Very Short Answer Type Questions

[1 Mark]

Que 1. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Sol. Since the perimeter and two sides are proportional
∴ The third side is proportional to the corresponding third side.
i.e., The two triangles will be similar by SSS criterion.

Que 2. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reason.

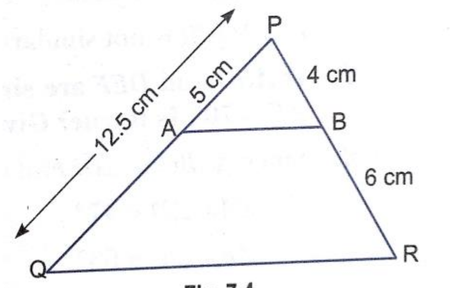

Fig. 7.4

Sol. Yes, $\frac{PA}{AQ} = \frac{5}{12.5 - 5} = \frac{5}{7.5} = \frac{2}{3}$

$$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$$

Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$

∴ AB||QR

Que 3. If ΔABC ~ ΔQRP, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}, AB = 18$ cm and BC = 15 cm, then find the length of PR.

Sol. $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QRP} = \frac{BC^2}{RP^2} \Rightarrow \frac{9}{4} = \frac{(15)^2}{RP^2}$

∴ $RP^2 = \frac{225 \times 4}{9} = \frac{9000}{9} = 100 \Rightarrow RP = 10\ cm$

Que 4. If it is given that ΔABC ~ ΔPQR with $\frac{BC}{QR} = \frac{1}{3}$, then find $\frac{ar(\Delta PQR)}{ar(\Delta ABC)}$.

Sol. $\frac{BC}{QR} = \frac{1}{3}$ (Given)

$$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{(QR)^2}{(BC)^2}$$

[∵ Ratio of area of similar triangles in equal to the ratio of square of its corresponding side]

$$= \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9:1$$

Que 5. ΔDEF ~ ΔABC, if DE: AB = 2: 3 and ar (ΔDEF) is equal to 44 square units. Find the area (ΔABC).

Sol. $\frac{ar(\Delta DEF)}{ar(\Delta ABC} = \frac{(DE)^2}{(AB)^2}$

[∵ Ratio of area of similar triangles in equal to the ratio of square of its corresponding side] Since ΔDEF ~ ΔABC

$$\frac{44}{ar(\Delta ABC)} = \left(\frac{2}{3}\right)^2 \quad \Rightarrow \quad ar(\Delta ABC) = \frac{44 \times 9}{4}$$

So, ar (ΔABC) = 99 cm^2

Que 6. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.

Sol. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 18^2$

∴ The given triangle is not a right triangle.

Short Answer Type Questions – I

[2 marks]

Que 1. In triangle PQR and TSM, ∠P = 55°, ∠Q = 25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Sol. Since, $\angle R = 180° - (\angle P + \angle Q)$

$= 180° - (55° + 25) = 100° = \angle M$

∠Q = ∠S = 25° (Given)

ΔQPR ~ ΔSTM

i.e., ΔQPR is not similar to ΔTSM.

Que 2. If ABC and DEF are similar triangles such that $\angle A = 47°$ and $\angle E = 63°$, then the measures of $\angle C = 70°$. Is it true? Give reason.

Sol. Since ΔABC ~ ΔDEF

∴ $\angle A = \angle D = 47°$

$\angle B = \angle E = 63°$

∴ $\angle C = 180° - (\angle A + \angle B) = 180° - (47° + 63°) = 70°$

∴ Given statement is true.

Que 3. Let ΔABC ~ ΔDEF and their areas be respectively 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Sol. We have, $\frac{area\ of\ \Delta ABC}{area\ of\ \Delta DEF} = \frac{BC^2}{EF^2}$ (as ΔABC ~ ΔDEF)

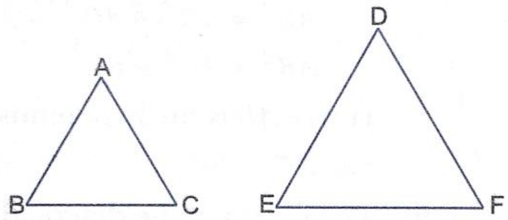

Fig. 7.5

$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \quad \frac{BC}{15.4} = \frac{8}{11}$

∴ BC $= \frac{8}{11} \times 15.4 = 11.2\ cm$

Que 4. ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.

Sol. ΔABC is right-angled at C.

$\therefore \quad AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow \quad AB^2 = AC^2 + AC^2$ [$\because$ AC = BC]

$\Rightarrow \quad AB^2 = 2AC^2$

Que 5. Sides of triangle are given below. Determine which of them are right triangles. In case of a right triangle, write the lenght of its hypotenuse.
(i) 7 cm, 24 cm, 25 cm **(ii) 3 cm, 8 cm, 6 cm**

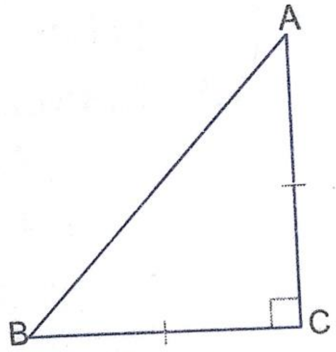

Fig. 7.6

Sol. (i) Let a = 7cm, b = 24 cm and c = 25 cm.
Here, largest side, c = 25 cm
We have, $a^2 + b^2 = (7)^2 + (24)^2 = 49 + 5769 = 625 = c^2$ [$\because c = 25$]

So, the triangle is a right triangle.
Hence, c is the hypotenuse of right triangle.

(ii) Let a = 3 cm, b = 8 cm and c = 6 cm
Here, largest side, b = 8 cm
We have, $a^2 + c^2 = (3)^2 + (6)^2 = 9 + 36 = 45 \neq b^2$
So, the triangle is not a right triangle.

Que 6. If triangle ABC is similar to triangle DEF such that 2AB = DE and BC = 8 cm. Then find the length of EF.

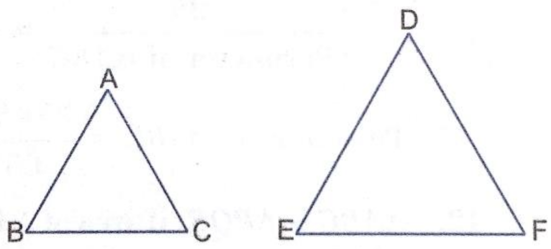

Fig. 7.7

Sol. $\Delta ABC \sim \Delta DEF$ (Given)

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF}$

$\frac{AB}{2AB} = \frac{8}{EF} \qquad (\because\ DE = 2AB)$

$\frac{1}{2} = \frac{8}{EF}$

$\therefore$ EF = 16 cm

Que 7. If the ratio of the perimeter of two similar triangles is 4: 25, then find the ratio of the similar triangles.

Sol. $\because$ Ratio of perimeter of 2 $\Delta's\ = 4:25$
Ratio of corresponding sides of the two $\Delta's = 4:25$

Now, The ratio of area of $2\ \Delta's$ = Ratio of square of its corresponding sides.

$$= \frac{(4)^2}{(25)^2} = \frac{16}{625}$$

Que 8. In an isosceles ΔABC, if AC = BC and $AB^2 = 2AC^2$ then find $\angle C$.

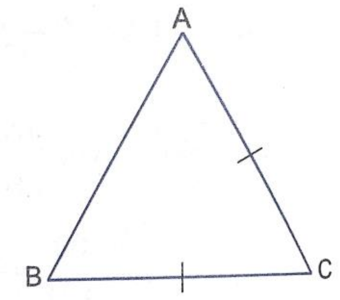

Fig. 7.8

Sol. $AB^2 = 2AC^2$ (Given)
$AB^2 = AC^2 + AC^2$
$AB^2 = AC^2 + BC^2$ ($\because$ AC = BC)
Hence AB is the hypotenuse and ΔABC is a right angle Δ.
So, $\angle C = 90°$

Que 9. The length of the diagonals of a rhombus are 16 cm and. Find the length of side of the rhombus.

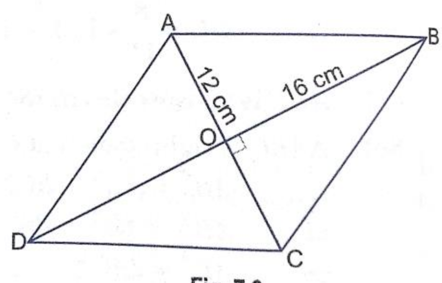

Fig. 7.9

Sol. ∵ The diagonals of rhombus bisect each other at 90°.
∴ In the right angle ΔBOC

BO = 8 cm

CO = 6 cm

∴ By Pythagoras Theorem

$BC^2 = BO^2 + CO^2 = 64 + 36$

$BC^2 = 100$

BC = 10 cm

Que 10. A man goes 24 m towards West and then 10 m towards North. How far is he from the starting point?

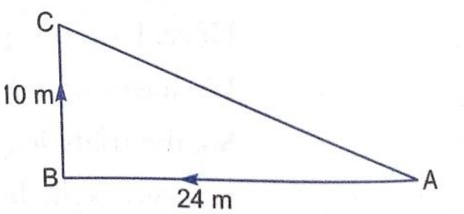

Fig. 7.10

Sol. By Pythagoras Theorem

$AC^2 = AB^2 + BC^2 = (24)^2 + (10)^2$

$AC^2 = 676$

AC = 26 m

∴ The man is 26 m away from the starting point.

Que 11. ΔABC ~ ΔDEF such that AB = 9.1 cm and DE = 6.5 cm. If the perimeter of ΔDEF is 25 cm, what is the perimeter of ΔABC?

Sol. Since ΔABC ~ ΔDEF

$$\frac{Perimeter\ of\ \Delta DEF}{Perimeter\ of\ \Delta ABC} = \frac{DE}{AE}$$

$$\frac{25}{Perimeter\ of\ \Delta ABC} = \frac{6.5}{9.1}$$

Perimeter of ΔABC = $\frac{25\times91}{65}$ = 35 cm

Que 12. ΔABC ~ ΔPQR; if area of ΔABC = 81 cm^2, area of ΔPQR = 169 cm^2 and AC = 7.2 cm, find the length of PR.

Sol. Since ΔABC ~ ΔPQR

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AC^2}{PR^2} \quad \Rightarrow \quad \frac{81}{169} = \frac{(7.2)^2}{PR^2}$$

$$\Rightarrow \quad PR^2 = \frac{(7.2)^2 \times 169}{81}$$

Taking square root both the sides

$$PR = \frac{7.2 \times 13}{9} = \frac{72 \times 13}{10 \times 9} = \frac{104}{10} = 10.4\ cm.$$

Short Answer Type Questions – II

[3 marks]

Que 1. In Fig. 7.11, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x 0- 1, find the value of x.

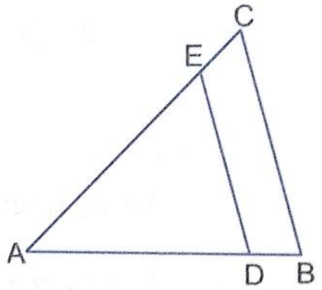

Fig. 7.11

Sol. In ΔABC, we have

DE|| BC.

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$ $\Rightarrow$ $x(x-1) = (x-2)(x+2)$

$\Rightarrow$ $x^2 - x = x^2 - 4$ $\Rightarrow x = 4$

Que 2. E and Fare points on the sides PQ and PR respectively of a ΔPQR. Show that EF || QR. If PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

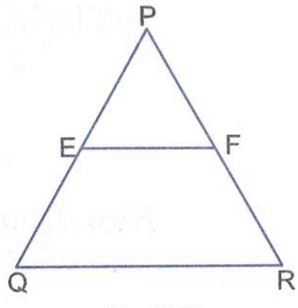

Fig. 7.12

Sol. We have, PQ = 1.28, PR = 2.56 cm

PE = 0.18 cm, PF = 0.36 cm

Now, EQ = PQ = PQ = 1.28 – 0.18 = 1.10 cm

And FR = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

And, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\therefore$ $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF||QR [By the converse of basic proportionality Theorem]

Que 3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. Let AB be a vertical pole of length 6 m and BC be its shadow and DE be tower and EF be its shadow.
Join AC and DF.

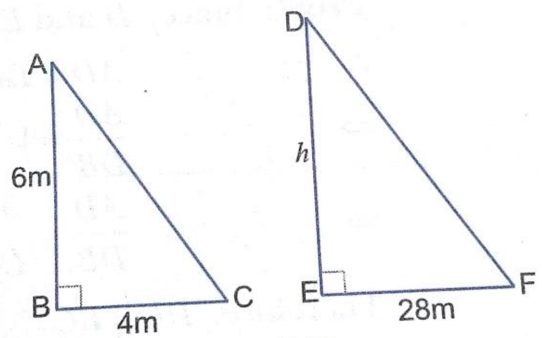

Fig. 7.13

Now, in ΔABC and ΔDEF, we have

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the sun)

$\therefore\ \Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{6}{h} = \frac{4}{28}$ (Let DE = h)

$\Rightarrow \frac{6}{h} = \frac{1}{7}$ $\Rightarrow$ h = 42

Hence, height of tower, DE = 42 m

Que 4. In Fig. 7.14, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

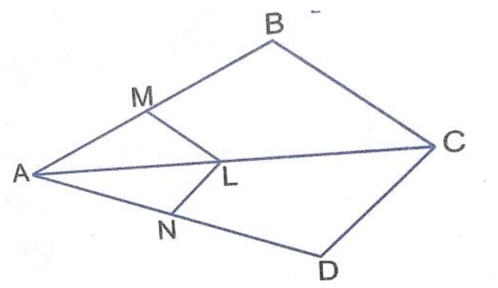

Fig. 7.14

Sol. Firstly, in ΔABC, we have

LM||CB (Given)

Therefore, by Basic proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad .....(i)$$

Again, in ΔACD, we have

LN||CD (Given)

∴ By Basic proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \quad .....(ii)$$

Now, from (i) and (ii), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

Que 5. In Fig. 7.15, DE || OQ and DF || OR, Show that EF || QR.

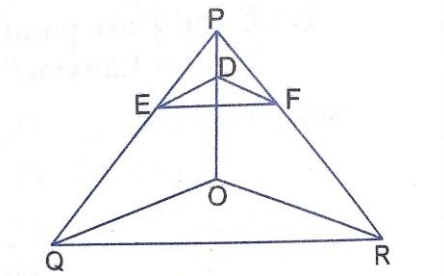

Fig. 7.15

Sol. In ΔPOQ, we have

DE || OQ (Given)

∴ By Basic proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad ....(i)$$

Similarly, in ΔPOR, we have

DF || OR (Given)

∴ $\frac{PD}{DO} = \frac{PF}{FR}$ ….(ii)

Now, from (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow EF \parallel QR$$

[Applying the converse of Basic proportionality Theorem in ΔPQR]

Que 6. Using converse of Basic proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

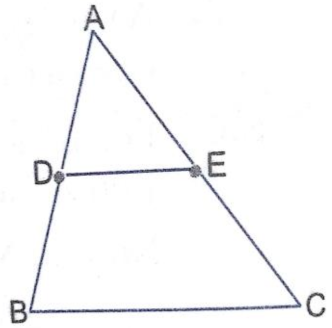

Fig. 7.16

Sol. Given: ΔABC in which D and E are the mid-points of sides AB and AC respectively.
To prove: DE || BC
Proof: Since, D and E are the mid-points of AB and AC respectively

∴ AD = DB and AE = EC

⇒ $\frac{AD}{DB} = 1 \quad and \quad \frac{AE}{EC} = 1$

⇒ $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, DE || BC (By the converse of Basic proportionality Theorem)

Que 7. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.

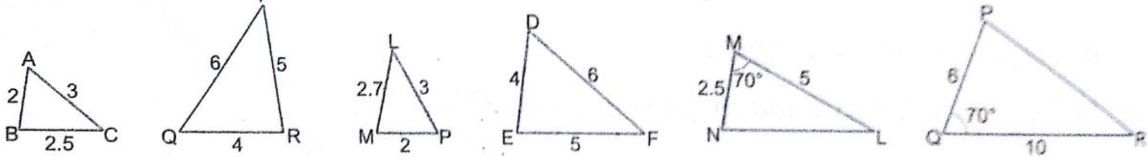

Fig. 7.17

Sol. (i) In ΔABC and ΔPQR, we have

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \quad \frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{PQ} = \frac{BC}{PR}$

$\therefore$ $\Delta ABC \sim \Delta QRP$ by SSS criterion of similarity.

(ii) In ΔLMP and ΔFED, we have

$$\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \quad \frac{LM}{EF} = \frac{27}{5}$$

Hence, $\frac{LP}{DF} = \frac{MP}{DE} \neq \frac{LM}{EF}$

$\therefore$ ΔLMP is not similar to ΔFED.

(iii) In ΔNML and ΔPQR, we have

$$\angle M = \angle Q = 70°$$

Now, $\frac{MN}{PQ} = \frac{2.5}{6} = \frac{5}{12}$ $\quad and \quad$ $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore \Delta NML \;\; is\ not\ similar\ to\ \Delta PQR.$

Que 8. In Fig. 7.18, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5 cm. Find the value of DC.

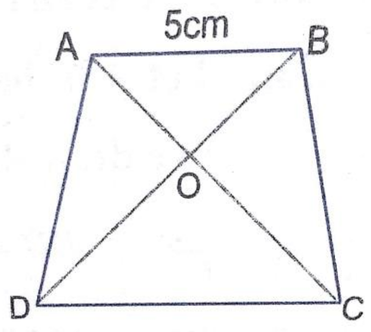

Fig. 7.18

Sol. In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\frac{AO}{OC} = \frac{BO}{OD}$ [Given]

So, by SAS criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$ $\quad \Rightarrow \frac{1}{2} = \frac{5}{DC}$ $\quad [\because\ AB\ =\ 5\ cm]$

⇒ DC = 10 cm

Que 9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE∼ ΔCFB.

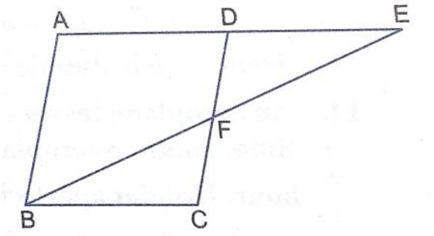

Fig. 7.19

Sol. In ΔABE and ΔCFB, we have

$\angle AEB = \angle CBF \quad (Alternate\ angles)$

$\angle A = \angle C$ (Opposite angles of a parallelogram)

∴ ΔABE∼ ΔCFB (By AA criterion of similarity)

Que 10. S and T are points on sides PR and QR if ΔPQR such that $\angle P = \angle RTS$. Show that ΔRPQ ∼ ΔRTS.

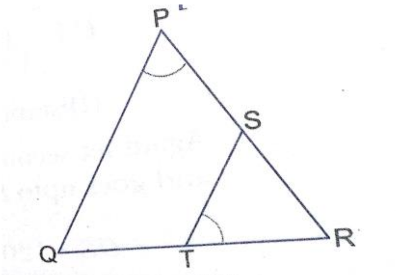

Fig. 7.20

Sol. In ΔRPQ and ΔRTS, we have

∠RPQ = ∠RTS (Given)

∠PRQ = ∠TRS = ∠R (Common)

∴ ∠RPQ ∼ ΔRTS (By AA criterion of similarity)

Que 11. In Fig. 7.21, ABC and AMP are two right triangles right-angled at B and M respectively. Prove that:

(i) ΔABC ∼ ΔAMP **(ii)** $\frac{CA}{PA} = \frac{BC}{MP}$

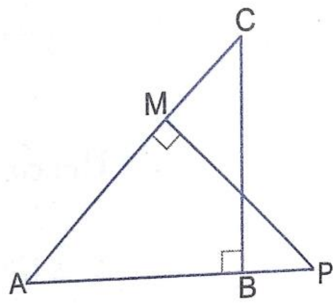

Fig. 7.21

Sol. (i) In ΔABC and ΔAMP, we have

∠ABC = ∠AMP = 90° (Given)

And, ∠BAC = ∠MAP (Common angle)

∴ ΔABC ~ ΔAMP (By AA criterion of similarity)

(ii) As ΔABC ~ ΔAMP (Proved above)

∴ $\frac{CA}{PA} = \frac{BC}{MP}$ (Sides of similar triangles are proportional)

Que 12. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD.

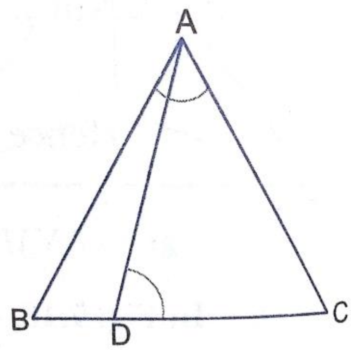

Fig. 7.22

Sol. In ΔABC and ΔDAC, we have

∠BAC = ∠ADC (Given)

and ∠C = ∠C (Common)

∴ ΔABC ~ ΔDAC (By AA criterion of similarity)

⇒ $\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

⇒ $\frac{CB}{CA} = \frac{CA}{CD}$ ⇒ $CA^2 = CB \times CD$

Que 13. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

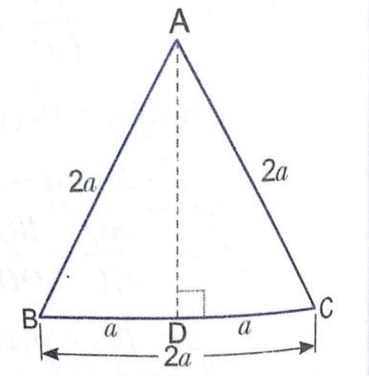

Fig. 7.23

Sol. Let ABC be an equilateral triangle of side 2a units.
We draw AD ⊥ BC. Then D is the mid-point of BC.

⇒ $BD = \frac{BC}{2} = \frac{2a}{2} = a$

Now, ABD is a right triangle right-angled at D.
∴ $AB^2 = AD^2 + BD^2$ $[By\ Pythagoras\ Theorem]$
⇒ $(2a)^2 = AD^2 + a^2$
⇒ $AD^2 = 4a^2 - a^2 = 3a^2$ ⇒ $AD = \sqrt{3}a$
Hence, each altitude = $\sqrt{3}a$ unit.

Que 14. An aeroplane leaves an airport an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

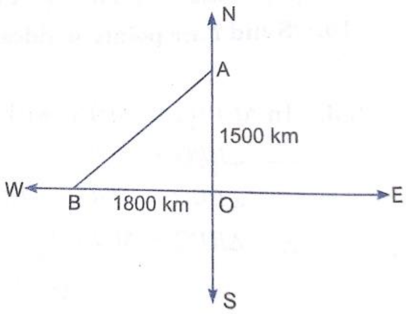

Fig. 7.24

Sol. Let the first aeroplane starts from O and goes upto A towards north where

$$OA = \left(1000 \times \frac{3}{2}\right) km = 1500\ km$$

(Distance = Speed x Time)
Again let second aeroplane starts from O at the same time and goes upto B towards west where

$$OB = 1200 \times \frac{3}{2} = 1800\ km$$

Now, we have to find AB.
In right angled ΔABO, we have

$AB^2 = OA^2 + OB^2$ [By using Pythagoras Theorem]

⇒ $AB^2 = (1500)^2 + (1800)^2$

⇒ $AB^2 = 2250000 + 3240000$ ⇒ $AB^2 = 5490000$

∴ $AB = 100\sqrt{549} = 100 \times 234307 = 2343.07$ km.

Que 15. In the given Fig. 7.25, ΔABC and ΔDBC are on the same base BC. If AD intersects BC at O. prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

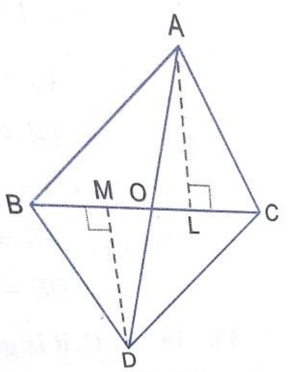

Fig. 7.25

Sol. Given: ΔABC and ΔDBC are on the same base BC and AD intersects BC at O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DP}$

Construction: Draw AL ⊥ BC and DM ⊥ BC
Proof: In ΔALO and ΔDMO, we have

∠ALO = ∠DMO = 90° and

∠AOL = ∠DOM (Vertically opposite angles)

∴ ΔALO ~ ΔDMO (By AA-Similarity)

⇒ $\frac{AL}{DM} = \frac{AO}{DO}$ …(i)

∴ $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}BC\times AL}{\frac{1}{2}BC\times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ (Using (i))

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Que 16. In Fig. 7.26, AB || PQ || CD, AB = x units, CD = y units and PQ = z units. Prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

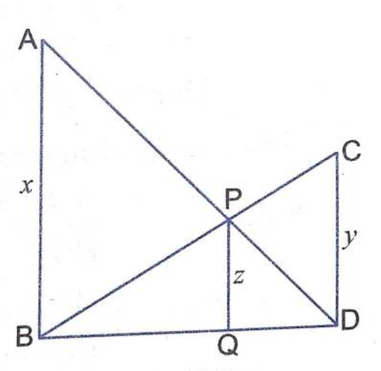

Fig. 7.26

Sol. In ΔADB and ΔPDQ,
Since AB || PQ

∠ABQ = ∠PQD (Corresponding∠'s)

∠ADB = ∠PDQ (Common)

By AA-Similarity

ΔADB ~ ΔPDQ

∴ $\frac{DQ}{DB} = \frac{PQ}{AB}$ ⇒ $\frac{DQ}{DB} = \frac{Z}{x}$ ..(i)

Similarly, ΔPBQ ~ ΔCBD

And $\frac{BQ}{DB} = \frac{z}{x}$ …(ii)

Adding (i) and (i), we get

$$\frac{z}{x} + \frac{z}{y} = \frac{DQ+BQ}{DB} = \frac{BD}{BD}$$

$$\frac{z}{x} + \frac{z}{y} = 1 \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

Que 17. In Fig. 7.27, if ΔABC ~ ΔDEF and their sides are of length (in cm) as marked along them, then find the length of the sides of each triangle.

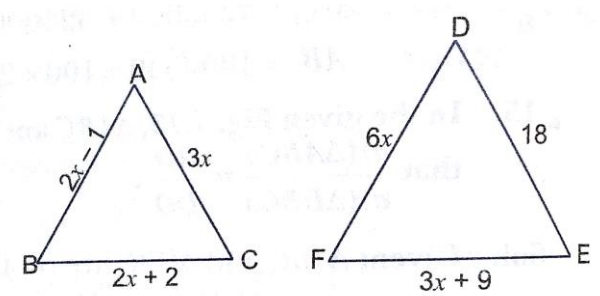

Fig. 7.27

Sol. ΔABC ~ ΔDEF (Given)

Therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

So, $\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

Now, taking $\frac{2x-1}{18} = \frac{1}{2}$

$\Rightarrow \quad 4x - 2 = 18 \quad \Rightarrow \quad x = 5$

$\therefore \quad AB = 2 \times 5 - 1 = 9, BC = 2 \times 5 + 2 = 12$

$CA = 3 \times 5 = 15, DE = 18, EF = 3 \times 5 + 9 = 24 \; and \; FD = 6 \times 5 = 30$

Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm
DE = 18 cm, EF = 24 cm, FD = 30 cm

Que 18. In ΔABC, it is given that $\frac{AB}{AC} = \frac{BD}{DC}$. If ∠B = 70° and ∠C = 50° thwen find ∠BAD.

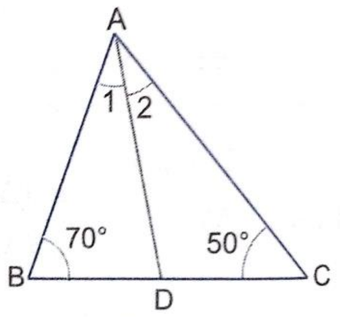

Fig. 7.28

Sol. In ΔABC

$\because \quad \angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\angle A + 70° + 50° = 180°$

$\Rightarrow \quad \angle A = 180° - 120° \quad \Rightarrow \quad \angle A = 60°$

$\because \quad \frac{AB}{AC} = \frac{BD}{DC}$ (Given)

$\therefore \quad \angle 1 = \angle 2$ ….(i)

[Because a line through one vertex of a triangle divides the opposite sides in the ratio of other two sides, then the line bisects the angle at the vertex.]

But $\angle 1 + \angle 2 = 60°$ ….(ii)

From (i) and (ii) we get,

$2\angle 1 = 60° \quad \Rightarrow \quad \angle 1 = \frac{60°}{2} = 30°$

Hence, ∠BAD = 30°

Que 19. If the diagonals of a quadrilateral divides each other proportionally, prove that it is a trapezium.

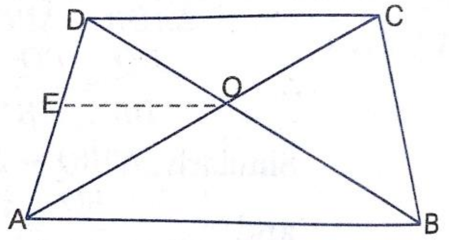

Fig. 7.29

Sol. $\frac{AO}{BO} = \frac{CO}{DO}$ (Given)

$\Rightarrow$ $\frac{AO}{CO} = \frac{BO}{DO}$ …(i)

In ΔABD, EQ || AB (Construction)

$\therefore$ $\frac{AE}{ED} = \frac{BO}{DO}$ (By BPT) …(ii)

From equations (i) and (ii)

$\frac{AE}{ED} = \frac{AO}{CO}$ $\Rightarrow$ EO || DC (Converse of BPT)

But EO || AB (Construction)
∴ AB || DC

⇒ In quad ABCDD AB || DC ⇒ ABCD is a trapezium.

Que 20. In the given Fig. 7.30, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle.

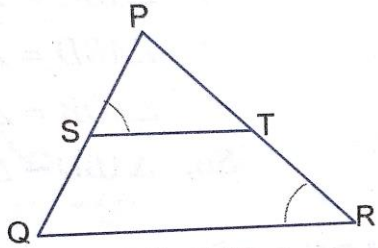

Fig. 7.30

Sol. Given: $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ

To Prove: PQR is isosceles triangle.

Proof: $\frac{PS}{SQ} = \frac{PT}{TR}$

By converse of BPT we get

ST || QR

∴ $\angle PST = \angle PQR$ (Corresponding angles) …(i)

But, ∠PST = ∠PRQ (Given) …(ii)

From equation (i) and (ii)

∠PQR = ∠PRQ ⇒ PR = PQ

So, ΔPQR is an isosceles triangle.

Que 21. The diagonals of a trapezium ABCD in which AB || DC, intersect at O. If AB = 2 cd then find the ratio of areas of triangles AOB and COD.

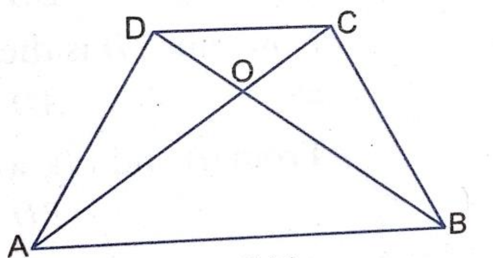

Fig. 7.31

Sol. In ΔAOB and ΔCOD

∠COD = ∠AOB (Vertically opposite angles)

∠CAB = ∠DCA (Alternate angles)

∴ ΔAOB ~ ΔCOD (B AA-similarity)

By area of theorem

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2} \quad \Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2CD)^2}{CD^2} = \frac{4}{1}$$

Hence, ar (ΔAOB): ar (ΔCOD) = 4: 1.

Que 22. In the given Fig. 7.32, find the value of x in terms of a, b and c.

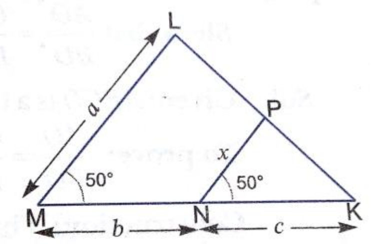

Fig. 7.32

Sol. In ΔLMK and ΔPNK

We have, ∠M = ∠N = 50° and ∠K = ∠K (Common)

ΔLMK ~ ΔPNK (AA – Similarity)

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c} \qquad \Rightarrow \quad x = \frac{ac}{b+c}$$

Que 23. In the given Fig. 7.33, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm.

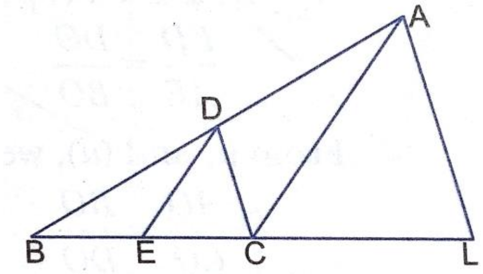

Fig. 7.33

Sol. In ΔABC, DE || AC (Given)

$\Rightarrow \quad \frac{BD}{DA} = \frac{BE}{EC}$ (By BPT) …(i)

In ΔABL DC || AL

$\Rightarrow \quad \frac{BD}{Da} = \frac{BC}{CL}$ (By BPT) …(ii)

From (i) and (ii) we get

$$\frac{BE}{EC} = \frac{BC}{CL} \quad \Rightarrow \quad \frac{4}{2} = \frac{6}{CL} \quad \Rightarrow \quad CL = 3\ cm$$

Que 24. In the given Fig. 7.34, AB = AC. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.

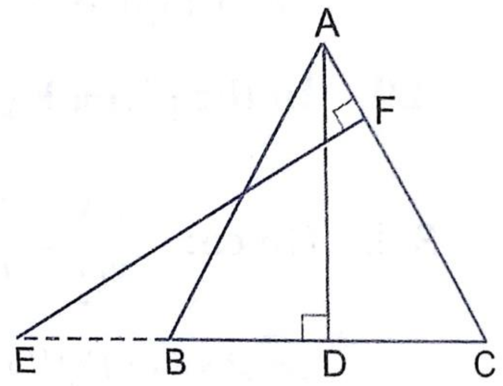

Fig. 7.34

Sol. In ΔABD and ΔCEF

AB = AC (Given)

$\Rightarrow \angle ABC = \angle ACB$ (Equal sides have equal opposite angles)

$\angle ABD = \angle ECF$

$\angle ADB = \angle EFC$ (Each 90°)

So, $\Delta ABD \sim \Delta CEF$ (AA – Similarity)

Long Answer Type Questions

[4 MARKS]

Que 1. Using Basic proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

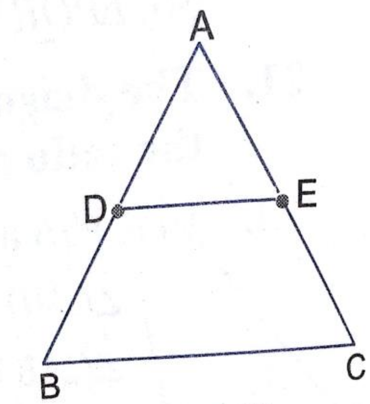

Fig. 7.35

Sol. Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: AE = EC

Proof: In ΔABC, DE || BC

∴ By Basic proportionality Theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \ldots(i)$$

Now, since D is the mid-point of AB

⇒ AD = BD …(ii)

From (i) and (ii), we have

$$\frac{BD}{BD} = \frac{AE}{EC} \quad \Rightarrow \quad 1 = \frac{AE}{EC}$$

⇒ AE = EC

Hence, E is the mid-point of AC.

Que 2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. Given: ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at the point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE || AB i.e., OE || DC.

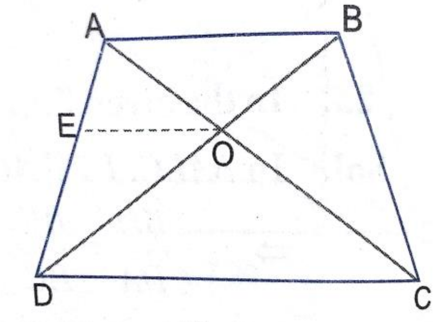

Fig. 7.36

Proof: In ΔADC, we have OE || AB (Construction)

∴ By Basic proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \qquad \text{....(i)}$$

Now, in ΔABD, we have OE || AB (Construction)

∴ By Basic proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \quad \Rightarrow \quad \frac{AE}{ED} = \frac{BO}{DO} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \quad \Rightarrow \quad \frac{AO}{BO} = \frac{CO}{DO}$$

Que 3. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

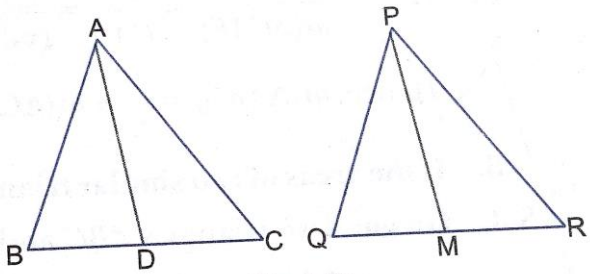

Fig. 7.37

Sol. In ΔABD and ΔPQM, we have

∠B = ∠Q (∵ $\Delta ABC \sim \Delta PQR$) ...(i)

$$\frac{AB}{PQ} = \frac{BC}{QR} (\because \ \Delta ABC \sim \Delta PQR)$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \quad \Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} \qquad \text{....(ii)}$$

[Since AD and PM are the medians of ΔABC and ΔPQR respectively]

From (i) and (ii) it is proved that

$\Delta ABD \sim \Delta PQM$ (By SAS criterion of similarity)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \qquad \Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$$

Que 4. In Fig. 7.38, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

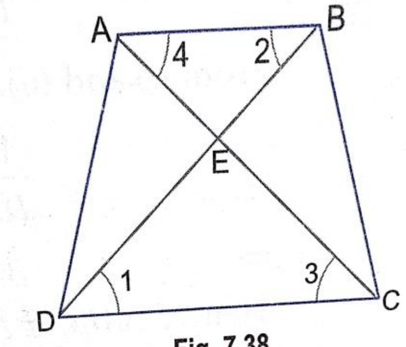

Fig. 7.38

Sol. In ΔEDC and ΔEBA, we have

∠1 = ∠2	[Alternate angles]
∠3 = ∠4	[Alternate angles]
and ∠CED = ∠AEB	[Vertically opposite angles]
∴ $\Delta EDC \sim \Delta EBA$	[By AA criterion of similarity]

$$\Rightarrow \quad \frac{ED}{EB} = \frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC} = \frac{EB}{EA} \qquad \text{….(i)}$$

It is given that ΔAED ~ ΔBEC

$$\therefore \quad \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \qquad \text{….(ii)}$$

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \quad \Rightarrow \quad (EB)^2 = (EA)^2 \quad \Rightarrow \quad EB = EA$$

Substituting EB = EA in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC} = 1 \quad \Rightarrow \quad AD = BC$$

Que 5. Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle describe on its hypotenuse.

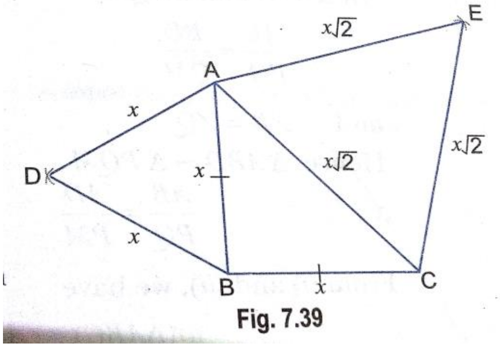

Fig. 7.39

Sol. Given: ΔABC in which $\angle ABC = 90° \ and \ AB = BC. \Delta ABD$ and ΔACE are equilateral triangles.

To Prove: ar (ΔABD) = $\frac{1}{2} \times ar(\Delta CAE)$

Proof: Let AB = BC = x units.

∴ $\quad hyp.\ CA = \sqrt{x^2 + x^2} \ = \ x\sqrt{2}\, units.$

Each of the ΔABD and ΔCAE being equilateral, has each angle equal to 60°

∴ $\quad \Delta ABD \sim \Delta CAE$

But, the ratio of the areas of two similar triangles in equal to the ratio of the squares of their corresponding sides.

∴ $$\frac{ar(\Delta ABD)}{ar(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{\left(x\sqrt{2}\right)^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, ar (ΔABD) = $\frac{1}{2} \times ar(\Delta CAE)$

Que 6. If the areas of two similar triangles are equal, prove that they are congruent.

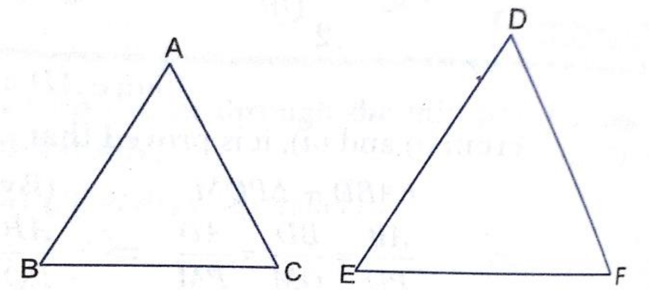

Fig. 7.40

Sol. Given: Two triangles ABC and DEF, such that
ΔABC ~ ΔDEF and area (ΔABC) = area (ΔDEF)
To prove: ΔABC ≅ ΔDEF
Proof: ΔABC ~ ΔDEF

⇒ $\quad \angle A = \angle D, \angle B = \angle C = \angle F$

And $\quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Now, ar (ΔABC) = ar (ΔDEF) (Given)

∴ $\quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$ ….(ii)

And $\quad \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)} (\because \Delta ABC - \Delta DEF)$ ….(ii)

From (i) and (ii), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \quad \Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

Hence, ΔABC ≅ ΔDEF (By SSS criterion of congruency)

Que 7. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

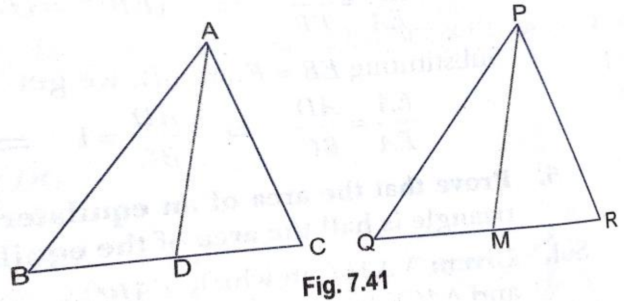

Fig. 7.41

Sol. Let ΔABC and ΔPQR be two similar triangles. AD and PM are the medians of ΔABC and ΔPQR resopectively.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since ΔABC ~ ΔPQR

∴ $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ …(i)

In ΔABD and ΔPQM

$\frac{AB}{PQ} = \frac{BD}{QM}$ $\left(\because \frac{AB}{PQ} = \frac{BC}{QR} = \frac{1/2\, BC}{1/2\, QR}\right)$

And $\angle B = \angle Q$ $(\because \Delta ABC \sim \Delta PQR)$

Hence, ΔABD ~ ΔPQM (By SAS Similarity criterion)

∴ $\frac{AB}{PQ} = \frac{AD}{PM}$ …(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

Que 8. In Fig.7.42, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that
(i) $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$
(ii) $AF^2 + BD^2 + CE^2 + AE^2 + CD^2 + BF^2$.

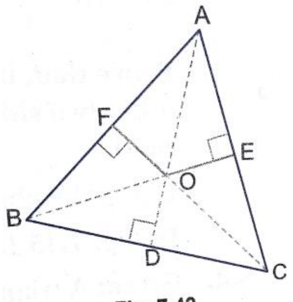

Fig. 7.42

Sol. Join OA, OB and OC.

(i) In right $\Delta's$ OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$ …(i)

$OB^2 = BD^2 + OD^2$ …(ii)

and $OC^2 = CE^2 + OE^2$ …(iii)

Adding (i), (ii) and (iii), we have

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow$ $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) We have, $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

$\Rightarrow$ $(OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$

$\Rightarrow$ $AE^2 + CD^2 + BF^2 = AF^2 + BD^2\ CE^2$

[Using Pythagoras Theorem in ΔAOE, ΔBOF and ΔCOD]

Que 9. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD
(see Fig. 7.43). Prove that 2 AB² = 2 AC² + BC².

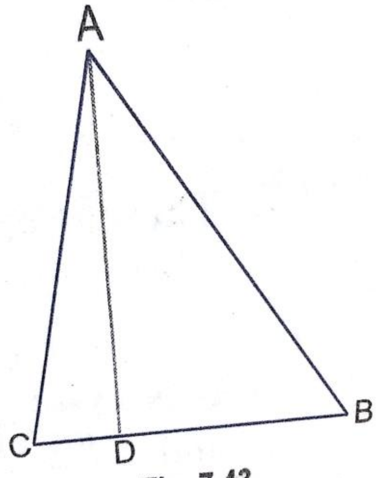

Fig. 7.43

Sol. We have, DB = 3 CD

Now, BC = BD + CD

$\Rightarrow$ BC = 3 CD + CD = 4 CD (Given DB = 3 CD)

$\therefore$ $CD = \frac{1}{4}BC$

And DB = 3 CD = $\frac{3}{4}BC$

Now, in right-angled triangle ABD, we have

$AB^2 = AD^2 + DB^2$ …(i)

Again, in right- angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ …(ii)

Subtracting (ii) from (i), we have

$AB^2 – AC^2 = DB^2 – CD^2$

$$\Rightarrow \; AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right) BC^2 = \frac{8}{16} BC^2$$

$$\Rightarrow \; AB^2 - AC^2 = \frac{1}{2} BC^2$$

$$\therefore \; 2AB^2 - 2AC^2 = BC^2 \qquad \Rightarrow \; 2AB^2 = 2AC^2 + BC^2$$

HOTS (Higher Order Thinking Skills)

Que 1. In Fig. 7.58, $\Delta FEC \cong \Delta GDB$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$.

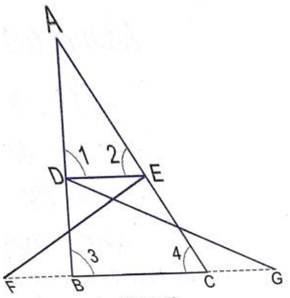

Fig. 7.58

Sol. Since $\Delta FEC \cong \Delta GDB$

⇒ EC = BD ….(i)

It is given that

$\angle 1 = \angle 2$

⇒ AE = AD [Sides opposite to equal angles are equal] ….(ii)

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

⇒ DE||BC [By the converse of basic proportionality theorem]

⇒ $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Corresponding angles]

Thus, in $\Delta' s\ ADE\ and\ ABC,$ we have

$\angle A = \angle A$ [Common]

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ [Proved above]

So, by AAA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$

Que 2. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Sol. Given: In ΔABC and $\Delta PQR, AD$ and PM are their medians respectively

Such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ …(i)

To prove: $\Delta ABC \sim \Delta PQR$

Construction: Produce AD to E such that AD = DE and produce PM to N such that PM = MN.

Join BE, CE, QN, RN.

Proof: Quadrilateral ABEC and PQNR are $||^{gm}$ because their diagonals bisect each other

at D and M respectively.

$\Rightarrow \quad BE = AC \quad and \quad QN = PR$

$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR} \quad \Rightarrow \quad \frac{BE}{QN} = \frac{AB}{PQ}$ [From (i)]

$i.e., \quad \frac{AB}{PQ} = \frac{BE}{QN}$ …(ii)

From (i) $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AE}{PN}$ …..(iii)

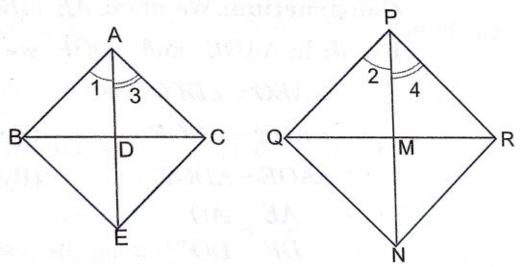

Fig. 7.59

From (ii) and (iii)

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow \quad \Delta ABE \sim \Delta PQN$ (SSS similarity criterion)

$\Rightarrow \quad \angle 1 = \angle 2$ ……(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \quad \Rightarrow \quad \angle 3 = \angle 4$ …….(v)

Adding (iv) and (v), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4 \quad \Rightarrow \quad \angle A = \angle P$

And $\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

$\therefore \quad \Delta ABC \sim \Delta PQR$ (By SAS criterion of similarity)

Que 3. In Fig. 7.60, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that RA = $\frac{1}{3}CA$.

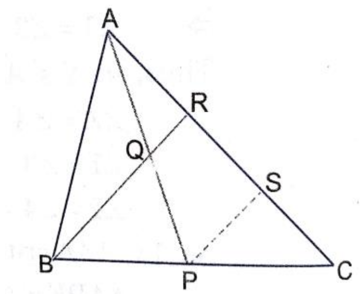

Fig. 7.60

Sol. Given: In ΔABC, P is the mid-point of BC, Q is the mid-point of AP such that BQ produced meets AC at R.

To prove: RA = $\frac{1}{3}CA$.

Construction: Draw PS||BR, meeting AC at S.
Proof: In $\Delta BCR, P$ is the mid-point of BC
and PS||BR.
∴ S is the mid-point of CR.
⇒ CS = SR ….(i)
In $\Delta APS, Q$ is the mid-point of AP and QR||PS.
∴ R is the mid-point of AS.
⇒ AR = RS ….(ii)
From (i) and (ii), we get
AR = RS = SC

⇒ $AC = AR + RS + SC = 3\ AR$ ⇒ $AR = \frac{1}{3}AC = \frac{1}{3}CA$

Que 4. In Fig. 7.61, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\mathbf{ar(\Delta ABC)}}{\mathbf{ar(\Delta DBC)}} = \frac{\mathbf{AO}}{\mathbf{DO}}$.

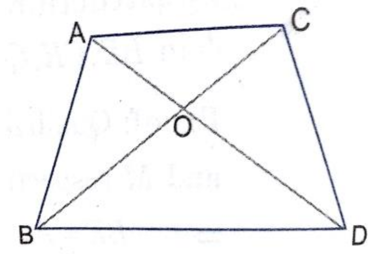

Fig. 7.61

Sol. Given: Two triangles ΔABC and ΔDBCwhich stand on the same base but on opposite sides of BC.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: We draw AE ⊥ BC and DF ⊥ BC.
Proof: In ΔAOE and ΔDOF, we have

$\angle AEO = \angle DFO = 90°$

$\angle AOE = \angle DOF$ (Vertically opposite angles)

∴ $\Delta AOE \sim \Delta DOF$ (By AA criterion of similarity)

⇒ $\frac{AE}{DF} = \frac{AO}{DO}$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF}$

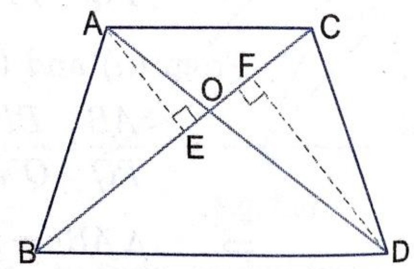

Fig. 7.62

⇒ $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DE}$ …(ii)

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Que 5. Two poles of height a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

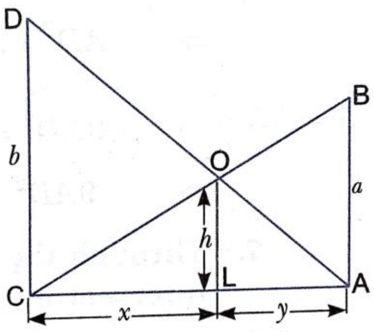

Fig. 7.63

Sol. Let AB and CD be two poles of height a and b metres respectively such that the poles are p metres apart i.e., AC = p metres. Suppose the lines AD and BC meet at O such that

OL = h metres.

Let CL = x and LA = y. Then, x + y = p.

In ΔABC and ΔLOC, we have

$\angle CAB = \angle CLO$ [Each equal to 90 °]

$\angle C = \angle C$ [Common]

$\therefore$ $\Delta ABC \sim \Delta LOC$ [By AA criterion of similarity]

$\Rightarrow$ $\frac{CA}{CL} = \frac{AB}{LO}$ $\Rightarrow$ $\frac{P}{x} = \frac{a}{h}$

$\Rightarrow$ $x = \frac{ph}{a}$ ....(i)

In ΔALO and ΔACD, we have

$\angle ALO = \angle ACD$ [Each equal to 90°]

$\angle A = \angle A$ [Common]

$\therefore$ $\Delta ALO \sim \Delta ACD$ [By AA criterion of similarity]

$\Rightarrow$ $\frac{AL}{AC} = \frac{OL}{DC}$ $\Rightarrow$ $\frac{y}{p} = \frac{h}{b}$

$\Rightarrow$ $y = \frac{ph}{b}$ .....(ii)

From (i) and (ii), we have

$x + y = \frac{ph}{a} + \frac{ph}{b}$ $\Rightarrow$ $p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ [$\because$ $x + y = p$]

$\Rightarrow$ $1 = h\left(\frac{a+b}{ab}\right)$ $\Rightarrow$ $h = \frac{ab}{a+b}$ $metres.$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

Value Based Questions

Que 1. Puneet prepared two posters on 'National Integration' for decoration on Independence Day on triangular sheets (say ABC and DEF). The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the perimeter P^2 of ΔDEF. Are the two triangular sheets similar? If yes, find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$.

What values can be inculcated through celebration of national festivals?

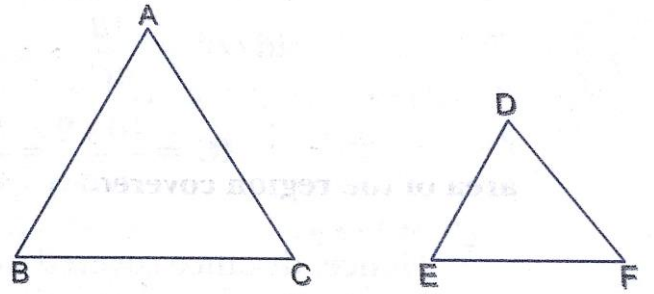

Fig. 4

Sol. In ΔABC and DEF

AB = 3 DE, AC = 3DF and $P_1 = 3p_2$

$\therefore \quad \frac{AB}{DE} = 3; \quad \frac{AC}{DF} = 3$

And $P_1 = 3p_2 \Rightarrow BC = 3EF$

$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$

$\Rightarrow$ ΔABC ~ΔDEF (By SSS similarity)

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9$

Unity of nation, fraternity, Patriotism.

Que 2. A man steadily goes 4 m due East and then 3 m due North.
(i) Find the distance from initial point to last point.
(ii) Which mathematical concept is used in this problem?
(iii) What is its value?

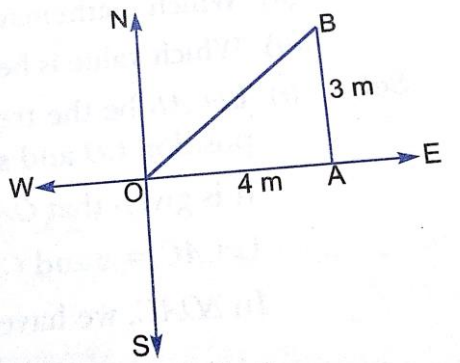

Fig. 5

Sol. (i) Let the initial position of the man be O and his final position be B. Since man goes 4 m due East and then 3 m due North. Therefore, △A0B is a right triangle right angled at A such that $OA = 4\ m\ and\ AB = 3m$

By Pythagoras Theorem, we have

$$OB^2 = OA^2 + AB^2$$

$$OB^2 = (4)^2 + (3)^2 = 16 + 9 = 25$$

$$OB = \sqrt{25} = 5\ m.$$

Hence, the man is at a distance of 5 m from the initial position.

(ii) Right-angled triangle, Pythagoras Theorem.

(iii) Knowledge of direction and speed saves the time.

Que 3. Two trees of height x and y are p metres apart.

(i) Prove that the height of the point of intersection of the line joining the top of each tree to the foot of the opposite tree is given by $\frac{xy}{x+y}m$.

(ii) Which mathematical concept is used in this problem?

(iii) What is its value?

Sol. (i) Similar to solution Q. 5, page 161.

(ii) Similarity of triangles.

(iii) Trees are helpful to maintain the balance in the environment. They should be saved at any cost.