.

Then $x + 50y = 9500$ ...(i)

$x + 75y = 13250$ ...(ii)

Subtracting (i) from (ii), we get

$25y = 3750$ or $y = 150$

From (i), $x = 9500 - 50y = 9500 - 50 \times 150 = 2000$

∴ Fixed amount donated = ₹ 2,000.

Amount donated for each child = ₹ 150.

Helpfulness, cooperation, happiness, caring.

Que 2. The fraction of people in a society using CNG in their vehicles becomes $\frac{9}{11}$, if 2 is added to both its numerator and denominator. If 3 is added to both its numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction. What does this fraction show?

Sol. Let the fraction be $\frac{x}{y}$.

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \Rightarrow \quad 11x - 9y = -4$$

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \Rightarrow \quad 6x - 5y = -3$$

Solving for x and y gives x = 7 and y = 9

$$\therefore \quad Fraction = \frac{x}{y} = \frac{7}{9}$$

More people are becoming aware about scarcity of petrol, so they are switching on to alternative resources.

Que 3. Reading book in a library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days. While Bunty paid ₹ 21 for the book kept for five days.

(i) Find the fixed charge.

(ii) Find how much additional charge Shristi and Bunty paid.
(iii) Which mathematical concept is used in this problem?
(iv) Which value does it depict?

Sol. Let fix charge for reading book be ₹ x and additional charge for each day be ₹ y.

Then, $x + 4y = 27$ ...(i)

On subtraction $\frac{\underline{\underline{x+2y=21}}}{2y=6}$ ....(ii)

$Y = 3$

Putting value of y in equation (i) we get

$x + 4 \times 3 = 27 \quad \Rightarrow \quad x + 12 = 27 \quad \Rightarrow \quad x = 15$

(i) Fixed charge on the book is ₹ 15.

(ii) Bunty paid additional ₹ 6 and Shristi paid ₹ 12.

(iii) Pair of linear equation in two variables.

(iv) Reading is a good habit.

Que 4. An honest person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But if he had interchanged amount invested, he would have received ₹ 4 more as interest.

(i) How much amount did he invest at different rates?

(ii) Which mathematical concept is used in this problem?

(iii) Which value is being emphasized here?

Sol. (i) Let the person invest ₹ x at rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

Yearly interest = $\frac{12x}{100} + \frac{10y}{100}$

$\therefore \quad \frac{12x}{100} + \frac{10y}{100} = 130 \quad \Rightarrow \quad 12x + 10y = 13000$ ...(i)

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$\frac{10x}{100} + \frac{12y}{100} = 134 \quad \Rightarrow \quad 10x + 12y = 13400$ ...(ii)

Adding eqn. (i) and (ii) we get

$22x + 22y = 26400$

$x + y = 1200$ ...(iii)

Subtracting (ii) from (i) we get

$2x - 2y = -400$

$x - y = -200$ ...(iv)

Solving (iii) and (iv), we get

$x = 500, y = 700$

Thus, person invested ₹ 500 at 12% per annum and ₹ 700 at 10% per annum.

(ii) Pair of linear equation in two variables.

(iii) Honesty is the best policy.

Que 5. If the price of petrol is increased by ₹ 2 per litre, a person will have to buy 1 litre less petrol for ₹ 1740. Find the original price of petrol at that time.

(a) Why do you think the price of petrol is increasing day-by-day?

(b) What should we do to save petrol?

Sol. Let the original price of the petrol be ₹ x per litre.

Then, amount of petrol that can be purchased = $\frac{1740}{x}$

According to question

$\frac{1740}{x} - \frac{1740}{x+2} = 1 \qquad \Rightarrow \quad 1740\,(x + 2 - x) = x\,(x + 2)$

$\Rightarrow \quad x^2 + 2x - 3480 = 0 \qquad \Rightarrow \quad x^2 + 60x - 58x - 3480 = 0$

$\Rightarrow \quad (x + 60)\,(x - 58) = 0 \qquad \Rightarrow \quad x = 58, -60\ (rejected)$

∴ Original cost of petrol was ₹ 58 per litre.

(a) Petrol is a natural resource which is depleting day-by-day. So, due to more demand and less supply, its price is increasing.

(b) We should use more of public, transport and substitute petrol with CNG or other renewable resource.

Que 6. One fourth of a group of people claim they are creative, twice the square root of the group claim to be caring and the remaining 15 claim they are optimistic. Find the total number of people in the group.

(a) How many persons in the group are creative?

(b) According to you, which one of the above three values is more important for development of a society?

Sol. Let x be the total

Then, number of creative persons = $\frac{x}{4}$

Number of caring persons = $2\sqrt{x}$

and number of optimistic persons = 15

Thus, total number of persons = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$

Let $\sqrt{x} = y, \quad then \quad x = y^2$

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \qquad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y - 6) + 10\,(y - 6) = 0 \qquad \Rightarrow \quad (3y + 10)\,(y - 6) = 0$

$\Rightarrow \quad y = 6 \ or \ y = -\frac{10}{3}$

Now, $\quad y = -\frac{10}{3} \qquad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9} (\because\ x = y^2)$

But, the number of persons cannot be a fraction.

∴ y = 6 $\qquad \Rightarrow \quad x = 6^2 = 36$

Hence, the number of people in the group = 36

(a) 9 persons

(b) All of these values have their own importance. A person having these values will certainly contribute to the development of society. However, the level of importance given to each of them depends upon a person's own attitude. Hence, any value with justification is correct. (Do yourself)

Que 7. In the centre of a rectangular plot of land of dimensions 120 m × 100 m, a rectangular portion is to be covered with trees so that the area of the remaining part of the plot is 10500 m². Find the dimensions of the area to be planted. Which social act is being discussed here? Give its advantages.

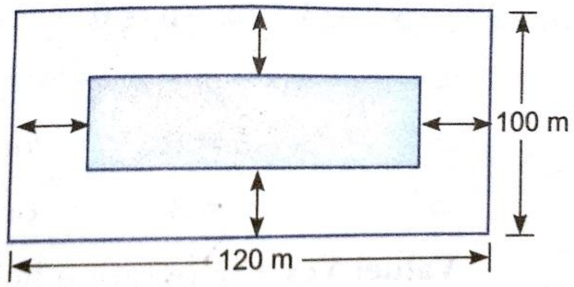

Fig. 1

Sol. Let the width of the unplanted area be x m

Then, dimension of area to be planted = (120 – 2x) and (100 – 2x)

∴ $(120-2x)(100-2x) = 120 \times 100 - 10500$

⇒ $12000 - 440x + 4x^2 = 1500$ or $x^2 - 110x + 2625 = 0$

⇒ $(x-75)(x-35) = 0$ or $x = 75, 35$

But x = 75 is not possible

∴ $x = 35$

Thus, dimension of area to be planted = (120 – 70) and (100 – 70) i.e., 50 m and 30 m.

Afforestation is being discussed here. Planting more trees helps in reducing air pollution and make the environment clean and green.

Que 8. Mr. Ahuja has to square plots of land which he utilises for two different purposes– one for providing free education to the children below the age of 14 years and the other to provide free medical services for the needy villagers. The sum of the areas of two square plots is 15425 m². If the difference of their perimeter is 60 m, find the sides of the two squares.

Which qualities of Mr. Ahuja are being depicted in the question?

Sol. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 15425$ ...(i)

Let x be the length of the side of the bigger square.

4x – 4y = 60

⇒ x – y = 15 or x = y + 15 ...(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y+15)^2 + y^2 = 15425$ ⇒ $2y^2 + 30y - 15200 = 0$

or $y^2 + 15y - 7600 = 0$ or $(y+95)(y-80) = 0$ ⇒ $y = -95, 80$

But, sides cannot be negative, so y = 80

Therefore, $x = 80 + 15 = 95$

Hence, sides of two squares are 80 m and 95 m.
Value: Caring, King, Social and generous.

Que 9. A takes 3 days longer than B to finish a work. But if they work together, then work is completed in 2 days. How long would each take to do it separately. Can you say cooperation helps to get more efficiency?

Sol. Let B finish a work in x days
then A finish a work in x + 3 days
According to question

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2} \quad \Rightarrow \quad \frac{(x+3)+x}{x(x+3)} = \frac{1}{2}$$

$\Rightarrow$ $2x + 6 + 2x = x^2 + 3x$ $\Rightarrow$ $x^2 + 3x - 2x - 6 - 2x = 0$
$\Rightarrow$ $x^2 - x - 6 = 0$
Solving the equation
$x^2 - (3 - 2)x - 6 = 0$ $\Rightarrow$ $x^2 - 3x + 2x - 6 = 0$
$\Rightarrow$ $x(x - 3) + 2(x - 3) = 0$ $\Rightarrow$ $(x - 3)(x + 2) = 0$
$\Rightarrow$ $x - 3 = 0$ or $x + 2 = 0$
$\Rightarrow$ $x = 3$ or $x = -2$ (Days cannot be negative)
Value: Yes, Cooperation helps in improving work efficiency.

Que 10. In a class of 48 students, the number of regular students is more than the number of irregular students. Had two irregular students been regular, the product of the number of two types of students would be 380. Find the number of each type of students.
(a) Why is regularity essential in life?
(b) Write values other than regularity that a student must possess.

Sol. Let the number of regular students be x
Then, the number of irregular students = 48 – x
According to the question,
$(x + 2)(48 - x - 2) = 380$
$\Rightarrow -x^2 + 44x + 92 - 380 = 0 \; or \; x^2 - 44x + 288 = 0$
$\Rightarrow$ $(x - 36)(x - 8) = 0$ or $x = 36, 8$
But $x > 48 - x$ (given)
$\therefore$ $x = 36$
i.e., The number of regular students = 36
and the number of irregular students = 48 – 36 = 12
(a) Regularity in any sphere gives confidence which, in turn, leads to the development of an individual and the society as well.
(b) Honesty, Creativity, Confidence, Punctuality. (you may add more to this list)

Que 11. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?

Sol. Number of plants planted by class 1 = 2 (class 1 × 2 sections) = 2 × 1 × 2 = 4 trees
Similarly, number of plants planted by class 2 = 2 (class 2 x 2 sections)
= 2 × 2 × 2 = 8 trees
We now know, a = 4 and d = 4
Number of classes n = 12
Total number of plants planted = Sum of AP = S_{12}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 4 + (12-1)4]$$

$$S^{12} = 6[8 + 44] = 6 \times 52 = 312\ trees$$

Values shown by the students:
(i) Envrionmental friendly
(ii) Social awareness
(iii) Sense of responsibility towards the society.

Que 12. A sum of ₹ 3150 is to be used to give six cash prizes to students of a school for overall academic performance, punctuality, regularity, cleanliness, confidence and creativity. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes.
(a) Which value according to you should be awarded with the maximum amount? Justify your answer.
(b) Can you add more values to the above ones which should be awarded?

Sol. Let the six prizes (1st, 2nd, 3rd …..6th) be a, a – 50, a – 100, a – 150, a – 200, a – 250 respectively
Then a + (a – 50) + (a – 100) + (a – 150) + (a – 200) + (a – 250) = 3150

$\Rightarrow\ 6a - 750 = 3150\ or\ a = \frac{3900}{6} = 650$

∴ The value of the 1st, 2nd, 3rd …..6th Prizes are 650, 600, 550, 500, 450, 400 respectively.
(a) Any value with justification is correct.
(b) Many more can be added like, honesty, good habits, friendship, respectively elders, loving youngers,….. etc.

Que 13. A person donates money to a trust working for education of children and women in some village. If the person donates ₹ 5,000 in the first year and his donation increases by ₹ 250 every year, find the amount donated by him in the eighth year and the total amount donated in eight years.
(a) Which mathematical concept is being used here?
(b) Write any two values the person mentioned here possess.
(c) Why do you think education of women is necessary for the development of a society?

Sol. The amount donated by the person each year forms an AP.
Here, a = 5,000, d = 250

We have to find a_8 and S_8

$$a_8 = a + 7d = 5{,}000 + 7 \times 250 = ₹\,6{,}750$$

$$S_8 = \frac{8}{2}[2a + 7d] = 4(2 \times 5{,}000 + 7 \times 250) = 4 \times 11{,}750 = ₹\,47{,}000$$

(a) Arithmetic progression.
(b) Socially aware and responsible citizen.
(c) Educating a woman means educating the whole family and an educated family makes development in society.

Que 14. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed. Find the usual speed of the plane.
What value is depicted in this question?

Sol. Let the usual speed of plane be x km/h.

$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$\Rightarrow \quad [1500(x + 250) - 1500x]\,2 = x(x + 250)$

$\Rightarrow \quad x^2 + 250x - 750000 = 0$

$\Rightarrow \quad (x + 1000)(x - 750) = 0 \Rightarrow x = 750 \text{ or } x = -1000$ (Which is neglected)

$\therefore$ Using speed of plane = 750 km/h

Values: Helping others

Que 15. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?
What value is reflected in this question?

Sol. Here a = ₹ 450, d = ₹ 20, n = 12

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20] = 6[1120] = 6720 > 6500$$

$\therefore$ Reshma will be able to send her daughter to school

Value: Encouraging efforts for girl education.