Very Short Answer Type Questions

[1 MARK]

Answer the following questions in one word, one sentence or as per the exact requirement of the question.

Que 1. What is the area of the triangle formed by the points O(0,0), A(-3,0) and B(5, 0)?

Sol. Area of $\Delta OAB = \frac{1}{2}[0(0-0) - 3(0-0) + 5(0-0)] = 0$

$\Rightarrow$ Given points are collinear

Que 2. If the centroid of triangle formed by points $P(a,b), Q(b,c)$ and $R(c,a)$ is at the origin, what is the value of $a+b+c$?

Sol. Centroid of $\Delta PQR = (\frac{a+b+c}{3}, \frac{b+c+a}{3})$

Given $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) = (0,0)$

$\Rightarrow$ $a+b+c=0$

Que 3. $AOBC$ is a rectangle whose three vertices are $A(0,3), O(0,0)$ and B $(5,0)$. Find the length of its diagonal.

Sol. diagonal $= AB = \sqrt{(5-0)^2} + (0-3)^2 = \sqrt{25} + 9 = \sqrt{34}$

Que 4. Find the value of a, so that the point $3,a)$ lie on the line $2x-3y=5$.

Sol. Since $(3,a)$ lies on the line $2x-3y=5$

Then $2(3) - 3(a) = 5$

$-3a = 5-6$

$-3a = -1$ $\Rightarrow$ $a = \frac{1}{3}$

Que 5. Find distance between the points (0,5) and (-5, 0).

Sol. Here $x_1 = 0, y_1 = 5, x_2 = -5$ and $y_2 = 0$

$\therefore$ $d = \sqrt{(x_2-x_1)^2} + (y_2-y_1)^2$

$= \sqrt{(-5-0)^2} + (0-5)^2$

$= \sqrt{25} + 25 = \sqrt{50} = 5\sqrt{2}$ units

Que 6. Find the distance of the point (-6,8) from the origin.

Sol. Here $x_1 = -6, y_1 = 8$

$x_2 = 0, y_2 = 0$

$d = \sqrt{(x_2 - x_1)^2} + (y_2 - y_1)^2$

$= \sqrt{[0 - (-6)^2] + (0 - 8)^2} = \sqrt{(6)^2 + (-8)^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ units

Que 7. If the points $\boldsymbol{A\ (1, 2), B(0, 0)}$ and $\boldsymbol{C\ (a, b)}$ are collinear, then what is the relation between $\boldsymbol{a}$ and ?

Sol. Points A, B and C are collinear

$\Rightarrow \quad 1(0 - b) + 0(b - 2) + a(2 - 0) = 0$

$\Rightarrow \quad -b + 2a = 0$ or $2a = b$

Que 8. Find the ratio in which the line segment joining the points $\boldsymbol{(-3, 10)}$ and $\boldsymbol{(6, -8)}$ is divided by $\boldsymbol{(-1, 6)}$.

Sol. In Fig. 6.6, let the point $P(-1,6)$ divides the line joining $A(-3,10)$ and $(6, -8)$ in the ratio $k: 1$

Then, the coordinates of P are $(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1})$

But, the coordinates of P are $(-1,6)$

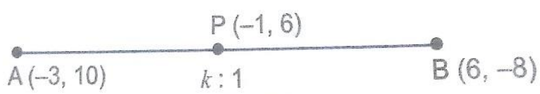

Fig. 6.6

$\therefore \frac{6k-3}{k+1} = -1 \Rightarrow \quad 6k - 3 = -k - 1$

$\Rightarrow 6k + k = 3 - 1 \Rightarrow 7k = 2$

$\therefore k = \frac{2}{7}$

Hence, the point P divides AB in the ratio 2:7.

Que 9. The coordinates of the points $\boldsymbol{P}$ and $\boldsymbol{Q}$ are respectively $\boldsymbol{(4, -3)}$ and $\boldsymbol{(-1, 7)}$. Find the abscissa of a point R on the line segment PQ such that $\boldsymbol{\frac{PR}{PQ} = \frac{3}{5}}$.

Sol

Fig. 6.7

$\frac{PQ}{PR} = \frac{5}{3}$ $\qquad\qquad$ $\frac{PQ-PR}{PR} = \frac{5-3}{3}$

$\Rightarrow \frac{RQ}{PR} = \frac{2}{3}$

$i.e.$, R divides PQ in the ratio 3: 2

Abscissa of $R = \frac{3\times-1+2\times4}{3+2} = \frac{-3+8}{5} = 1$

Que 10. A line segment is of length 10 units. If the coordinates of its one end are $(2, -3)$ and the abscissa of the other end is 10, find its ordinate.

Sol. Let required ordinate be x.

Then $(10-2)^2 + (x+3)^2 = 10^2$

$\Rightarrow \quad 64 + x^2 + 6x + 9 = 100 \; or \; x^2 + 6x - 27 = 0$

$\Rightarrow \quad (x-3)(x+9) = 0$ or $x = 3, -9$

Short Answer Type Questions – I

[2 MARKS]

Que 1. Write the coordinates of a point on $x-\text{axis}$ which is equidistant from the points $(-3,4)$ and $(2,5)$.

Sol. Let the required point be $(x,0)$.

Since, $(x,0)$ is equidistant from the points $(-3,4)$ and $(2,5)$

$\therefore \quad \sqrt{(-3-x)^2+(4-0)^2}=\sqrt{(2-x)^2+(5-0)^2}$

$\Rightarrow \quad \sqrt{9+x^2+6x+16}=\sqrt{4+x^2-4x+25}$

$\therefore \quad x^2+6x+25=x^2-4x+29 \quad \Rightarrow \quad 10x=4$ or $x=\frac{4}{10}=\frac{2}{5}$

$\therefore$ Required point is $\left(\frac{2}{5},0\right)$

Que 2. Find the values of x for which the distance between the points $P(2,3)$ and $Q(x,5)$ is 10.

Sol. Distance between the given points $=\sqrt{(x-2)^2+(5+3)^2}$

$\Rightarrow \quad 10=\sqrt{x^2+4-4x+64}$

$\Rightarrow \quad 100=x^2-4x+68$

$\Rightarrow \quad x^2-4x-32=0$

$\Rightarrow \quad x^2-8x+4x-32=0$

$\Rightarrow \quad (x-8)(x+4)=0 \quad \Rightarrow \quad x=8,-4$

Que 3. What is the distance between the points $(10\cos 30^0, 0)$ and $0, 10\cos 60^0)$?

Sol. Distance between the given points $=\sqrt{(0-10\cos 30^0)^2+(10\cos 60^0-0)^2}$

$=\sqrt{100cos^2\, 30^0+100cos^2\, 60^0}$

$=\sqrt{100\left[\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]}=\sqrt{100\left(\frac{3}{4}+\frac{1}{4}\right)}=\sqrt{100}=10$ units.

Que 4. In Fig.6.8, if $A(-1,3), B(1,-1)$ and $C(5,1)$ are the vertices of a triangle ABC, what is the length of the median through vertex A?

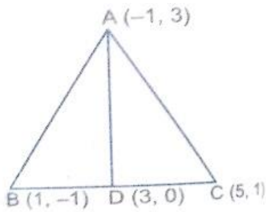

Fig. 6.8

Sol. Coordinates of the mid-point of $BC = \left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$

∴ Length of the median through $A = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units.

Que 5. Find the ratio in which the line segment joining the points $P(3, -6)$ and $Q(5, 3)$ is divided by the x −axis.

Sol. Let the required ratio be $\lambda : 1$

Then, the point of division is $\left(\frac{5\lambda+3}{\lambda+1}, \frac{3\lambda+6}{\lambda+1}\right)$

Given that this point lies on the x-axis

∴ $\frac{3\lambda-6}{\lambda+1} = 0$ or $3\lambda = 6$ or $\lambda = 2$

Thus, the required ratio is 2: 1.

Que 6. Point $P(5, -3)$ is one of the two points of trisection of the line segment joining the points $A(7, -2)$ and $B(1, -5)$. State true or false and justify your answer.

Sol. Points of trisection of line segment AB are given by

$$= \left(\frac{2\times1+1\times7}{3}, \frac{2\times-5+1\times-2}{3}\right) \text{ and } \left(\frac{1\times1+2\times7}{3}, \frac{1\times-5+2\times-2}{3}\right)$$

$$= \left(\frac{9}{3}, \frac{-12}{3}\right) \text{ and } \left(\frac{15}{3}, \frac{-9}{3}\right) \text{ or } (3, -4) \text{ and } (5, -3)$$

∴ Given statement is true.

Que 7. ΔABC with vertices $A(-2, 0), B(2, 0)$ and $C(0, 2)$ is similar to ΔDEF with vertices $D(-4, 0), E(4, 0)$ and $F(0, 4)$. State true or false and justify your answer.

Sol. $AB = \sqrt{(2+2)^2 + 0} = \sqrt{16} = 4$

$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{8} = 2\sqrt{2}$

$CA = \sqrt{(-2-0)^2 + (0-2)^2} = \sqrt{8} = 2\sqrt{2}$

$DE = \sqrt{(4+4)^2 + 0} = \sqrt{64} = 8$

$EF = \sqrt{(0-4)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$

$FD = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2} \quad \Rightarrow \quad \Delta ABC \sim \Delta DEF$

Que 8. Point $P(0, 2)$ is the point of intersection of y-axis and perpendicular bisector of line segment joining the points, $A(-1, 1)$ and $B(3, 3)$. State true or false and justify your answer.

Sol. The point $P(0,2)$ lies on y-axis

Also, $AP = \sqrt{(0+1)^2 + (2-1)^2} = \sqrt{2}$

$BP = \sqrt{(0-3)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$\because \quad AP \neq BP$

$\therefore$ $P(0,2)$ Does not lie on the perpendicular bisector of AB. So, given statement is false.

Que 9. Check whether $(5, 2), B\ (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

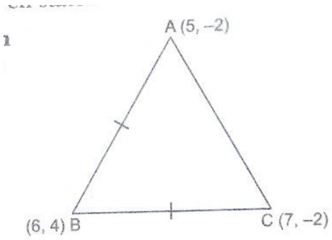

Fig. 6.9

Sol. Let $A\ (5, -2), B(6, 4)$ and $C(7, -2)$ be the vertices of a triangle

Then we have,

$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$

$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4} = 2$

Here, $AB = BC$

$\therefore \quad \Delta ABC$ is an isosceles triangle.

Que 10. If $(1, 2), (4, y), (x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y.

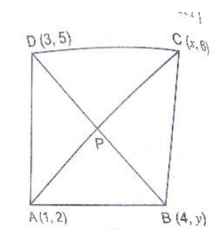

Fig. 6.10

Sol. Let $A(1,2), B(4,y), C(x,6)$ and $D(3,5)$ be the vertices of a parallelogram $ABCD$.

Since, the diagonals of a parallelogram bisect each other.

$\therefore \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2}$

$\Rightarrow \quad x + 1 = 7$ or $x = 6$

$\Rightarrow \quad 4 = \frac{5+y}{2}$ $\quad 5 + y = 8$ or $y = 8 - 5 = 3$

Hence, $x = 6$ and $y = 3$.

Que 11. Find the ratio in which y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the coordinates of the point of division.

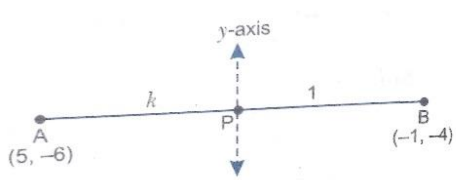

Fig. 6.11

Sol. Let the point on y-axis be $P(0, y)$ and $AP: PB = K: 1$

Therefore $\frac{5-k}{k+1} = 0$ gives $k = 5$

Hence required ratio is $5: 1$.

$$y = \frac{-4(5)-6}{5+1} = \frac{-13}{3}$$

Hence, point on y-axis is $P(0, \frac{-13}{3})$.

Que 12. Let P and Q be the points of trisection of the line segment joining the points $A(2,-2)$ and $B(-7,4)$ such that P is nearer to A. Find the coordinates of P and Q.

Fig. 6.12

Sol. $\because$ P divides AB in the ratio $1:2$

$\because$ Coordinates of $P = \left(\frac{1\times(-7)+2\times2}{1+2}\right), \left(\frac{1\times4+2\times-(2)}{1+2}\right)$

$$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = (-1,0)$$

$\therefore$ Q is the midpoint of PB

$\therefore$ Coordinates of Q = $\frac{-1+(-7)}{2}, \frac{0+4}{2}$

$$= \frac{-8}{2}, 2 = -4,2$$

Que 13. Find the ratio in which the point (-3, k) divides the line-segment joining the points (-5,-4) and (-2, 3). Also find the value of k.

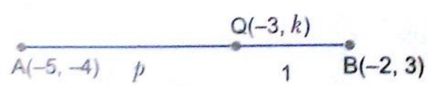

Fig. 6.13

Sol. Let Q divide AB in the ratio of p: 1

$$-3 = \frac{-2p-5}{p+1}$$

$\Rightarrow$ -3p – 3 = - 2p – 5 $\Rightarrow$ p = 2

$\therefore$ Ratio is 2: 1

$$K = \frac{2\times3-4}{2+1} = \frac{2}{3}$$

Que 14. The x-coordinate of a point p is twice its y-coordinate. If p is equidistant from Q (2,-5) and R (-3, 6), find the coordinate of p.

Sol. Let the point p be (2y, y)

PQ = PR $\Rightarrow$ $\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

$\Rightarrow$ $4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$$\Rightarrow \quad 2y + 29 = 45 \quad \Rightarrow y = 8$$

Hence, coordinates of point P are (16,8).

Short Answer Type Questions – II

[3 MARKS]

Que 1. Determine, if the points (1,5), (2,3) and (-2, - 11) are collinear.

Sol. Let A (1,5), B (2,3) and C (-2, - 11) be the given points. Then we have

AB = $\sqrt{(2-1)^2+(3-5)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{(2-2)^2+(-11-3)^2} = \sqrt{16+196} = \sqrt{4\times 53} = 2\sqrt{53}$

AC = $\sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{9+256} = \sqrt{265}$

Clearly, AB + BC ≠ AC

∴ A, B, C are not collinear.

Que 2. Find the distance between the following pairs of points:
(i) (-5, 7), (-1, 3) (ii) (a, b), (-a, - b)

Sol. (i) let two given points be A (-5, 7) and B (-1, 3).

Thus, we have x_1 = -5 and x_2 = - 1

Y_1 = 7 and y_2 = 3

∴ AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

⇒ AB = $\sqrt{(-1+5)^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units.

(ii) Let two given points be A (a, b) and B (-a,-b)

Here, x_1 = a and x_2 = - a; y_1 = b and y_2 = - b

∴ AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2}$

= $\sqrt{4a^2+4b^2} = 2\sqrt{a^2+b^2}$ units.

Que 3. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:
(i) (-1, - 2), (1, 0), (-1, 2), (-3, 0)
(ii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (i) Let A (-1, - 2), B (1,0), C (-1, 2) and D (-3, 0) be the four given points.

Then, using distance formula, we have,

AB = $\sqrt{(1+1)^2+(0+2)^2}$ $\sqrt{4+4}$ = $\sqrt{8}$ = 2 $\sqrt{2}$

BC = $\sqrt{(-1-1)^2+(2-0)^2}$ = $\sqrt{4+4}$ = $\sqrt{8}$ = 2 $\sqrt{2}$

CD = 2 $\sqrt{(-3+1)^2+(0-2)^2}$ = $\sqrt{4+4}$ = $\sqrt{8}$ = 2 $\sqrt{2}$

DA = $\sqrt{(-1+3)^2+(-2+0)^2}$= $\sqrt{4+4}$ = $\sqrt{8}$ = 2 $\sqrt{2}$

AC = $\sqrt{(-1+1)^2+(2+2)^2}$ = $\sqrt{0+16}$ = 4

And BD = $\sqrt{(-3-1)^2+(0-0)^2}$ = $\sqrt{16}$ = 4

Hence, four sides of quadrilateral are equal and diagonal AC and BD are also equal.

∴ Quadrilateral ABCD is a square.

(ii) Let A (4, 5), B (7, 6), C (4, 3) and D (1, 2) be the given points. Then,

AB = $\sqrt{(7-4)^2+(6-5)^2}$ = $\sqrt{9+1}$ = $\sqrt{10}$

BC = $\sqrt{(4-7)^2+(3-6)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$ = 3 $\sqrt{2}$

CD = $\sqrt{(1-4)^2+(2-3)^2}$ = $\sqrt{9+1}$ = $\sqrt{10}$

DA = $\sqrt{(4-1)^2+(5-2)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$ = 3 $\sqrt{2}$

AC = $\sqrt{(4-4)^2+(3-5)^2}$ = $\sqrt{0+4}$ = 2

And BD = $\sqrt{(1-7)^2+(2-6)^2}$ = $\sqrt{36+16}$ = $\sqrt{52}$ = 2 $\sqrt{13}$

Cleary, AB = CD, BC = DA and AC ≠ BD

∴ ABCD is a parallelogram.

Que 4. Find the value of y for which the distance between the points p (2, 3) and Q (10, y) is 10 units.

Sol. We have, PQ = 10

⇒ $\sqrt{(10-2)^2+(y+3)^2}$ = 10

Squaring both sides, we have

⇒ $(8)^2 + (y + 3)^2 = 100$ ⇒ $(y + 3)^2 = 100 – 64$

⇒ $(y + 3)^2 = 36$ or $y + 3 = \pm 6$

⇒ y + 3 = 6, y + 3 = - 6 or y = 3, y = - 9

Hence, values of y are – 9 and 3.

Que 5. If Q (0, 1) is equidistant from P (5, - 3) and R (x, 6), find the value of x. Also, find the distances QR and PR.

Sol. Since, point Q (0, 1) is equidistant from P (5, - 3) and R (x, 6).

Therefore, QP = QR

Squaring both sides, we have, $QP^2 = QR^2$

$\Rightarrow (5 - 0)^2 + (-3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16 \qquad \therefore x = \pm 4$

Thus, R is (4, 6) or (-4, 6).

Now, QR = $\sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Or, QR = $\sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

And PR = $\sqrt{(4-5)^2 + (6+3)^2} = \sqrt{1+81} = \sqrt{82}$

Or, PR = $\sqrt{(-4-5)^2 + (6+3)^2} = \sqrt{81+81} = 9\sqrt{2}$

Que 6. Find the point on the x – axis which is equidistant from (2, - 5) and (-2, 9).

Sol. Let P (x, 0) be any point on x-axis.

Now, P (x, 0) is equidistant from point A (2, -5) and B (-2, 9)

$\therefore$ AP = BP

$\Rightarrow \sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$

Squaring both sides, we have

$(x - 2)^2 + 25 = (x + 2)^2 + 81$

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81 \qquad \Rightarrow \qquad -8x = 56$

$\therefore x = \frac{56}{-8} = -7$

$\therefore$ The point on the x-axis equidistant from given points is (-7, 0).

Que 7. Find the relation between x and y, if the points (x, y), (1, 2) and (7, 0) are collinear.

Sol. Given points are A (x, y), B (1, 2) and C (7, 0)

These points will be collinear if the area of the triangle formed by them is zero.

Now, ar (ΔABC) = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Rightarrow 0 = \frac{1}{2}[x(2 - 0) + 1(0 - y) + 7(y - 2)]$

$\Rightarrow 0 = \frac{1}{2}(2x - y + 7y - 14) \qquad \Rightarrow \qquad 2x + 6y - 14 = 0$

$\Rightarrow x + 3y = 7$, which is the required relation between x and y.

Que 8. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

Sol. Let P (x, y) be equidistant from the point A (3, 6) and B (-3, 4)

i.e., PA = PB

Squaring both sides, we get

$AP^2 = BP^2$

$\Rightarrow (x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow -12x - 4y + 20 = 0 \qquad \Rightarrow 3x + y - 5 = 0$, which is the required relation.

Que 9. Find the coordinates of the point which divides the line joining of (-1, 7) and (4, -3) in the ratio 2:3.

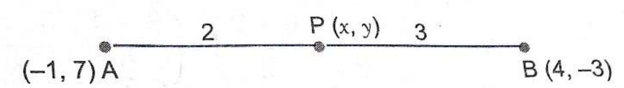

Fig. 6.14

Sol. Let P (x, y) be the required point. Thus, we have

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

Therefore,

$$x = \frac{2\times4+3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1 \quad \text{and,} \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$Y = \frac{2\times(-3)+3\times7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

So, the coordinates of P are (1, 3).

Que 10. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

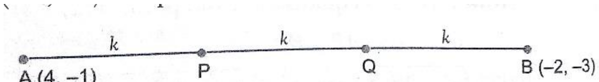

Fig. 6.15

Sol. Let the given points be A (4, - 1) and B (-2, - 3) and points of trisection be P and Q.

Let AP = PQ = QB = k

∴ PB = PQ + QB = k + k = 2k

AP: PB = k: 2k = 1: 2.

Therefore, coordinates of P are

$$\left(\frac{1\times-2+2\times4}{3},\frac{1\times-3+2\times-1}{3}\right)=\left(2,-\frac{5}{3}\right)$$

Now, AQ = AP + PQ = k + k = 2k

∴ AQ: QB = 2k: k = 2: 1

And, coordinates of Q are

$$\left(\frac{2\times-2+1\times4}{3},\frac{2\times-3+1\times-1}{3}\right)=\left(0,-\frac{7}{3}\right)$$

Hence, points of trisection are $\left(2,-\frac{5}{3}\right)$ and $\left(0,-\frac{7}{3}\right)$

Que 11. Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Sol. Let the required ratio be k: 1. Then, the coordinates of the point of division is

$$P\quad\left(\frac{-4k+1}{k+1},\frac{5k-5}{k+1}\right)$$

Since, this point lies on x-axis. Therefore its y-coordinates is zero.

i.e., $\frac{5k-5}{k+1}=0 \Rightarrow 5k-5=0$

$\Rightarrow$ 5k = 5 or $k=\frac{5}{5}=1$

Thus, the required ratio is 1: 1 and the point of division is P $\left(\frac{-4\times1+1}{1+1},\frac{5\times1-5}{1+1}\right)$

i.e., P $\left(-\frac{3}{2},0\right)$

Que 12. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

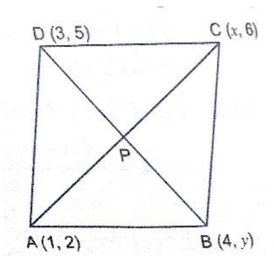

Fig. 6.16

Sol. Let A (1, 2), B (4, y), C (x, 6) and D (3, 5) be the vertices of a parallelogram ABCD.

Since, the diagonals of a parallelogram bisect each other.

$\therefore \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{x+1}{2}, \frac{7}{2} \quad \Rightarrow \quad x + 1 = 7$ or $x = 6$

And $\quad 4 = \frac{5+y}{2}$

$\Rightarrow \quad 5 + y = 8$ or $y = 8 – 5 = 3$

Hence, $x = 6$ and y = 3.

Que 13. Find the coordinates of a points A, where AB is the diameter of a circle whose center is (2,-3) and B is (1, 4).

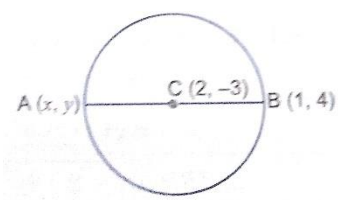

Fig. 6.17

Sol. Let the coordinates of A be (x,y).

Now, C is the center of circle therefore, the coordinates of

C $= \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$ but coordinates of C are given as (2,-3)

$\therefore \quad \frac{x+1}{2} = 2 \quad \Rightarrow \quad x + 1 = 4 \quad \therefore\ x = 3$

And $\quad \frac{y+4}{2} = -3 \quad \Rightarrow y + 4 = -6 \quad \therefore\ y = -10$

Hence, coordinates of A are (3,-10).

Que 14. If A and B are (-2,-2) and (2,-4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

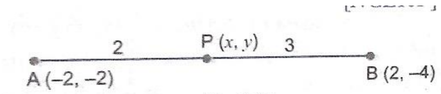

Fig. 6.18

Sol. In Fig. 8.10, we have, AP = $\frac{3}{7}$ AB

$\Rightarrow \frac{AB}{AB} = \frac{3}{7}$ $\quad\Rightarrow\quad$ $\frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \frac{AP+PB}{AP} = \frac{7}{3}$ $\quad\Rightarrow\quad$ $\frac{AP}{AP} + \frac{PB}{AP}\ \frac{7}{3}$

$\Rightarrow 1 + \frac{PB}{AP} = \frac{7}{3}$ $\quad\Rightarrow\quad$ $\frac{PB}{AP} + \frac{7}{3} - 1 = \frac{4}{3}$

$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$ $\quad\Rightarrow\quad$ AP: PB = 3: 4

Let P (x, y) be the point which divides the join of A (-2, -2) and B (2, - 4) in the ratio 3: 4.

$\therefore x = \frac{3\times2+4\times-2}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$ and $y = \frac{3\times-4+4\times-2}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$

Hence, the coordinates of the point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

Que 15. Find the coordinates of the points which divide the line segment joining A (-2,2) and B (2, 8) into four equal parts (Fig. 6.19).

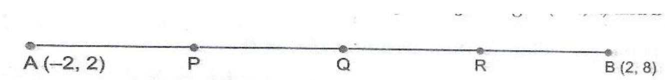

Fig. 6.19

Sol. Let P, Q, R be the points that divide the line segment joining A (-2,2) and B (2,8) into four equal parts.

Since, Q divides the line segment AB into two equal parts, i.e., Q is the mid-point of AB.

$\therefore$ Coordinates of Q are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ i.e., (0,5)

Now, P divides AQ into two equal parts i.e., P is the mid-point of AQ.

$\therefore$ Coordinates of P are $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$ i.e., $\left(-1, \frac{7}{2}\right)$

Again, R is the mid-point of QB.

$\therefore$ Coordinates of R are $\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$ i.e., $\left(1, \frac{13}{2}\right)$.

Que 16. Find the area of a rhombus if its vertices (3, 0), (4, 5), (-1, 4) and (-2,-1) are taken in order.

Sol. Let A (3, 0), B (4, 5), C (-1, 4) and D (-2,-1) be the vertices of a rhombus.

Therefore, its diagonals

$AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

And $BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$\therefore$ Area of rhombus ABCD = $\frac{1}{2}$ x (Product of length of diagonals)

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ sq units.

Que 17. Find the area of the triangle whose vertices are: (-5,-1), (3,-5), (5, 2)

Sol. Let $A(x_1, y_1) = (-5,-1)$, $B(x_2, y_2) = (3, -5)$, $C(x_3, y_3) = (5, 2)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2+1) + 5(-1+5)]$

$= \frac{1}{2}(35 + 9 + 20) = \frac{1}{2} \times 64 = 32$ sq units.

Que 18. If the points A (0, 2) is equidistant from the points B (3, p) and C (p, 5), find p. Also find the length of AB.

Sol. Given that A (0, 2) is equidistant from B (3, p) and C (p, 5)

$\therefore \quad AB = AC$

Or $\quad AB^2 = AC^2$

$\Rightarrow \quad (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow \quad 3^2 + p^2 + 4 - 4p = p^2 + 9 \quad \Rightarrow \quad 4 - 4p = 0$

$\Rightarrow \quad 4p = 4 \quad \Rightarrow \quad p = 1$

Length of $AB = \sqrt{(3-0)^2 + (1-2)^2}$

$= \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

Que 19. If the points A (-2, 1), B (a, b) and C (4, - 1) are collinear and a – b = 1, find the value of a and b.

Sol. Since the given points are collinear, then area of $\Delta ABC = 0$

$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

Given, $x_1 = -2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$, $x_3 = 4$, $y_3 = -1$

Putting the value,

$$\frac{1}{2}[-2(b + 1) + a(-1 - 1) + 4(1 - b)] = 0$$

$\Rightarrow \quad -2b - 2 - 2a + 4 - 4ab = 0 \quad \Rightarrow \quad 2a + 6b = 2$

$\Rightarrow \quad a + 3b = 1 \quad \ldots(i)$

Given, $\quad a - b = 1 \quad \ldots(ii)$

Subtracting (i) from (ii), we have

$$-4b = 0 \Rightarrow b = 0$$

Subtracting the value of b in (ii), we have a = 1

Que 20. If the point P (k – 1, 2) is equidistant from the points A (3, k) and B (k, 5), find the value of k.

Sol. Since P is equidistant from A and B.

$$AP = BP \text{ or } AP^2 = BP^2$$

$$[3 - (k - 1)]^2 + (k - 2)^2 = [k - (k - 1)]^2 + (5 -)^2$$

$$(3 - k + 1)^2 + (k - 2)^2 = [k - k + 1)^2 + (3)^2$$

$$(4 - k)^2 + (k - 2)^2 = (1)^2 + (3)^2 \quad \Rightarrow \quad 16 + k^2 - 8k + k^2 + 4 - 4k = 1 + 9$$

$$2k^2 - 12k + 20 = 10 \quad \Rightarrow \quad k^2 - 6k + 10 = 5$$

$$K^2 - 6k + 5 = 0 \quad \Rightarrow \quad k^2 - 5k - k + 5 = 0$$

$$K(k - 5) - 1(k - 5) = 0 \quad \Rightarrow \quad k = 1 \text{ or } k = 5$$

Que 21. Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x-axis. Also find the coordinates of the points of division.

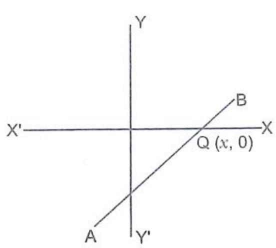

Fig. 6.20

Sol. Here, points Q is on x axis so its ordinate is *O.*

Let ratio be k: 1 and coordinates of points Q be (x, 0)

So, $Q_y = \frac{(my_2+my_1)}{m+n}$

We are given that A (3, -3) and B (-2,7)

$\therefore$ $0 = \frac{k\times7+1\times(-3)}{k+1}$

$0 = \frac{7k-3}{k+1}$ $\Rightarrow$ 0 (K + 1) = 7K -3

$\Rightarrow$ 7K = 3 $\Rightarrow$ K = $\frac{3}{7}$

K: 1 = 3: 7

Now, $Q_x = \frac{(mx_2+nx_1)}{m+n}$

$\Rightarrow$ $Q_x = \frac{\left(\frac{3}{7}\times-2\right)+1\times3}{\frac{3}{7}+1} = 1.5$

Que 22. Find the value of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.

Sol. Points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear

$\therefore$ Area of ΔABC = 0

$\Rightarrow$ $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow$ $\frac{1}{2}$ [(k + 1) (2k + 3 - 5k) + 3k (5k – 2k) + (5k – 1) {2k – (2k + 3)}] = 0

$\Rightarrow$ $\frac{1}{2}$ [(k + 1) (-3k + 3) + 3k (3k) + (5k – 1) (2k – 2k - 3)] = 0

$\Rightarrow$ $\frac{1}{2}\ [-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3] = 0$

$\Rightarrow$ $\frac{1}{2}\ [6k^2 - 15k + 6] = 0$ $\Rightarrow$ $6k^2 - 15k + 6 = 0$

$\Rightarrow$ $2k^2 - 5k + 2 = 0$ $\Rightarrow$ $2k^2 - 4k - k + 2 = 0$

$\Rightarrow$ (k – 2)(2k – 1) = 0

If k – 2 = 0. Then k = 2

If 2k – 1 = 0. Then k = $\frac{1}{2}$

$\therefore \quad k = 2, \frac{1}{2}$

Que 23. If the points P (x, y) is equidistant from the points A (a+b, b-a) and B (a-b, a+b). Prove that bx = ay.

Sol. Given. PA = PB or $(PA)^2 = (PB)^2$

$$(a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$$

$\Rightarrow (a + b)^2 + x^2 – 2ax – 2bx + (b – a)^2 + y^2 – 2by + 2ay$

$= (a – b)^2 + x^2 – 2ax + 2bx + (a + b)^2 + y^2 – 2ay – 2by$

$\Rightarrow \quad 4ay = 4bx$ or $bx = ay$

Hence proved.

Que 24. If the point C (-1, 2) divides internally the line segment joining the points A (2, 5) and B (x, y) in the ratio of 3: 4, find the value of $x^2 + y^2$.

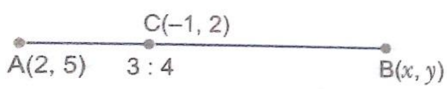

Fig. 6.21

Sol. $\frac{3x+4(2)}{7} = -1 \quad \Rightarrow x = -5$

$\frac{3y+4(5)}{7} = 2 \quad \Rightarrow y = -2$

$\therefore x^2 + y^2 = (-5)^2 + (-2)^2 = 29$

Long Answer Type Questions

[4 MARKS]

Que 1. Find the value of 'k', for which the points are collinear: (7, - 2), (5, 1), (3, k).

Sol. Let the given points be

A (x_1, y_1) = (7, - 2), B (x_2, y_2) = (5, 1) and C (x_3, y_3) = (3, k)

Since these points are collinear therefore area (ΔABC) = 0

$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad 7(1 - k) + 5(k + 2) + 3(-2 - 1) = 0$

$\Rightarrow \quad 7 - 7k + 5k + 10 - 9 = 0$

$\Rightarrow \quad -2k + 8 = 0 \quad \Rightarrow \quad 2k = 8$

$\Rightarrow \quad k = 4$

Hence, given points are collinear for k = 4.

Que 2. Find the area of the triangle formed by joining the mid-points of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

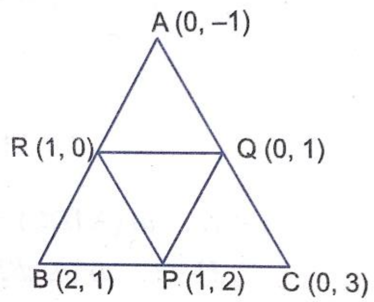

Fig. 6.22

Sol. Let A (x_1, y_1) = (0, - 1), B (x_2, y_2) = (2, 1), C (x_3, y_3) = (0, 3) be the vertices of ΔABC.

Now, let P, Q R be the mid-point of BC, CA and AB, respectively.

So, coordinates of P, Q, R are

$P = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$

$R = \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$

Therefore, ar (ΔPQR) = $\frac{1}{2}$ [1(1 – 0) + 0(0 – 2) + 1(2 – 1)] = $\frac{1}{2}$ (1 + 1) = 1 sq. unit

Now, ar (ΔABC) = $\frac{1}{2}$ [0(1 – 3) + 2(3 + 1) + 0(- 1 – 1)]

= $\frac{1}{2}$ [0 + 8 + 0] = $\frac{8}{2}$ = 4 sq. units

∴ Ratio of ar (ΔPQR) to the ar (ΔABC) = 1: 4.

Que 3. Find the area of the quadrilateral whose vertices, taken in order, are (- 4, - 2), (- 3, - 5), (3, -2) and (2, 3).

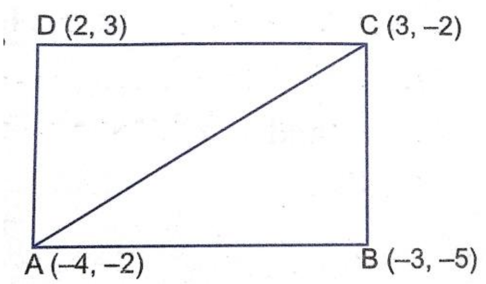

Fig. 6.23

Sol. Let A (- 4, - 2), B (- 3, - 5), C (3, -2) and D (2, 3) be the vertices of the quadrilateral ABCD.

Now, area of quadrilateral ABCD

= area of ΔABC + area of ΔADC

= $\frac{1}{2}$ [- 4 (- 5 + 2) – 3 (- 2 + 2) + 3 (- 2 + 5)]

+ $\frac{1}{2}$ [- 4 (- 2 – 3) + 3 (3 + 2) + 2 (- 2 + 2)]

= $\frac{1}{2}$ [12 – 0 + 9] + $\frac{1}{2}$ [20 + 15 + 0]

= $\frac{1}{2}$ [21 + 35] = $\frac{1}{2}$ x 56 = 28 sq. units.

Que 4. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, - 6), B (3, - 2), and C (5, 2).

Sol. Since AD is the median of ΔABC, therefore, D is the mid-point of BC.

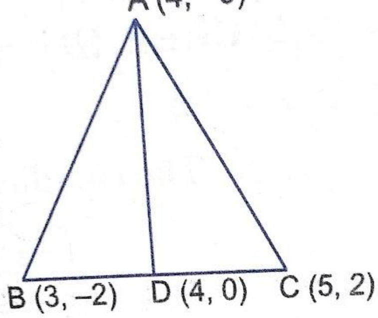

Fig. 6.24

Coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ $i.e., (4, 0)$

Now, area of ΔABD

$$= \frac{1}{2}[4(-2-0) + 3(0+6) + 4(-6-2)]$$

$$= \frac{1}{2}(-8+18-16) = \frac{1}{2} \times (-6) = -3$$

Since area is a measure, it cannot be negative.

Therefore, ar (ΔABD) = 3 sq. units

And area of ΔADC $= \frac{1}{2}[4(0-2) + 4(2+6) + 5(-6-0)]$

$$= \frac{1}{2}(-8+32-30)$$

$$= \frac{1}{2}(-6) = -3, \textit{which cannot be negative}.$$

∴ ar (ΔADC = 3 sq. units

Here, ar (ΔABD) = ar (ΔADC)
Hence, the median divides it into two triangles of areas.

Que 5. Find the ratio in which the point P (x, 2), divides the line segment joining the points A (12, 5) and B (4, 3). Also find the value of x.

Sol.

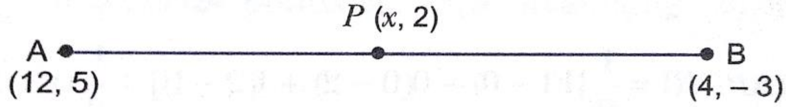

Fig. 6.25

Let the ratio in which point P divides the line segment be k: 1.

Then, coordinates of P: $\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$

Given, the coordinates of P as (x, 2)

∴ $\frac{4k+12}{k+1} = x$ …… (i)

And $\frac{-3k+5}{k+1} = 2$ ……. (ii)

-3k + 5 = 2k + 2

5k = 3 ⇒ k = $\frac{3}{5}$

Putting the value of k in (i), we have

$$\frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1} = x \quad \Rightarrow \quad \frac{12 + 60}{3 + 5} = x$$

$$x = \frac{72}{8} \quad \Rightarrow \quad x = 9$$

The ratio in which p divides the line segment is $\frac{3}{5}, i.e., 3: 5$.

Que 6. If A (4, 2), B (7, 6) and C (1, 4) are the vertices of a ΔABC and AD is its median, prove that the median AD divides ΔABC into two triangles of equal areas.

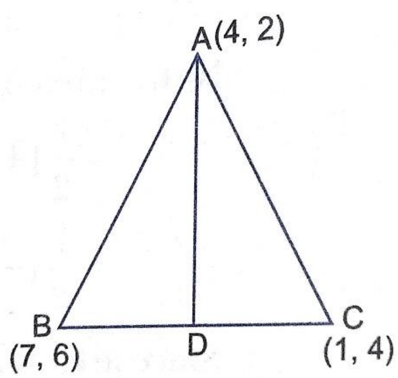

Fig. 6.26

Sol. Given: AD is the median on BC.
⇒ BD = DC
The coordinates of mid-point D are given by.

$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$ $i.e.,$ $\left(\frac{1+7}{2}, \frac{4+6}{2}\right)$

Coordinates of D are (4, 5).

Now, Area of triangle ABD = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[4(6-5) + 7(5-2) + 4(2-6)] = \frac{1}{2}[4 + 21 - 16] = \frac{9}{2} \text{ sq. units}$$

Area of ΔACD = $\frac{1}{2}[4(4-5) + 1(5-2) + 4(2-4)]$

$$= \frac{1}{2}[-4 + 3 - 8] = -\frac{9}{2} = \frac{9}{2} \text{ sq. units}$$

Hence, AD divides ΔABC into two equal areas.

Que 7. If the point A (2, 4) is equidistant from P (3, 8) and Q (-10, y), find the values of y. Also find distance PQ.

Sol. Given points are A (2, - 4), P (3, 8) and Q (-10, y)

According to the question,

$$PA = QA$$

$$\sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(2+10)^2 + (-4-y)^2}$$

$$\sqrt{(-1)^2 + (-12)^2} = \sqrt{(12)^2 + (4+y)^2}$$

$$\sqrt{1 + 144} = \sqrt{144 + 16 + y^2 + 8y}$$

$$\sqrt{145} = \sqrt{160 + y^2 + 8y}$$

On squaring both sides, we get

$145 = 160 + y^2 + 8y$

$y^2 + 8y + 160 - 145 = 0$

$y^2 + 8y + 15 = 0$

$y^2 + 5y + 3y + 15 = 0$

$y(y + 5) + 3(y + 5) = 0$

$\Rightarrow$ $(y + 5)(y + 3) = 0$

$\Rightarrow$ $y + 5 = 0$ $\Rightarrow y = -5$

and $y + 3 = 0$ $\Rightarrow y = -3$

$\therefore$ $y = -3, -5$

Now, PQ = $\sqrt{(-10-3)^2 + (y-8)^2}$

For y = - 3 PQ = $\sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169 + 121} = 290 \, units$

And for y = - 5 PQ = $\sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169 + 169} = \sqrt{338} \, units$

Hence, values of y are – 3 and – 5, PQ = $\sqrt{290}$ and $\sqrt{338}$

Que 8. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, - 3). The origin is the mid-point of the base. Find the coordinates

of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.

Sol. ∵ O is the mid-point of the base BC.

∵ Coordinates of point B are (0, 3).

So, BC = 6 units

Let the coordinates of point A be (x, 0).

Using distance formula,

$$AB = \sqrt{(0-x)^2 + (3-0)^2} = \sqrt{x^2+9}$$
$$BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{36}$$

Also, $Ab = BC$ (∴ Δ ABC is an equilateral triangle)

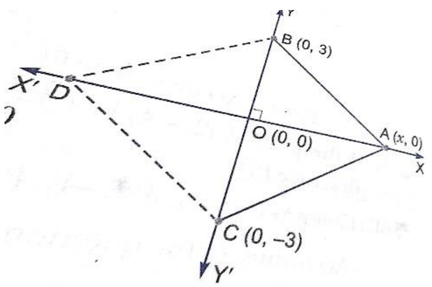

Fig. 6.27

$$\sqrt{x^2+9} = \sqrt{36}$$
$$x^2 + 9 = 36$$
$$x^2 = 27 \quad \Rightarrow \quad x^2 - 27 = 0$$

$$x^2 - \left(3\sqrt{3}\right)^2 = 0 \quad \Rightarrow \quad \left(x+3\sqrt{3}\right)\left(x-3\sqrt{3}\right) = 0$$
$$x = -3\sqrt{3} \ or\ x = 3\sqrt{3}$$

⇒ $x = \mp 3\sqrt{3}$

∴ Coordinates of point D = $\left(-3\sqrt{3},\ 0\right)$

Que 9. Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Sol. Area of a triangle = $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

Area of the triangle = $\frac{1}{2}|t(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)|$

$$= \frac{1}{2}|2t + 2t + 4 - 4t - 12|$$

$$= 4 \text{ Sq. units}$$

Which is independent of t. Hence proved.

Que 10. The coordinates of the points A, B and c are (6, 3), (-3, 5) and (4, -2) respectively. P(x, y) is any point in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \frac{x+y-2}{7}$

Sol. P(x, y), B (-3, 5), C (4, -2), A (6, 3)

$\therefore\ ar(\Delta PBC) = \frac{1}{2}|x\,(7) - 3\,(-2-y) + 4\,(y-5)| = \frac{1}{2}|7x + 7y - 14|$

$ar(\Delta ABC) = \frac{1}{2}|6 \times 7 - 3(-5) + 4(3-5)| = \frac{1}{2}|42 + 15 - 8| = \frac{49}{2}$

$LHS = \left|\frac{ar(\Delta PBC)}{ar(\Delta ABC)}\right| = \frac{\frac{1}{2}|7x + 7y - 14|}{\frac{1}{2} \times 49}\left|\frac{x+y+-2}{7}\right| = \frac{x+y+-2}{7} = RHS$

Que 11. In fig. 6.28, the vertices of ∆ ABC are A (4, 6), B (1, 5) and C (7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ∆ADE and compare it with area of ∆ABC.

Sol. Since $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

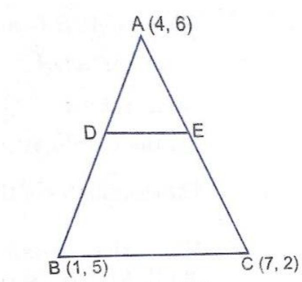

Fig. 6.28

⇒ D and E divide AB and AC respectively in the ratio 1: 2

Coordinates of D are $\left(\frac{1(1)+2(4)}{3}\right), \left(\frac{1(5)+2(6)}{3}\right)\ i.e., \left(3, \frac{17}{3}\right)$

Coordinates of E are $\left(\frac{1(7)+2(4)}{3}, \frac{1(2+2(6))}{3}\right)\ i.e., \left(5, \frac{14}{3}\right)$

Area of ∆ADE = $\frac{1}{2}\left[4\left(\frac{17}{3} - \frac{14}{3}\right) + 3\left(\frac{14}{3} - 6\right) + 5\left(6 - \frac{17}{3}\right)\right]$

$= \frac{1}{2}\left(4 + (-4) + \frac{5}{3}\right) = \frac{5}{6}\ sq\ unit$

Area ∆ABC $= \frac{1}{2}[4(3) + 1(-4) + 7(1)] = \frac{15}{2}\ sq\ unit$

Area ∆ADE: area ∆ABC = $\frac{5}{6} : \frac{15}{2}\ or\ 1:9$

HOTS (Higher Order Thinking Skills)

Que 1. The line joining the points (2, 1) and (5, – 8) is trisected by the points P and Q. If the point P lies on the line 2x – y + k = 0, find the value of k.

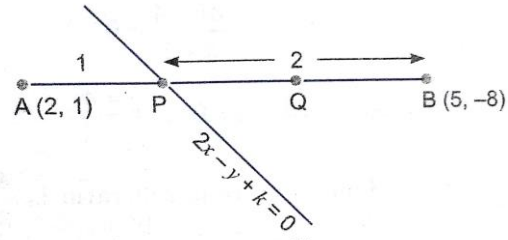

Fig. 6.29

Sol. As line segment AB is trisected by the points P and Q. Therefore,

Case I: When AP: PB = 1: 2.

Then, coordinates of P are $\left\{\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+1\times2}{1+2}\right\}$

$\Rightarrow \quad P\,(3, -2)$

Since the point P (3, – 2) lies on the line

2x – y + k = 0

$\Rightarrow \; 2\times3-(-2)+k=0$

$\Rightarrow$ k = – 8

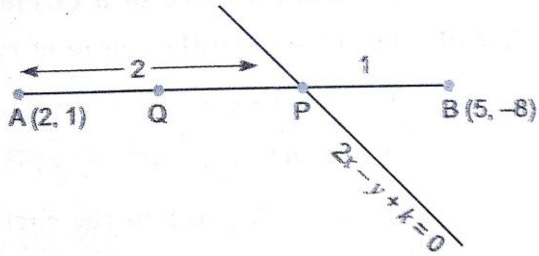

Fig. 6.30

Case II: When AP: PB = 2: 1.

Coordinates of point P are

$\left\{\frac{2\times5+1\times2}{2+1}, \frac{2\times-8+1\times1}{2+1}\right\} = \{4, -5\}$

Since the point P (4, -5) lies on the line

2x – y + k = 0

$\therefore$ 2 × 4 – (– 5) + k = 0 $\Rightarrow$ $k = -13.$

Que 2. Prove that the diagonals of a rectangle bisect each other and are equal.

Sol. Let OACB be a rectangle such that OA is along x-axis and OB is along y-axis.
Let OA = a and OB = b.
Then, the coordinates of A and B are (a, 0) and (0, b) respectively.

Since, OACB is a rectangle. Therefore,

AC = OB $\Rightarrow$ $AC = b$

Also, OA = a $\Rightarrow$ BC = a

So, the coordinates of C are (a, b)

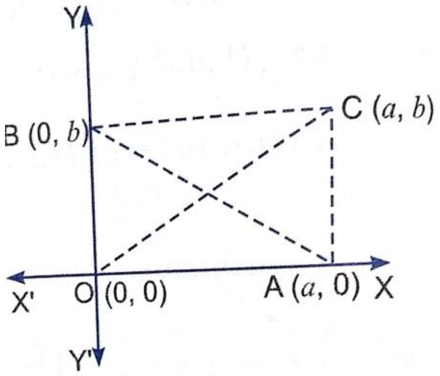

Fig. 6.31

The coordinates of the mid-points of OC are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, the coordinates of the mid-points of AB are $\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$.

Clearly, coordinates of the mid-point of OC and AB are same.
Hence, OC and AB bisect each other.

Also, $OC = \sqrt{a^2 + b^2}$ and $BA = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$\therefore$ OC = AB

Que 3. In what ratio does the y-axis divides the line segment joining the point P (– 4, 5) and Q (3, – 7)? Also, find the coordinates of the point of intersection.

Sol. Suppose y-axis divides PQ in the ration k: 1. Then, the coordinates of the point of

divides are

$R\left(\frac{3k-4}{k+1}, \frac{7k+5}{k+1}\right)$

Since, R lies on y-axis and x-coordinate of every point on y-axis is zero.

$\therefore \quad \frac{3k-4}{k+1} = 0$

$\Rightarrow \quad 3k - 4 = 0 \Rightarrow k = \frac{4}{3}$

Hence, the required ratio is $\frac{4}{3}$: 1 i.e., 4: 3.

Putting k = 4/3 in the coordinates of R, we find that its coordinates are $\left(0, \frac{-13}{7}\right)$.

Que 4. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Sol. Let O (x, y) be the centre of circle. Given points are A (6, – 6), B (3, – 7) and C (3, 3).

Then, OA = $\sqrt{(x-6)^2+(y+6)^2}$; $OB = \sqrt{(x-3)^2+(y+7)^2}$

and OC = $\sqrt{(x-3)^2+(y-3)^2}$

Since, each point on the circle is equidistant from centre.

∴ OA = OB = OC = Radius

Since, OA = OB

⇒ $\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$

Squaring both sides, we get,

$(x-6)^2+(y+6)^2 = (x-3)^2+(y+7)^2$

⇒ $x^2-12x+36+y^2+12y+36 = x^2-6x+9+y^2+14y+49$

or $-6x-2y = -14$

or 3x + y = 7

Similarly, OB = OC

⇒ $\sqrt{(x-3)^2+(y+7)^2} = \sqrt{(x-3)^2+(y-3)^2}$

Squaring both sides, we get,

$(x-3)^2+(y+7)^2 = (x-3)^2+(y-3)^2 \quad \Rightarrow \quad (y+7)^2 = (y-3)^2$

⇒ $y^2+14y+49 = y^2-6y+9 \quad or \quad 20y = -40$

⇒ $y = -2$

Putting y = – 2 in (i), we get

3x – 2 = 7 ⇒ $x = 3$

Hence, the coordinates of the centre of circle are (3, – 2).

Que 5. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, – 3) and (3, 4). Find its centroid.

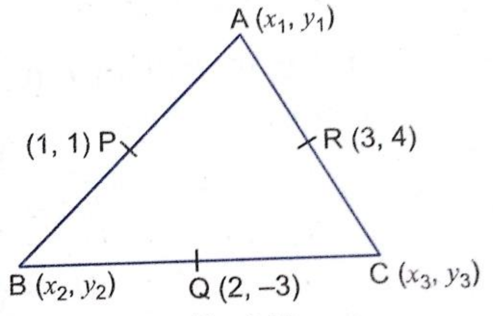

Fig. 6.33

Sol. Let P (1, 1), Q (2, – 3), R (3, 4) be the mid-points of sides AB, BC and CA respectively, of

triangle ABC. Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of triangle ABC.

Then, P is the mid-point of AB.

⇒ $\frac{x_1+x_2}{2} = 1, \qquad \frac{y_1+y_2}{2} = 1$

⇒ $x_1+x_2 = 2$ And $y_1+y_2 = 2$ …(i)

Q is the mid-point of BC

$\Rightarrow \quad \frac{x_2+x_3}{2} = 2, \frac{y_2+y_3}{2} = -3 \Rightarrow \quad x_2 + x_3 = 4 \; and \; y_2 + y_3 = -6$ …(ii)

R is the mid-point of AC

$\Rightarrow \quad \frac{x_1+x_3}{2} = 3 \; and \; \frac{y_1+y_3}{2} = 4 \quad \Rightarrow \quad x_1 + x_3 = 6 \; and \; y_1 + y_3 = 8$ …(iii)

From (i), (ii) and (iii), we get

$$x_1 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$$

And $\quad y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \; and \; y_1 + y_2 + y_3 = 2$ …..(iv)

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right)$$ [Using (iv)]

Value Based Questions

Que 1. Aadya and Nitya planted some trees in a square garden as shown in the Fig. 2, both arguing that they have planted them in a straight line. Find out who is correct? Justify your decision. (N stands for Nitya and A for Aadya)

Sol. Aadya planted the tress at (2, 1), (4, 3) and (6, 5)

Area of the triangle (if any) formed by these points

$$= \frac{1}{2}[2(3-5) + 4(5-1) + 6(1-3)]$$

$$= \frac{1}{2}(-4 + 16 - 12) = 0$$

$\therefore$ Given points are collinear

Nitya planted the trees at (2, 3), (3, 4), (4, 6)

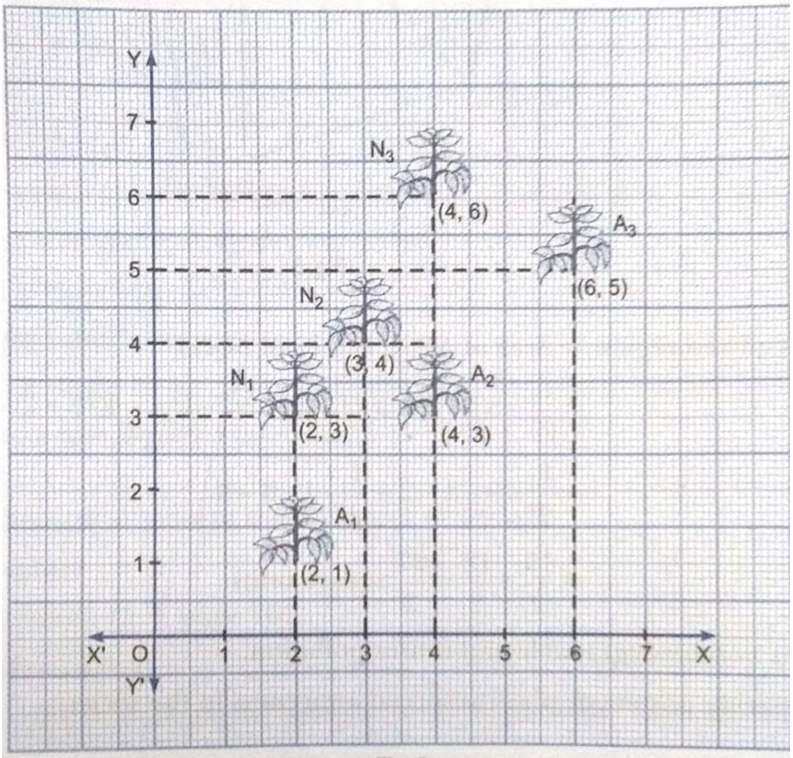

Fig. 2

Area of the triangle (if any) formed by these points = $\frac{1}{2}[2(4-6) + 3(6-3) + 4(3-4)]$

$$= \frac{1}{2}(-4 + 9 - 4) = \frac{1}{2} sq.unit$$

$\therefore$ *Given points are not collinear*.

Hence, only Aadya planted the trees in a line.

Plating more trees helps in making the environment clean. So, the two girls are giving healthy surrounding to the society.

Que 2. The students of class X of a school undertook to work for the campaign 'Say No to plastic' in a city. They took the map of the city and form coordinate

plane on it to divide their areas. Group A took the region covered between the coordinates (1, 1), (– 3, 2), (–2, –2) and (1, –3) taken in order. Find the area of the region covered by group A.

(i) What are the harmful effects of using plastic?

(ii) How can you contribute in spreading awareness for such campaign?

Sol. The region covered by group A is divided into the triangles PQS and QRS.

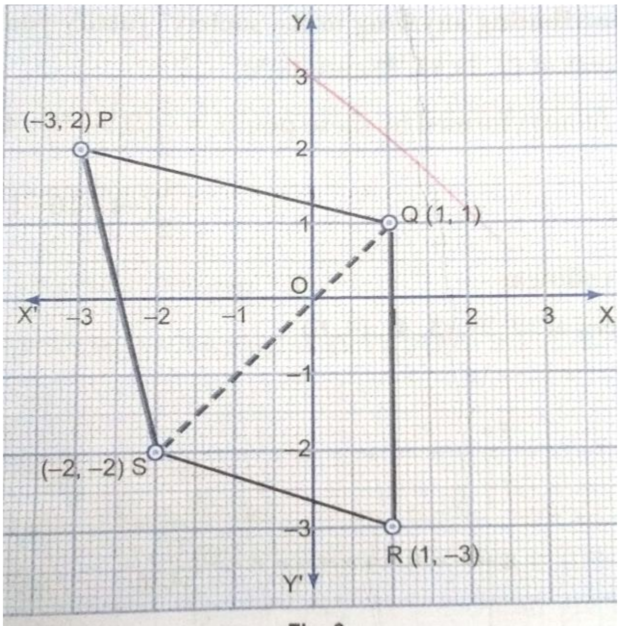

Fig. 3

∴ Area of required region = Area of ΔPQS + Area of ΔPRS

$$= \frac{1}{2}|-3(1+2) + 1(-2-2) - 2(2-1)| + \frac{1}{2}|1(-3+2) + 1(-2-1) + (-2)(1+3)|$$

$$= \frac{1}{2}|-9-4-2| + \frac{1}{2}|-1-3-8|$$

$$= \frac{15}{2} + \frac{12}{2} = \frac{27}{2} \; square\ units$$

(i) Plastic is non-biodegradable and causes pollution.

(ii) By preparing posters or plays to spread awareness in the society.