Very Short Answer Type Questions

[1 mark]

Que 1. Which of the following can be the nth term of an AP?
$4n + 3, 3n^2 + 5, n^3 + 1$ give reason.

Sol. $4n + 3$ because nth term of an AP can only be a linear relation in n as $a_n = a + (n - 1) d$.

Que 2. Is 144 a term of the AP: 3, 7, 11,…? Justify your answer.

Sol. No, because here a = 3 an odd number and d = 4 which is even. So, sum of odd and even must be odd whereas 144 is an even number.

Que 3. The first term of an AP is p and its common difference is q. Find its 10^{th} term.

Sol. $a_{10} = a + 9d = p + 9q$.

Que 4. For what value of k: 2k, k+10 and 3k + 2 are in AP?

Sol. Given numbers are in AP

$\therefore \quad (k + 10) - 2k = (3k + 2) - (k + 10)$

$\Rightarrow \quad -k + 10 = 2k - 8 \quad \text{or} \quad 3k = 18 \quad \text{or} \quad k = 6.$

Que 5. If $a_n = 5 - 11n$, find the common difference.

Sol. we have $a_n = 5 - 11n$

Let d be the common difference

$d = a_{n+1} - a_n$

$= 5 - 11(n + 1) - (5 - 11n) = 5 - 11n - 11 - 5 + 11n = -11$

Que 6. If n^{th} term of an AP is $\frac{3+n}{4}$, find its 8^{th} term.

Sol. $a_n = \frac{3+n}{4}$; so, $a_8 = \frac{3+8}{4} = \frac{11}{4}$

Que 7. For what value of p are 2p + 1, 13, 5p – 3, three consecutive terms of AP?

Sol. since 2p + 1, 13, 5p – 3 are in AP.

$\therefore$ Second terms – First terms = Third term – Second term.

$13 - (2p + 1) = 5p - 3 - 13$

$13 - 2p - 1 = 5p - 16 \quad \Rightarrow \quad 12 - 2p = 5p - 16$

$-7p = -28 \quad \Rightarrow \quad p = 4$

Que 8. In an AP, if $d = -4$, $n = 7$, $a_n = 4$ them find a.

Sol. We know, $a_n = a + (n - 1) d$

Putting the value given, we get

⇒ $4 = a + (7 – 1) (-4)$ or $a = 4 + 24$ ⇒ $a = 28$

Que 9. Find the 25th term of the AP: -5, $\frac{-5}{2}$,0, $\frac{5}{2}$ …………

Sol. Here, $a = -5$, $d = -\frac{5}{2} – (-5) = \frac{5}{2}$

We know,

$a_{25} = a + (25-) d$

$= (-5) + 24 \left(\frac{5}{2}\right) = -5 + 60 = 55$

Que 10. Find the common difference of an AP in which $a_{18} – a_{14} = 32$.

Sol. Given, $a_{18} – a_{14} = 32$

⇒ $(a + 17d) – (a + 13d) = 32$

⇒ $17d – 13d = 32$ or $d = \frac{32}{4} = 8$

Que 11. If 7 times the 7th term of an AP is equal to 11 times its 11th terms, then find its 18th term.

Sol. Given, $7a_7 = 11a_{11}$

⇒ $7 (a + 6d) = 11(a + 10d)$ or $7a + 42d = 11a + 110d$

⇒ $4a + 68d = 0$ or $a + 17d = 0$

Now, $a_{18} = a + 17d = 0$

Que 12. In an AP, if a = 1, $a_n = 20$ and $S_n = 399$, then find n.

Sol. Given, $a_n = 20$

⇒ $1 + (n + 1) d = 20$ ⇒ $(n – 1) d = 19$

⇒ $S_n = \frac{n}{2} \{2a + (n – 1) d\}$ ⇒ $399 = \frac{n}{2} \{2 \times 1 + 19\}$

⇒ $\frac{399 \times 2}{21} = n$ ⇒ $n = 38$

Que 13. Find the 9th term from the end (toward the first term) of the AP 5, 9, 13, …, 185.

Sol. $l = 185$, $d = -4$

$L_9 = l – (n – 1) d = 185 – 8 \times 4 = 153$

Short Answer Type Questions – I

[2 marks]

Que 1. In which of the following situations, does the list of numbers involved to make an AP? If yes, give reason.
(i) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.
(ii) The amount of money in the account every year, when ₹ 10,000 is deposited at simple interest at 8% per annum.

Sol. (i) The numbers imvolved are 150, 200, 250, 300, …
Here 200 - 150 = 250 - 200 = 300 - 250 and so on
∴ It forms an AP with a = 150, d = 50
(ii) The numbers involved are 10,800, 11,600, 12,400, …
which forms an AP with a = 10,800 and d = 800.

Que 2. Find the 20th term from the last term of the AP, 3, 8, 13,…, 253.

Sol. We have, last term = l = 253
And, common difference = d = 2nd term - 1st term = 8 – 3 = 5
Therefore, 20th term from end = l - (20 - 1) × d = 253 – 19 x 5 = 253 – 95 = 158.

Que 3. If the sum of the first p terms of an AP is $ap^2 + bp$, find its common difference.

Sol. $a_p = S_{p-1} = (ap^2 + bp) – [a(p – 1)^2 + b(p – 1)]$

$= ap^2 + bp – (ap^2 + a – 2ap + bp – b)$

$= ap^2 + bp – ap^2 – a + 2ap – bp + b = 2ap + b – a$

∴ $a_1 = 2a + b – a = a + b$ and $a_2 = 4a + b – a = 3a + b$

⇒ $d = a_2 – a_1 = (3a + b) – (a + b) = 2a$

Que 4. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Sol. Let the first term be 'a' and common difference be ‘d’.
Given, a = 5, T_n, = 45, S_n = 400
$T_n = a + (n – 1)d$ ⇒ $45 = 5 + (n – 1)d$
⇒ $(n – 1)d = 40$ … (i)
$S_n = \frac{n}{2}(a + T_n)$ ⇒ $400 = \frac{n}{2}(5 + 45)$
⇒ $n = 2 \times 8 = 16$

Substituting the value of n in (i)
$(16 – 1)d = 40$ ⇒ $d = \frac{40}{15} = \frac{8}{3}$

Que 5. Find the number of natural number between 101 and 999 which are divisible by both 2 and 5.

Sol. Given: $a_1 = 110$, $d = 10$, $a_n = 990$

We know, $a_n = a_1 + (n – 1) d$

$990 = 110 + (n - 1) 10$

$(n – 1) = \frac{990-110}{10}$ ⇒ $n = 88 + 1 = 89$

Que 6. The sum of the first n terms of an AP is $3n^2 + 6n$. Find the nth terms of the AP.

Sol. Given: $S_n = 3n^2 + 6n$

$S_{n-1} = 3(n – 1)^2 + 6(n – 1) = 3(n^2 + 1 – 2n) + 6n – 6$

$= 3n^2 + 3 – 6n + 6n – 6 = 3n^2 – 3$

The n^{th} terms will be a_n

$Sn = S_{n-1} + a_n$

$a_n = S_n – S_{n-1} = 3n^2 + 6n – 3n^2 + 3 = 6n + 3$

Que 7. How many terms of the AP 18, 16, 14, ... be taken so that their sum is zero?

Sol. Here, $a = 18$, $d = -2$, $S_n = 0$

Therefore, $\frac{n}{2}[36 + (n – 1) – 2] = 0$

⇒ $n (36 – 2n + 2) = 0$ ⇒ $n =19$

Que 8. The 4th term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term.

Sol. ∴ $a_4 = 0$ (Given)

⇒ $a + 3d = 0$ ⇒ $a = -3d$

$a_{25} = a + 24d = - 3d + 24d = 21d$

$3a_{11} = 3(a + 10d) = 3(7d) = 21d$

∴ $a_{25} = 3a_{11}$ Hence proved.

Que 9. If the ratio of sum of the first m and n term of an AP is $m^2 : n^2$, show that ratio of its mth and nth terms is (2m – 1) : (2n – 1).

Sol. $\frac{s_m}{s_n} = \frac{m^2}{n^2} = \frac{\frac{m}{2}}{\frac{n}{2}} \frac{(2a+(m-1)d)}{(2a+(n-1)d)}$

$\Rightarrow \frac{m}{n} = \frac{2a+(m-1)d}{2a+(n-1)d}$ $\Rightarrow$ 2am + mnd – md = 2an + mnd – nd

$\Rightarrow$ a (2m – 2n) = d (m – n) $\Rightarrow$ 2a = d

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+2(m-1)a}{a+2(n-1)a} = \frac{2m-1}{2n-1}$$

Hence proved.

Short Answer Type Questions – II

[3 marks]

Que 1. Which term of the AP: 3, 8, 13, 18, …, is 78?

Sol. let a_n be the required term and we have given AP

3, 8, 13, 18, …

Here, a = 3, d = 8 – 3 = 5 and a_n = 78

Now, $a_n = a + (n - 1)d \Rightarrow \quad 78 = 3 + (N - 1) \times 5$

$\Rightarrow \quad 78 - 3 = (N - 1) \times 5 \quad \Rightarrow \quad 75 = (n - 1) \times 5$

$\Rightarrow \quad \frac{75}{5} = n - 1 \quad \Rightarrow \quad 15 = n - 1 \quad \Rightarrow \quad n = 15 + 1 = 16$

Hence, 16th term of given AP is 78.

Que 2. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. Let the first term be a and common difference be d.

Now, we have

$a_{11} = 38 \quad \Rightarrow \quad a = (11 - 1)\,d = 38$

$\Rightarrow \quad a + 10d = 38 \quad \ldots(i)$

and $a_{16} = 73 \quad \Rightarrow \quad a = (16 - 1)\,d = 73$

$\Rightarrow \quad a = 15d = 73 \quad \ldots(ii)$

Now subtracting (ii) from (i), we have

Now,

$$a + 10d = 38$$
$$\underline{-a \pm 15d = -73}$$
$$-5d = -35 \quad \text{or} \quad 5d = 35$$
$$d = \frac{35}{5} = 7$$

Putting the value of d in equation (i), we have

$a + 10 \times 7 = 38 \quad \Rightarrow \quad a + 70 = 38$

$\Rightarrow \quad a = 38 - 70 \quad \Rightarrow \quad a = -32$

We have, a = - 32 and d = 7

Therefore, $a_{31} = a + (31 - 1)\,d$

$\Rightarrow \quad a_{31} = a + 30d = (-32) + 30 \times 7 = -32 + 210$

$\Rightarrow \quad a_{31} = 178$

Que 3. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Let a be the first term and d be the common difference.

Since, give AP consist of 50 terms, so n = 50

$a_3 = 12 \Rightarrow a + 2d = 12$ … (i)

$i.e.,$ $a_{50} = 106 \Rightarrow a + 49d = 106$ … (ii)

Subtracting (i) from (ii), we have

$47d = 94 \Rightarrow d = \frac{94}{47} = 2$

Putting the value of d in equation (i), we have

$a + 2 \times 2 = 12 \Rightarrow a = 12 - 4 = 8$

Here, a = 8, d = 2

∴ 29th term is given by

$a_{29} = a + (29 - 1) d = 8 + 28 \times 2$

$\Rightarrow a_{29} = 8 + 56 \Rightarrow a_{29} = 64$

Que 4. If the 8th term of an AP is 31 and the 15th term is 16 more than the 11th term, find the AP.

Sol. Let a be the first term and d be the common difference of the AP.

We have, $a_8 = 31$ and $a_{15} = 16 + a_{11}$

$\Rightarrow a + 7d = 31$ and $a + 14d = 16 + a + 10d$

$\Rightarrow a + 7d = 31$ and $4d = 16$

$\Rightarrow a + 7d = 31$ and $d = 4 \Rightarrow a + 7 \times 4 = 31$

$\Rightarrow a + 28 = 31 \Rightarrow a = 3$

Hence, the AP is a, a + d, a + 2d, a + 3d,…

$i.e.,$ 3, 7, 11, 15, 19, …

Que 5. Which term of the arithmetic progression 5, 15, 25, … will be 130 more than its 31st term?

Sol. We have, a = 5 and d = 10

∴ $a_{31} = a + 30d = 5 + 30 \times 10 = 305$

Let nth term of the given AP be 130 more than its 31st term. Then,

$a_n = 130 + a_{31}$

∴ $a + (n - 1) d = 130 + 305$

$\Rightarrow$ 5 + 10 (n – 1) = 435 $\Rightarrow$ 10 (n – 1) = 430

$\Rightarrow$ n – 1 = 43 $\Rightarrow$ n = 44

Hence, 44th term of the given AP is 130 more than its 31st term.

Que 6. Find the sum given below:

$$\mathbf{7 + 10\frac{1}{2} + 14 + \ldots + 84}$$

Sol. let a be the first term, d be the common difference and a_n be the last term of given AP.

Thus, a = 7, d = $10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$ and a_n = 84

Now, $a_n = a + (n-1)\,d$

$\Rightarrow$ $84 = 7 + (n-1) \times \frac{7}{2}$ $\Rightarrow$ $84 - 7 = (n-1) \times \frac{7}{2}$

$\Rightarrow$ $77 = (n-1) \times \frac{7}{2}$ $\Rightarrow$ $11 \times 2 = (n-1)$ $\Rightarrow$ $22 = n - 1$

$\therefore$ n = 22 + 1 = 23

Now, $S_n = \frac{n}{2}\,[2a + (n-1).\,d]$

$\Rightarrow$ $S_{23} = \frac{23}{2}\left[2 \times 7 + (23-1) \times \frac{7}{2}\right]$ $\Rightarrow$ $S_{23} = \frac{23}{2}\left[14 + 22 \times \frac{7}{2}\right]$

$= \frac{23}{2}\,[14 + 77] = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

Que 7. In an AP: given l = 28, S = 144, and there are total 9 terms. Find a.

Sol. We have, l = 28, S = 144 and n = 9

Now, $l = a_n = 28$

28 = a + (n – 1) d $\Rightarrow$ 28 = a + (9 – 1) d

$\Rightarrow$ 28 = a + 8d … (i)

And S = 144

$\Rightarrow$ $144 = \frac{n}{2}\,[2a + (n-1)\,d]$ $\Rightarrow$ $144 = \frac{9}{2}\,[2a + (9-1)\,d]$

$\Rightarrow$ $\frac{144 \times 2}{9} = 2a + 8d$ $\Rightarrow$ 32 = 2a + 8d

$\Rightarrow$ 16 = a + 4d … (ii)

Now, subtracting equation (ii) from (i), we get

4d = 12 or d = 3

Putting the value of d in equation (i), we have

a + 8 x 3 = 28

⇒ a + 24 = 28 ⇒ a = 28 – 24

∴ a = 4.

Que 8. How many terms of the AP : 9, 17, 25, … must be taken to give a sum of 636?

Sol. let sum of n term be 636.

Sn = 636, a = 9, d = 7 – 9 = 8

⇒ $\frac{n}{2}$ [2a + (n – 1) d] = 636 ⇒ $\frac{n}{2}$ [2 x 9 + (n – 1) x 8] = 636

⇒ $\frac{n}{2}$ x 2 [9 + (n – 1) x 4] = 636 ⇒ n [9 + 4n – 4] = 636

⇒ n [5 + 4n] = 636 ⇒ 5n + $4n^2$ = 636

⇒ $4n^2$ + 5n – 636

∴ n = $\frac{-5 \pm \sqrt{(5)^2 - 4 \times (-636)}}{2 \times 4} = \frac{-5 \pm \sqrt{25+10176}}{8}$

$= \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8} = \frac{96}{8}, \frac{-106}{8}$ = 12, - $\frac{53}{4}$

But n $\neq \frac{-53}{4}$ So, n = 12

Thus, the sum of 12 terms of given AP is 636.

Que 9. How many terms of the series 54, 51, 48 …… be taken so that, their sum is 513? Explain the double answer.

Sol. Clearly, the given sequence is an AP with first term a = 54 and common difference d = - 3. Let the sum of n terms be 513. Then,

Sn = 513

⇒ $\frac{n}{2}$ {2a + (n – 1) d} = 513 ⇒ $\frac{n}{2}$ [108 + (n – 1) x – 3] = 513

⇒ n^2 – 37n + 342 = 0 ⇒ (n – 18) (n – 19) = 0

⇒ n = 18 or 19

Here, the common difference is negative. So, 19th term is given by

$a_{19} = 54 + (19 – 1) \times – 3 = 0$

Thus, the sum of 18 terms as well as that of 19 terms is 513.

Que 10. The first term, common difference and last term of an AP are 12, 6 and 252 respectively. Find the sum of all terms of this AP.

Sol. We have, a = 12, d = 6 and l = 252

Now, $l = 252 \quad \Rightarrow \quad a_n = 252$

$\Rightarrow \quad l = a + (n – 1) d \quad \Rightarrow \quad 252 = 12 + (n – 1) \times 6$

$\Rightarrow \quad 240 = (n – 1) \times 6 \quad \Rightarrow \quad n – 1 = 40$ or $n = 41$

Thus, $Sn = \frac{n}{2}(a + l)$

$\Rightarrow \quad S_{41} = \frac{41}{2}(12 + 252) = \frac{41}{2}(264) = 41 \times 132 = 5412$

Que 11. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol. We have, $S_7 = 49$

$\Rightarrow 49 = \frac{7}{2}[2a + (7 – 1) \times d] \quad \Rightarrow \quad 49 \times \frac{2}{7} = 2a + 6d$

$\Rightarrow 14 = 2a + 6d \quad \Rightarrow \quad a + 3d = 7 \qquad \ldots(i)$

And $S_{17} = 289$

$\Rightarrow 289 = \frac{17}{2}[2a + (17 – 1) d] \quad \Rightarrow \quad 2a + 16d = \frac{289 \times 2}{17} = 34$

$\Rightarrow a + 8d = 17 \qquad \ldots(ii)$

Now, subtracting equation (i) from (ii), we have

$5d = 10 \quad \Rightarrow \quad d = 2$

Putting the value of d in equation (i), we have

$a + 3 \times 2 = 7 \quad \Rightarrow \quad a = 7 – 6 = 1$

Here, a = 1 and d = 2

Now, $S_n = \frac{n}{2}[2a + (n – 1) d]$

$= \frac{n}{2}[2 \times 1 + (n – 1) \times 2] = \frac{n}{2}[2 + 2n – 2] = \frac{n}{2} \times 2n = n^2$

Que 12. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. We have, $a = 5$ and $S_n = 400$

Now, $l = 45$

$\Rightarrow$ $a_n = 45$ $\Rightarrow$ $a + (n - 1)d = 45$

$\Rightarrow$ $5 + (n - 1) \times d = 45$ $\Rightarrow$ $(n - 1)d = 40$ …(i)

Again $S_n = 400$ $\Rightarrow$ $\frac{n}{2}[2a + (n - 1)d] = 400$

$\Rightarrow$ $\frac{n}{2}[2 \times 5 + (n - 1)d] = 400$ …(ii)

$\frac{n}{2}[10 + 40] = 400$ (Using equation (i)

$\Rightarrow$ $\frac{n}{2} \times 50 = 400$ $\Rightarrow$ $n = \frac{400}{25} = 16$

Now, putting the value of n in equation (i), we have

$(16 - 1)d = 40$ $\Rightarrow$ $15d = 40$

$\therefore$ $d = \frac{40}{15} = \frac{8}{3}$

Hence, number of terms is 16 and common difference is $\frac{8}{3}$.

Que 13. If the seventh terms of an AP is $\frac{1}{9}$ and its ninth terms is $\frac{1}{7}$, find its 63rd term.

Sol. Given, $a_7 = \frac{1}{9}$ and $a_9 = \frac{1}{7}$

$a_7 = a + (7 - 1)d = \frac{1}{9}$

$a + 6d = \frac{1}{9}$ …(i)

$a_9 = a + (9 - 1)d = \frac{1}{7}$

$a + 8d = \frac{1}{7}$ …(ii)

Subtracting (i) from (ii) we get

$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63}$ $\Rightarrow$ $d = \frac{1}{63}$

Putting the value of d in (i)

$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$ $\Rightarrow$ $a = \frac{1}{9} - \frac{1}{63} = \frac{7-6}{63}$ $\Rightarrow$ $a = \frac{1}{63}$

$\therefore$ $a_{63} = a + (63 – 1)\, d$

$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{63}{63} = 1$

Que 14. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP.

Sol. $a_5 + a_9 = 30$

$\Rightarrow$ (a + 4d) + (a + 8d) = 30

$\Rightarrow$ 2a + 12d = 30 $\Rightarrow$ a + 6d = 15

$\Rightarrow$ a = 15 – 6d …(i)

$a_{25} = 3a_8$ $\Rightarrow$ a + 24d = 3(a + 7d)

a + 24d = 3a + 21d $\Rightarrow$ 2a = 3d

Putting the value of a form (i), we have

2 (15 – 6d) = 3d $\Rightarrow$ 30 – 12d = 3d

$\Rightarrow$ 15d = 30 $\Rightarrow$ d = 2

So, a = 15 – 6 x 2 = 15 – 12 [From equation (i)]

$\Rightarrow$ a = 3

The AP will be 3, 5, 7, 9…

Que 15. The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find the 28th terms of this AP.

Sol. Sum of first seven terms,

$S_n = \frac{n}{2} [2a + (n – 1)\, d]$

$S_7 = \frac{7}{2} [2a + (7 – 1)\, d] = \frac{7}{2} [2a + 6d]$

$\Rightarrow$ 63 = 7a + 21a $\Rightarrow$ $a = \frac{63 - 21d}{7}$ …(i)

$S_{14} = \frac{14}{2} [2a + 13d]$ $\Rightarrow$ $S_{14} = 7 [2a + 13d] = 14a + 91d$

But ATQ, $S_{1-7} + S_{8-14} = S_{14}$

63 + 161 = 14a + 91d $\Rightarrow$ 224 = 14a + 91d

$2a + 13d = 32$ ...(ii)

$2\left(\frac{63 - 21d}{7}\right) + 13d = 32 \quad \Rightarrow \quad 126 - 42d + 91d = 224$

$\Rightarrow \quad 49d = 98 \quad \Rightarrow \quad d = 2$

$\Rightarrow \quad a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = \frac{21}{7} = 3$

$\Rightarrow \quad a_{28} = a + 27d = 3 + 27 \times 2 \quad \Rightarrow \quad a_{28} = 3 + 54 = 57$

Que 16. If the ratio of the sum of first in terms of two AP's is (7n + 1): (4n + 27), find the ratio of their mth terms.

Sol. $\frac{S_n}{S'_n} = \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n+1}{4n+27} = \frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'} = \frac{7n+1}{4n+27}$ ...(i)

Since $\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'}$, = So replacing $\frac{n-1}{2}$ by m – 1 and n by 2m – 1 in (i)

$\frac{a+(m-1)d}{a'+(m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27} \quad \Rightarrow \quad \frac{t_m}{t'_m} = \frac{14m-6}{8m+23}$

Long Answer Type Questions

[4 marks]

Que 1. The sum of the 4th and 8th term of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.

Sol. We have, $a_4 + a_8 = 24$

$\Rightarrow \quad a + (4 - 1)d + a + (8 - 1)d = 24 \quad \Rightarrow \quad 2a + 3d + 7d = 24$

$\Rightarrow \quad 2a + 10d = 24 \quad \Rightarrow \quad 2(a + 5d) = 24$

$\therefore \quad a + 5d = 12 \quad$ …(i)

and, $a_6 + a_{10} = 44$

$\Rightarrow \quad a + (6 - 1)d + a + (10 - 1)d = 44 \quad \Rightarrow \quad 2a + 5d + 9d = 44$

$\Rightarrow \quad 2a + 14d = 44 \quad \Rightarrow \quad a + 7d = 22 \quad$ …(ii)

Subtracting (i) from (ii), we have

$2d = 10$

$\therefore \quad d = \frac{10}{2} = 5$

Putting the value of d in equation (i), we have

$a + 5 \times 5 = 12 \quad \Rightarrow \quad a = 12 - 25 = -13$

Here, $a = -13, d = 5$

Hence, first three terms are

$-13, -13 + 5, -13 + 2 \times 5 \quad i.e., \quad -13, -8, -3$

Que 2. The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and 12th term.

Sol. We have, $S_n = 3n^2 - 4n \quad (i)$

Replacing n by $n - 1$, we get

$$S_{n-1} = 3(n-1)^2 - 4(n-1) \quad (ii)$$

We know,

$$a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$$

$$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$$

$$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$$

So, nth term $a_n = 6n - 7$ (iii)

To get the AP, substituting $n = 1,2,3, \ldots$ respectively in(iii), we get

$a_1 = 6 \times 1 - 7 = -1, a_2 = 6 \times 2 - 7 = 5$

$a_3 = 6 \times 3 - 7 = 11, \ldots$

Hence, AP is $-1, 5, 11, \ldots$

Also, to get 12th term, substituting $n = 12$in(iii), we get

$a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$

Que 3. Divide 56 into four parts which are in AP such that the ratio of product of extremes to the product of means is $5:6$.

Sol. Let the four parts be $a - 3d, a - d, a + d, a + 3d$.

Given, $(a - 3d) + (a - d) + (a + d) + (a + 3d) = 56$

$\Rightarrow \quad 4a = 56$ or $a = 14$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$

$\Rightarrow \quad \frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6} \quad \Rightarrow \quad 6(196 - 9d^2) = 5(196 - d^2) \quad [\because a = 14]$

$\Rightarrow \quad 6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

$\Rightarrow \quad 49d^2 = 6 \times 196 - 5 \times 196 = 196$

$\Rightarrow \quad d^2 = 4$ or $d = \pm 2$

$\therefore$ Required parts are $14 - 3 \times 2, 14 - 2, 14 + 2, 14 + 3 \times 2$

Or $14 - 3(-2), 14 + 2, 14 - 2, 14 + 3(-2)$

$i.e.,$ $8, 12, 16, 20$

Que 4. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

Sol. Let $'a'$ be the first term and $'d'$ be the common difference.

nth term of AP is $a_n = a + (n - 1)d$

And sum of AP is $S_n = \frac{n}{2}[2a + 9d]$

Sum of first 10 terms $= 210 = \frac{10}{2}[2a + 9d]$

$\Rightarrow \quad 42 = 2a + 9d \Rightarrow \quad 2a + 9d = 42 \quad \ldots(i)$

15^{th} term from the last = $(50 - 15 + 1th) = 36th$ term

$\Rightarrow \quad a_{36} = a + 35d$

Sum of last 15 terms = $2565 = \frac{15}{2}[2a_{36} + (15 - 1)d]$

$\Rightarrow \quad 2565 = \frac{15}{2}[2(a + 35d) + 14d$

$\Rightarrow \quad 2565 = 15[a + 35d + 7d]$

$\Rightarrow \quad a + 42d = 171$

Subtracting (ii) from (i) we get

$9d - 84d = 42 - 342 \quad \Rightarrow \quad 75d = 300$

$\Rightarrow \quad d = \frac{300}{75} = 4$

Putting the value of d in (ii)

$42 \times 4 + a = 171 \quad \Rightarrow \quad a = 171 - 168$

$\Rightarrow \quad a = 3$

So, the AP formed is $3, 7, 11, 15, \dots \dots$ and 199.

Que 5. If S_n denotes the sum of the first n terms of an AP, prove that $S_{30} = 3(S_{30} = 3(S_{20} - S_{10})$.

Sol. $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \quad \Rightarrow \quad S_{30} = 30a + 435d \quad \dots(i)$

$\Rightarrow \; S_{20} = \frac{20}{2}[2a + 19d] \quad \Rightarrow \quad S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \quad \Rightarrow \quad S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30} \quad$ [From)(i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

Que 6. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?

Sol. Let total time be n minutes

Total distance covered by thief = $100n$ metres

Total distance covered by policeman $= 100 + 110 + 120 + \cdots + (n-1)$ terms

$\therefore \qquad 100n = \frac{n-1}{2}[100(2) + (n-2)10]$

$\Rightarrow \qquad 200n = (n-1)(180 + 10n) \qquad \Rightarrow \qquad 10n^2 - 30n - 180 = 0$

$\Rightarrow \qquad n = 6$

Policeman took $(n-1) = (6-1) = 5$ minutes to catch the thief.

Que 7. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceeding the house numbered X is equal to sum of the numbers of houses following X. Find value of X.

Sol. The numbers of houses are 1, 2, 3, 4 ……….49.

The numbers of the houses are in AP, where $a = 1$ and $d = 1$

Sum of n terms of an $AP = \frac{n}{2}[2a + (n-1)d]$

Let Xth number house be the required house.

Sum of number of houses preceding Xth house is equal to $S_{x-1} i.e.,$

$S_{X-1} = \frac{X-1}{2}[2a + (X-1-1)d] \qquad \Rightarrow \qquad S_{X-1} = \frac{X-1}{2}[2 + (X-2)]$

$\Rightarrow \qquad S_{X-1} = \frac{X-1}{2}[2 + X - 2 \qquad \Rightarrow \qquad S_{X-1} = \frac{X(X-1)}{2}$

Sum of numbers of houses following X^{th} house is equal to $S_{49} - S_X$

$= \frac{49}{2}[2a + (49-1)d] - \frac{X}{2}(2a + (X-1)d$

$= \frac{49}{2}[2 + 48] - \frac{X}{2}(2 + X - 1) = \frac{49}{2}(50) - \frac{X}{2}(X+1)$

$= 25(49) - \frac{X}{2}(X+1)$

Now, we are given that

Sum of number of houses before X is equal to sum of number of houses after X.

$i.e., S_{X-1} = S_{49} - S_X$

$\Rightarrow \qquad \frac{X(X-1)}{2} = 25(49) - X\frac{(X+1)}{2} \qquad \Rightarrow \qquad \frac{X^2}{2} - \frac{X}{2} = 1225 - \frac{X^2}{2} - \frac{X}{2}$

$\Rightarrow \qquad X^2 = 1225 \qquad \Rightarrow \qquad X = \sqrt{1225}$

$\Rightarrow \qquad X = \pm 35$

Since number of houses is positive integer, $\therefore X = 35$

Long Answer Type Questions

[4 marks]

Que 1. The sum of the 4th and 8th term of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.

Sol. We have, $a_4 + a_8 = 24$

$\Rightarrow$ a + (4 – 1) d + a + (8 – 1) d = 24 $\Rightarrow$ 2a + 3d + 7d = 24

$\Rightarrow$ 2a + 10d = 24 $\Rightarrow$ 2 (a + 5d) = 24

$\therefore$ a + 5d = 12 …(i)

and, $a_6 + a_{10} = 44$

$\Rightarrow$ a + (6 – 1) d + a + (10 – 1) d = 44 $\Rightarrow$ 2a + 5d + 9d = 44

$\Rightarrow$ 2a + 14d = 44 $\Rightarrow$ a + 7d = 22 …(ii)

Subtracting (i) from (ii), we have

2d = 10

$\therefore$ $d = \frac{10}{2} = 5$

Putting the value of d in equation (i), we have

a + 5 x 5 = 12 $\Rightarrow$ a = 12 – 25 = -13

Here, $a = -13, d = 5$

Hence, first three terms are

$-13, -13 + 5, -13 + 2 \times 5$ $i.e.,$ $-13, -8, -3$

Que 2. The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and 12th term.

Sol. We have, $S_n = 3n^2 - 4n$ (i)

Replacing n by $n - 1$, we get

$S_{n-1} = 3(n-1)^2 - 4(n-1)$ (ii)

We know,

$a_n = S_n - S_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$

$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$

$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$

So, nth term $a_n = 6n - 7$ (iii)

To get the AP, substituting $n = 1,2,3, \ldots$ respectively in(iii), we get

$a_1 = 6 \times 1 - 7 = -1, a_2 = 6 \times 2 - 7 = 5$

$a_3 = 6 \times 3 - 7 = 11, \ldots$

Hence, AP is $-1, 5, 11, \ldots$

Also, to get 12th term, substituting $n = 12$in(iii), we get

$a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$

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$\Rightarrow \qquad X = \pm 35$

Since number of houses is positive integer, $\therefore X = 35$

Value Based Questions

Que 1. Some people collected money to be donated in some orphanages. A part of the donation is fixed and remaining depends on the number of children in the orphanage. If they donated ₹ 9,500 in the orphanage having 50 children and ₹ 13,250 with 75 children, find the fixed part of the donation and the amount donated for each child.
What values do these people posses?

Sol. Let the fixed donation be ₹ x and amount donated for each child be ₹ y.

Then $x + 50y = 9500$ …(i)

$x + 75y = 13250$ …(ii)

Subtracting (i) from (ii), we get

$25y = 3750$ or $y = 150$

From (i), $x = 9500 - 50y = 9500 - 50 \times 150 = 2000$

∴ Fixed amount donated = ₹ 2,000.

Amount donated for each child = ₹ 150.

Helpfulness, cooperation, happiness, caring.

Que 2. The fraction of people in a society using CNG in their vehicles becomes $\frac{9}{11}$, if 2 is added to both its numerator and denominator. If 3 is added to both its numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction. What does this fraction show?

Sol. Let the fraction be $\frac{x}{y}$.

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \Rightarrow \quad 11x - 9y = -4$$

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \Rightarrow \quad 6x - 5y = -3$$

Solving for x and y gives x = 7 and y = 9

$$\therefore \quad Fraction = \frac{x}{y} = \frac{7}{9}$$

More people are becoming aware about scarcity of petrol, so they are switching on to alternative resources.

Que 3. Reading book in a library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid ₹ 27 for a book kept for seven days. While Bunty paid ₹ 21 for the book kept for five days.

(i) Find the fixed charge.

(ii) Find how much additional charge Shristi and Bunty paid.
(iii) Which mathematical concept is used in this problem?
(iv) Which value does it depict?

Sol. Let fix charge for reading book be ₹ x and additional charge for each day be ₹ y.

Then, x + 4y = 27 …(i)

On subtraction $\frac{x + 2y = 21}{2y = 6}$ ….(ii)

Y = 3

Putting value of y in equation (i) we get

x + 4 × 3 = 27 ⇒ x + 12 = 27 ⇒ x = 15

(i) Fixed charge on the book is ₹ 15.
(ii) Bunty paid additional ₹ 6 and Shristi paid ₹ 12.
(iii) Pair of linear equation in two variables.
(iv) Reading is a good habit.

Que 4. An honest person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But if he had interchanged amount invested, he would have received ₹ 4 more as interest.
(i) How much amount did he invest at different rates?
(ii) Which mathematical concept is used in this problem?
(iii) Which value is being emphasized here?

Sol. (i) Let the person invest ₹ x at rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

Yearly interest = $\frac{12x}{100} + \frac{10y}{100}$

∴ $\frac{12x}{100} + \frac{10y}{100} = 130 \Rightarrow 12x + 10y = 13000$ …(i)

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$\frac{10x}{100} + \frac{12y}{100} = 134 \Rightarrow 10x + 12y = 13400$ …(ii)

Adding eqn. (i) and (ii) we get

22x + 22y = 26400

x + y = 1200 …(iii)

Subtracting (ii) from (i) we get

2x – 2y = – 400

x – y = – 200 …(iv)

Solving (iii) and (iv), we get

x = 500, y = 700

Thus, person invested ₹ 500 at 12% per annum and ₹ 700 at 10% per annum.

(ii) Pair of linear equation in two variables.
(iii) Honesty is the best policy.

Que 5. If the price of petrol is increased by ₹ 2 per litre, a person will have to buy 1 litre less petrol for ₹ 1740. Find the original price of petrol at that time.

(a) Why do you think the price of petrol is increasing day-by-day?
(b) What should we do to save petrol?

Sol. Let the original price of the petrol be ₹ x per litre.

Then, amount of petrol that can be purchased = $\frac{1740}{x}$

According to question

$$\frac{1740}{x} - \frac{1740}{x+2} = 1 \qquad \Rightarrow \quad 1740\,(x + 2 - x) = x\,(x + 2)$$

$\Rightarrow \quad x^2 + 2x - 3480 = 0 \qquad \Rightarrow \quad x^2 + 60x - 58x - 3480 = 0$

$\Rightarrow \quad (x + 60)\,(x - 58) = 0 \qquad \Rightarrow \quad x = 58, -60\ (rejected)$

∴ Original cost of petrol was ₹ 58 per litre.

(a) Petrol is a natural resource which is depleting day-by-day. So, due to more demand and less supply, its price is increasing.
(b) We should use more of public, transport and substitute petrol with CNG or other renewable resource.

Que 6. One fourth of a group of people claim they are creative, twice the square root of the group claim to be caring and the remaining 15 claim they are optimistic. Find the total number of people in the group.
(a) How many persons in the group are creative?
(b) According to you, which one of the above three values is more important for development of a society?

Sol. Let x be the total

Then, number of creative persons = $\frac{x}{4}$

Number of caring persons = $2\sqrt{x}$
and number of optimistic persons = 15

Thus, total number of persons = $\frac{x}{4} + 2\sqrt{x} + 15$

Now, by hypothesis, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y, \quad then \quad x = y^2$

$\Rightarrow \quad 3y^2 - 8y - 60 = 0 \qquad \Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y - 6) + 10\,(y - 6) = 0 \qquad \Rightarrow \quad (3y + 10)\,(y - 6) = 0$

$\Rightarrow \quad y = 6 \ or \ y = -\frac{10}{3}$

Now, $\quad y = -\frac{10}{3} \qquad \Rightarrow \quad x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9} (\because\ x = y^2)$

But, the number of persons cannot be a fraction.

∴ y = 6 $\qquad \Rightarrow \quad x = 6^2 = 36$

Hence, the number of people in the group = 36

(a) 9 persons

(b) All of these values have their own importance. A person having these values will certainly contribute to the development of society. However, the level of importance given to each of them depends upon a person's own attitude. Hence, any value with justification is correct. (Do yourself)

Que 7. In the centre of a rectangular plot of land of dimensions 120 m × 100 m, a rectangular portion is to be covered with trees so that the area of the remaining part of the plot is 10500 m^2. Find the dimensions of the area to be planted. Which social act is being discussed here? Give its advantages.

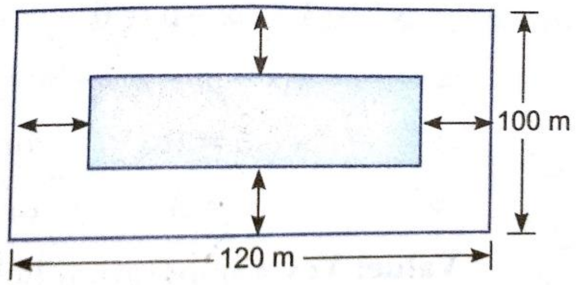

Fig. 1

Sol. Let the width of the unplanted area be x m

Then, dimension of area to be planted = (120 – 2x) and (100 – 2x)

∴ $(120 - 2x)\,(100 - 2x) = 120 \times 100 - 10500$

⇒ $12000 - 440x + 4x^2 = 1500 \; or \; x^2 - 110x + 2625 = 0$

⇒ $(x - 75)\,(x - 35) = 0 \; or \; x = 75, 35$

But x = 75 is not possible

∴ $x = 35$

Thus, dimension of area to be planted = (120 – 70) and (100 – 70) i.e., 50 m and 30 m.

Afforestation is being discussed here. Planting more trees helps in reducing air pollution and make the environment clean and green.

Que 8. Mr. Ahuja has to square plots of land which he utilises for two different purposes–one for providing free education to the children below the age of 14 years and the other to provide free medical services for the needy villagers. The sum of the areas of two square plots is 15425 m^2. If the difference of their perimeter is 60 m, find the sides of the two squares.
Which qualities of Mr. Ahuja are being depicted in the question?

Sol. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 15425$ …(i)

Let x be the length of the side of the bigger square.

4x – 4y = 60

⇒ x – y = 15 or x = y + 15 …(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 15)^2 + y^2 = 15425$ ⇒ $2y^2 + 30y - 15200 = 0$

or $y^2 + 15y - 7600 = 0$ or $(y + 95)(y - 80) = 0$ ⇒ $y =$

$-95, 80$

But, sides cannot be negative, so y = 80

Therefore, $x = 80 + 15 = 95$

Hence, sides of two squares are 80 m and 95 m.

Value: Caring, King, Social and generous.

Que 9. A takes 3 days longer than B to finish a work. But if they work together, then work is completed in 2 days. How long would each take to do it separately. Can you say cooperation helps to get more efficiency?

Sol. Let B finish a work in x days

then A finish a work in x + 3 days

According to question

$$\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2} \quad \Rightarrow \quad \frac{(x+3)+x}{x(x+3)} = \frac{1}{2}$$

$\Rightarrow$ $2x + 6 + 2x = x^2 + 3x$ $\Rightarrow$ $x^2 + 3x - 2x - 6 - 2x = 0$

$\Rightarrow$ $x^2 - x - 6 = 0$

Solving the equation

$x^2 - (3 - 2)x - 6 = 0$ $\Rightarrow$ $x^2 - 3x + 2x - 6 = 0$

$\Rightarrow$ $x(x - 3) + 2(x - 3) = 0$ $\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow$ $x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow$ x = 3 or x = – 2 (Days cannot be negative)

Value: Yes, Cooperation helps in improving work efficiency.

Que 10. In a class of 48 students, the number of regular students is more than the number of irregular students. Had two irregular students been regular, the product of the number of two types of students would be 380. Find the number of each type of students.

(a) Why is regularity essential in life?

(b) Write values other than regularity that a student must possess.

Sol. Let the number of regular students be x

Then, the number of irregular students = 48 – x

According to the question,

$(x + 2)(48 - x - 2) = 380$

$\Rightarrow$ $-x^2 + 44x + 92 - 380 = 0 \ or \ x^2 - 44x + 288 = 0$

$\Rightarrow$ $(x - 36)(x - 8) = 0$ or x = 36, 8

But x > 48 – x (given)

$\therefore$ x = 36

i.e., The number of regular students = 36

and the number of irregular students = 48 – 36 = 12

(a) Regularity in any sphere gives confidence which, in turn, leads to the development of an individual and the society as well.

(b) Honesty, Creativity, Confidence, Punctuality. (you may add more to this list)

Que 11. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are

studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?

Sol. Number of plants planted by class 1 = 2 (class 1 × 2 sections) = 2 × 1 × 2 = 4 trees Similarly, number of plants planted by class 2 = 2 (class 2 x 2 sections)
= 2 × 2 × 2 = 8 trees

We now know, a = 4 and d = 4
Number of classes n = 12
Total number of plants planted = Sum of AP = S_{12}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 4 + (12-1)4]$$

$$S^{12} = 6[8 + 44] = 6 \times 52 = 312\, trees$$

Values shown by the students:
(i) Envrionmental friendly
(ii) Social awareness
(iii) Sense of responsibility towards the society.

Que 12. A sum of ₹ 3150 is to be used to give six cash prizes to students of a school for overall academic performance, punctuality, regularity, cleanliness, confidence and creativity. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes.
(a) Which value according to you should be awarded with the maximum amount? Justify your answer.
(b) Can you add more values to the above ones which should be awarded?

Sol. Let the six prizes (1st, 2nd, 3rd …..6th) be a, a – 50, a – 100, a – 150, a – 200, a – 250 respectively
Then a + (a – 50) + (a – 100) + (a – 150) + (a – 200) + (a – 250) = 3150

$$\Rightarrow\ 6a - 750 = 3150\ or\ a = \frac{3900}{6} = 650$$

∴ The value of the 1st, 2nd, 3rd …..6th Prizes are 650, 600, 550, 500, 450, 400 respectively.
(a) Any value with justification is correct.
(b) Many more can be added like, honesty, good habits, friendship, respectively elders, loving youngers,….. etc.

Que 13. A person donates money to a trust working for education of children and women in some village. If the person donates ₹ 5,000 in the first year and his donation increases by ₹ 250 every year, find the amount donated by him in the eighth year and the total amount donated in eight years.
(a) Which mathematical concept is being used here?
(b) Write any two values the person mentioned here possess.
(c) Why do you think education of women is necessary for the development of a society?

Sol. The amount donated by the person each year forms an AP.

Here, a = 5,000, d = 250

We have to find a_8 and S_8

$$a_8 = a + 7d = 5{,}000 + 7 \times 250 = ₹\,6{,}750$$

$$S_8 = \frac{8}{2}[2a + 7d] = 4(2 \times 5{,}000 + 7 \times 250) = 4 \times 11{,}750 = ₹\,47{,}000$$

(a) Arithmetic progression.
(b) Socially aware and responsible citizen.
(c) Educating a woman means educating the whole family and an educated family makes development in society.

Que 14. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed. Find the usual speed of the plane.
What value is depicted in this question?

Sol. Let the usual speed of plane be x km/h.

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$\Rightarrow \quad [1500(x + 250) - 1500x]\,2 = x(x + 250)$
$\Rightarrow x^2 + 250x - 750000 = 0$
$\Rightarrow \quad (x + 1000)(x - 750) = 0 \quad \Rightarrow \quad x = 750 \text{ or } x = -1000$ (Which is neglected)
∴ Using speed of plane = 750 km/h
Values: Helping others

Que 15. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?
What value is reflected in this question?

Sol. Here a = ₹ 450, d = ₹ 20, n = 12

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20] = 6[1120] = 6720 > 6500$$

∴ Reshma will be able to send her daughter to school
Value: Encouraging efforts for girl education.