Very Short Answer Type Questions

[1 Marks]

Que 1. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Sol. sin 67° + cos 75°

= sin (90° - 23°) +cos (90° - 15°) = cos 23° + sin 15°

Que 2. Evaluate:

$(\boldsymbol{i})\ \frac{\sin 18°}{\cos 72°}$ $\qquad$ $(\boldsymbol{ii})\ \frac{\tan 26°}{cot\, 64°}$

$(\boldsymbol{iii})\ \cos 48° - \sin 42°$ $\qquad$ $(\boldsymbol{iv})\ \text{co}sec\ 31° - sec\ 59°$

Sol. $(i)\ \frac{\sin 18°}{\cos 72°} = \frac{\sin(90°-72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1$

$(ii)\ \frac{\tan 26°}{cot\ 64°} = \frac{\tan(90°-64°)}{cot\ 64°} = \frac{cot\ 64°}{cot\ 64°} = 1$

$(iii)\ \cos 48° - \sin 42° = \cos(90° - 42°)\ -\ \sin 42°$

$(iv)\ \text{co}sec\ 31° - sec\ 59° = \text{co}sec\ (90° - 59°) - sec\ 59°$

$= sec\ 59° - sec\ 59°\ = 0.$

Que 3. In ΔABC right angled at C, find the value of cos (A + B).

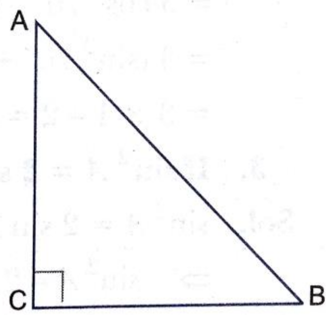

Fig. 10.2

Sol. In Fig. 10.2

$\because \qquad \angle C = 90°$

$\therefore\ \angle A + \angle B = 90°$ (By angle sum property)

Hence, cos (A + B) = cos (90°) = 0.

Que 4. Can the value of the expression (cos 80° - sin 80°) be negative? Justify your answer.

Sol. True, for $\theta\ >\ 45°, \sin\theta\ > \cos\theta, so$ $\cos 80° - \sin 80°$ has a negative value.

Que 5. If sin A + sin^2 A = 1, then show that cos^2 A + cos^4 A = 1.

Sol. sin A + sin^2 A = 1 $\Rightarrow$ sin A = 1 – sin^2 A = cos^2 A

$\therefore \quad \cos^2 A\ +\ \cos^4 A = \sin A\ +\ \sin^2 A = 1.$

Que 6. Write the value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$.

Sol. $cot^2\, \theta \;-\; \frac{1}{\sin^2 \theta} = cot^2\, \theta \;-\; \text{cos}ec^2\, \theta = 1.$

Que 7. If $\sin \theta = \frac{1}{3}$, then find the value of $2 \cot^2 \theta + 2$.

Sol. $2\; cot^2\, \theta + 2 \;=\; 2\;(cot^2\, \theta + 1) \;=\; 2\; \text{cos}ec^2\, \theta = \frac{2}{\sin^2 \theta} = \frac{2}{\left(\frac{1}{3}\right)^2} = 2 \times 9 = 18.$

Short Answer Type Questions – I

[2 marks]

Que 1. Evaluate cos 48° cos 42° - sin 48° sin 42°.

Sol. cos 48° cos 42° - sin 48° sin 42° = cos (90° - 42°) cos (90° - 48°) – sin 48° sin 42°
= sin 42° sin 48° - sin 48° sin 42° $[\because \cos(90-\theta) = \sin\theta]$
= 0

Que 2. Find the value of: 3 sin² 20° - 2 tan² 45° + 3 sin² 70°

Sol. $3\sin^2 20° - 2\tan^2 45° + 3\sin^2 70°$
$= 3\sin^2(90° - 70°) - 2(1)^2 + 3\sin^2 70°$ $[\because \tan 45° = 1]$
$= 3\cos^2 70° - 2 + 3\sin^2 70°$ $[\because \sin(90-\theta) = \cos\theta]$
$= 3(\sin^2 70° + \cos^2 70°) - 2$
$= 3 \times 1 - 2 = 3 - 2 = 1.$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Que 3. If sin² A = 2 sin A then find the value of A.

Sol. $\sin^2$ A = 2 sin A.
$\Rightarrow \sin^2 A - 2\sin A = 0 \Rightarrow \sin A(\sin A - 2) = 0$
⇒ either sin A = 0 or sin A – 2 = 0.
$\Rightarrow A = 0°$ $[\sin A = 2, Not\ possible]$
∴ Value of $\angle A = 0°$

Que 4. Find maximum value of $\frac{1}{\sec\theta}, 0° \le \theta \le 90°$.

Sol. $\frac{1}{\sec\theta}, (0° \le \theta \le 90°)$ (Given)

∵ sec θ is in the denominator

∴ The min. value of sec θ will return max. Value for $\frac{1}{\sec\theta}$.

But the min. Value of sec θ is sec 0° = 1.

Hence, the max. Value of $\frac{1}{\sec 0°} = \frac{1}{1} = 1.$

Que 5. Given that sin $\theta = \frac{a}{b}$, find the value of tanθ.

Sol. $\sin\theta = \frac{a}{b}$

$$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{\frac{b^2 - a^2}{b^2}} = \sqrt{\frac{b^2 - a^2}{b}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a \mid b}{\sqrt{\frac{b^2 - a^2}{b}}} = \frac{a}{\sqrt{b^2 - a^2}}$$

Que 6. If sin θ = cos θ, then find the value of 2 tan θ + cos^2 θ.

Sol. sin θ = cos θ (Given)

It means value of θ = 45°

Now, $2\tan\theta + \cos^2\theta = 2\,tan\,45° + \cos^2 45°$

$$= 2 \times 1 + \left(\frac{1}{\sqrt{2}}\right)^2 \quad \left(\because\ \tan 45° = 1, \cos 45° = \frac{1}{\sqrt{2}}\right)$$

$$= 2 + \frac{1}{2} = \frac{4+1}{2} = \frac{5}{2}$$

Que 7. I sin (x - 20°) = cos (3x - 10)°, then find the value of x.

Sol. $\sin(x-20)° = \cos(3x-10)°$

$\Rightarrow\ \cos[90° - (x-20)°] = \cos(3x-10)°$

By comparing the coefficient

$90° - x° + 20° = 3x° - 10° \quad \Rightarrow\ 110° + 10° = 3x° + x°$

$$120° = 4x° \qquad \Rightarrow\ x° = \frac{120°}{4} = 30°$$

Que 8. If sin^2 A =$\frac{1}{2}\tan^2 45°$, where A is an acute angle, then find the value of A.

Sol. $\sin^2 A = \frac{1}{2}\tan^2 45° \qquad \Rightarrow \qquad \sin^2 A = \frac{1}{2}(1)^2 \qquad [\because\ \tan 45° = 1]$

$\Rightarrow\ \sin^2 A = \frac{1}{2} \qquad \Rightarrow \qquad \sin A = \frac{1}{\sqrt{2}}$

Hence, $\angle A = 45°$

Short Answer Type Questions – II

[3 marks]

Que 1. If sin A = $\frac{3}{4}$, calculate cos A and tan A.

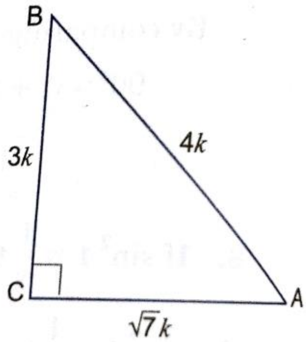

Fig. 10.3

Sol. Let us first draw a right ΔABC in which $\angle C = 90°$.

Now, we know that

$$\sin A = \frac{perpendicular}{Hypotenuse} = \frac{BC}{AB} = \frac{3}{4}$$

Let BC = 3k and AB = 4k, where k is a positive number.

Then, by Pythagoras Theorem, we have

$AB^2 = BC^2 + AC^2 \quad \Rightarrow (4k)^2 = (3k)^2 + AC^2$

$\Rightarrow \quad 16k^2 - 9k^2 = AC^2 \quad \Rightarrow 7k^2 = AC^2$

$\therefore \quad AC = \sqrt{7}k$

$\therefore \quad \cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$.

Que 2. Given 15 cot A = 8, find sin A and sec A.

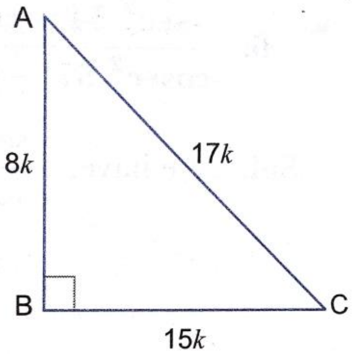

Fig. 10.4

Sol. Let us first draw a rightΔABC, in which $\angle B = 90°$.

Now, we have, 15 cot A = 8

$$\therefore \quad cot\ A = \frac{8}{15} = \frac{AB}{BC} = \frac{Base}{Perpendicular}$$

Let AB = 8k and BC = 15k

Then, AC = $\sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras Theorem)

$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$

$$\therefore \quad \sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

And, sec A = $\frac{Hypotenuse}{Base} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$.

Que 3. In Fig. 10.5, find tan P – cot R.

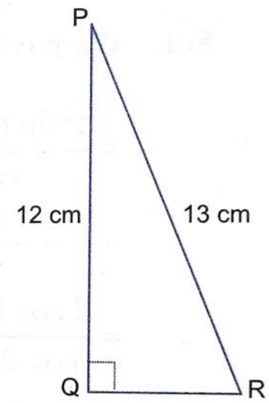

Fig. 10.5

Sol. Using Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow 169 = 144 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25 \quad \Rightarrow QR = 5$ cm

Now, tan P = $\frac{QR}{PO} = \frac{5}{12}$ and cot R = $\frac{QR}{PQ} = \frac{5}{12}$

$$\therefore \quad \tan P - cot\ R = \frac{5}{12} - \frac{5}{12} = 0.$$

Que 4. If sin $\boldsymbol{\theta} + \cos \boldsymbol{\theta} = \sqrt{3}$, then prove that tan $\boldsymbol{\theta}$ + cot $\boldsymbol{\theta}$ = 1.

Sol. sin θ + cos θ = $\sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\theta\cos\theta = 2 \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sin\theta.\ \text{Cos}\ \theta = 1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow \qquad 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \qquad 1 = \tan\theta + \cot\theta$

Therefore $\tan\theta + \cot\theta = 1$

Que 5. Prove that $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

Sol. LHS $= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$ [Rationalising the denominator]

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\vartheta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= (sec\,\theta - \tan\theta)^2 = RHS$

Without using tables, evaluate the following (6 to 10).

Que 6. $\frac{sec^2\,54° - cot^2\,36°}{cosec^2\,57° - \tan^2 33°} + 2\sin^2 38°.sec^2\,52° - \sin^2 45°.$

Sol. We have, $\frac{sec^2\,54° - cot^2\,36°}{cosec^2\,57° - \tan^2 33°} + 2\sin^2 38°.sec^2\,52° - \sin^2 45°.$

$= \frac{sec^2\,(90°-36°) - cot^2\,36°}{cosec^2\,(90°-33°) - \tan^2 33°} + 2\sin^2 38°.sec^2\,(90° - 38°) - \sin^2 45°$

$= \frac{cosec^2\,36° - cot^2\,36°}{sec^2\,33° - \tan^2 33°} + 2\sin^2 38°.cosec^2\,38 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{1} + 2.1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}.$

Que 7. $\frac{2\sin 68°}{\cos 22°} - \frac{2\,cot\,15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}.$

Sol. We have $\frac{2\sin 68°}{\cos 22°} - \frac{2\,cot\,15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}$

$= \frac{2\sin(90°-22°)}{\cos 22°} - \frac{2\,cot\,15°}{5\tan(90°-15°)}$

$- \frac{3\tan 45°.\tan 20°.\tan 40°.\tan(90°-40°).\tan(90°-20°)}{5}$

$= \frac{2\cos 22°}{\cos 22°} - \frac{2\,cot\,15°}{5\,cot\,15°} - \frac{3\tan 45°,\tan 20°.\tan 40°.cot\,40°.cot\,20°}{5}$

$= 2 - \frac{2}{5} - \frac{3\tan 45°.(\tan 20°.cot\,20°).(\tan 40°.cot\,40°)}{5}$

$$2 - \frac{2}{5} - \frac{3}{5}.1.1.1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1.$$

Que 8. $\frac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\frac{\sin(90°-\theta\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta).\cos\theta}{cot\ \theta}\right].$

Sol. We have $\frac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\frac{\sin(90°-\theta\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta).\cos\theta}{cot\ \theta}\right]$

$$= \frac{\sin^2 20° + \sin^2 (90°-20°)}{\cos^2 20° + \cos^2 (90°-20°)} + \left[\frac{\cos\theta\sin\theta}{\tan\theta} + \frac{\cos\theta\sin\theta}{cot\ \theta}\right]$$

$$= \frac{\sin^2 20° + \cos^2 20°}{\cos^2 20° + \sin^2 20°} + \left[\frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta\sin\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$= \frac{1}{1} + \left[\cos^{2\,\theta\,+\,\sin^2\theta}\right] = 1 + 1 = 2.$$

Que 9. Evaluate sin 25° cos 65° + cos 25° sin 65°.

Sol. sin 25°. Cos 65° + cos 25°. Sin 65°

$= \sin(90° - 65°).\cos 65° + \cos(90° - 65°).\sin 65°$

$= \cos 65°.\cos 65° + \sin 65°.\sin 65°$

$= \cos^2 65° + \sin^2 65° = 1.$

Que 10. Without using tables, evaluate the following:

$$\mathbf{3\cos 68°.cosec\ 22° - \frac{1}{2}\tan 43°.\tan 47°.\tan 12°.\tan 60°.\tan 78°.}$$

Sol. We have,

$3\cos 68°.cosec\ 22° - \frac{1}{2}\tan 43°.\tan 47°.\tan 12°.\tan 60°.\tan 78°.$

$= 3\cos(90° - 22°).cosec\ 22° - \frac{1}{2}.\{\tan 43°.\tan(90° - 43°)\}$

$.\{\tan 12°.\tan(90° - 12°).\tan 60°$

$= 3\sin 22°.cosec\ 22° - \frac{1}{2}(\tan 43°.cot\ 43°).(\tan 12°.cot\ 12°).\tan 60°$

$= 3 \times 1 - \frac{1}{2} \times 1 \times 1 \times \sqrt{3} = 3 - \frac{\sqrt{3}}{2} = \frac{6-\sqrt{3}}{2}.$

Que 11. If sin 3 θ = cos (θ – 6°) where 3 θ and θ – 6° are both acute angles, find the value of θ.

Sol. According to question:

sin 3θ = cos (θ – 6°)

$\Rightarrow \cos(90° - 3\theta) = \cos(\theta - 6°)$ $[\because \cos(90° - \theta) = \sin\theta]$

$\Rightarrow \quad 90° - 3\theta = \theta - 6°$ $\quad\quad [comparing\ the\ angles]$

$\Rightarrow \quad 4\theta = 90° + 6 = 96° \quad \Rightarrow \quad \theta = \frac{96}{4} = 24°$

Hence, θ = 24°

Que 12. If sec $\boldsymbol{\theta} = \boldsymbol{x} + \frac{\mathbf{1}}{\mathbf{4x}}$, prove that sec $\boldsymbol{\theta}$ + tan $\boldsymbol{\theta}$ = 2x or $\frac{\mathbf{1}}{\mathbf{2x}}$.

Sol. Let sec θ + tan θ = λ …(i)

We know that, $\sec^2 \theta - \tan^2 \theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1 \quad \Rightarrow \lambda\,(sec\ \theta - \tan\theta = 1$

$\sec\theta - \tan\theta = \frac{1}{\lambda}$ …(ii)

Adding equations (i) and (i), we get

$2\ sec\ \theta = \gamma + \frac{1}{\lambda} \quad \Rightarrow \quad 2\left(x + \frac{1}{4x}\right) = \lambda + \frac{1}{\lambda}$

$\Rightarrow 2x + \frac{1}{2x} = \lambda + \frac{1}{\lambda}$

On comparing, we get λ = 2x or $\lambda = \frac{1}{2x}$

$\Rightarrow \ sec\ \theta + \tan\theta = 2x\ \ or\ \frac{1}{2x}$

Que 13. Find an acute angle$\boldsymbol{\theta}$, when $\frac{\mathbf{cos}\ \boldsymbol{\theta} - \mathbf{sin}\ \boldsymbol{\theta}}{\mathbf{cos}\ \boldsymbol{\theta} + \mathbf{sin}\ \boldsymbol{\theta}} = \frac{\mathbf{1} - \sqrt{\mathbf{3}}}{\mathbf{1} + \sqrt{\mathbf{5}}}$.

Sol. We have,

$$\frac{\cos\theta - sin\ \theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \quad \Rightarrow \quad \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

[Dividing numerator & denominator of the LHS by cosθ]

$$\Rightarrow \quad \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

On comparing we get

$\Rightarrow \quad \tan\theta = \sqrt{3} \quad \Rightarrow \tan\theta = \tan 60° \quad \Rightarrow \theta = 60°$

Que 14. The altitude AD of a ΔABC, in which $\angle A$ is an obtuse angle has length 10 cm. If BD = 10 cm and CD = 10$\sqrt{3}$ cm, determine $\angle A$.

Sol. ΔABD is a right triangle right angled at D, such that AD = 10 cm and BD = 10 cm.

Let $\angle BAD = \theta$

$\therefore \quad \tan\theta = \frac{BD}{AD} \qquad \Rightarrow \tan\theta = \frac{10}{10} = 1$

$\Rightarrow \tan\theta = \tan 45° \quad \Rightarrow \theta = \angle BAD = 45°$ …(i)

ΔACD is a right triangle right angled at D such that AD = 10 cm and DC = $10\sqrt{3}\ cm$.
Let $\angle CAD = \phi$

$\therefore \quad \tan\phi = \frac{CD}{AD} \qquad \Rightarrow \tan\phi = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\Rightarrow \tan\phi = \tan 60° \quad \Rightarrow \phi = \angle CAD = 60°$ …(ii)

From (i) & (ii), we have

$\angle BAC = \angle BAD + \angle CAD = 45° + 60° = 105°$

Que 15. If cosec $\boldsymbol{\theta} = \frac{13}{12}$, evaluate $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Sol. Given cosec $\theta = \frac{13}{12}$, then sin $\theta = \frac{12}{13}$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = \frac{169-144}{169} = \frac{25}{169}$$

$$\cos\theta = \frac{5}{13}$$

Now,
$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} = \frac{24-15}{48-45} = \frac{9}{3} = 3$$

Long Answer Type Questions

[4 MARKS]

Que 1. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

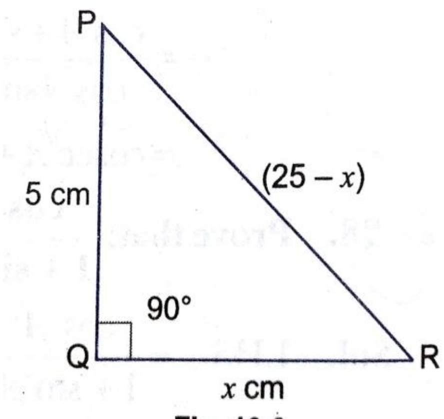

Fig. 10.6

Sol. We have a right-angled ΔPQR in which $\angle Q = 90°$.

Let QR = x cm

Therefore, PR = (25 – x) cm

By Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow \quad (25-x)^2 = 5^2 + x^2 \qquad \Rightarrow \quad (25-x)^2 - x^2 = 5^2$

$\Rightarrow \quad (25-x-x)(25-x+x) = 25$

$\Rightarrow \quad (25-2x)\,25 = 25 \qquad \Rightarrow \quad 25-2x = 1$

$\Rightarrow \quad 25-1 = 2x \qquad \Rightarrow \quad 24 = 2x$

$\therefore \quad x = 12\ cm.$

Hence, QR =12 cm

PR (25 – x) cm = 25 – 12 = 13 cm

PQ = 5 cm

$\therefore \quad \sin P = \frac{QR}{PR} = \frac{12}{13}$, Cos P = $\frac{PQ}{PR} = \frac{5}{13}$; $\quad \tan P = \frac{QR}{PQ} = \frac{12}{5}$

Que 2. In triangle ABC, right-angled at B, if tan $A = \frac{1}{\sqrt{3}}$, find the value of:

(i) sin A cos C + cos A sin C **(ii) cos A cos C – sin A sin C.**

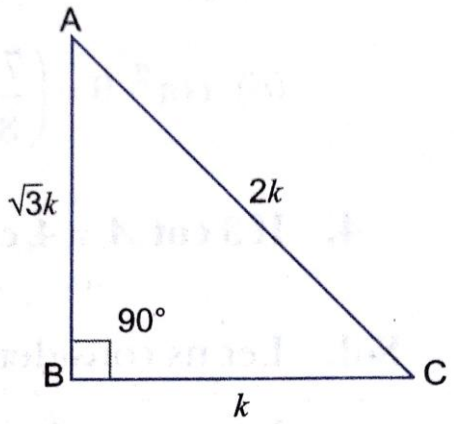

Fig. 10.7

Sol. We have a right-angled ΔABC in which $\angle B = 90°$.

And, $\tan A = \frac{1}{\sqrt{3}}$

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let BC = k and AB = $\sqrt{3}\,k$

$\therefore$ By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$

$\Rightarrow \quad AC^2 = 4k^2 \qquad \therefore \quad AC = 2k$

Now, $\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{k}{2k} = \frac{1}{2}$; $\qquad \cos A = \frac{Base}{Hypotenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$sin\ C = \frac{Perpendicular}{Hypotenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$, $\qquad \cos C = \frac{Base}{Hypotenuse} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A\,.\cos C + \cos A.\sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$

(ii) $\cos A.\cos C - \sin A.\sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$

Que 3. If $\cot\theta = \frac{7}{8}$, evaluate: (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, (ii) $\cot^2\theta$.

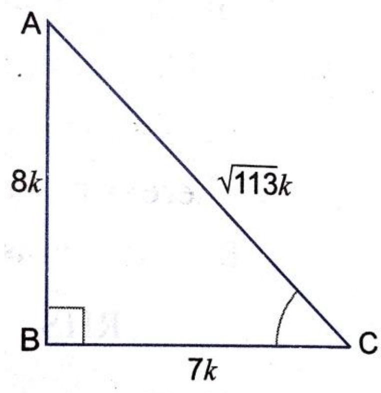

Fig. 10.8

Sol. Let us draw a right triangle ABC in which $\angle B = 90°$ and $\angle c = \theta$.

We have, cot $\theta = \frac{7}{8} = \frac{Base}{Perpendicular} = \frac{BC}{AB}$ (given)

Let BC = 7k and AB = 8k
Therefore, by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$AC^2 = 113k^2 \qquad \therefore \quad AC = \sqrt{113}k$$

$$\therefore \quad \sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{AB}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

And $\cos\theta = \frac{Base}{Hypotenuse} = \frac{BC}{AC} = \frac{7}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}.$$

Alternate method:

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = cot^2\,\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

(ii) $$cot^2\,\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

Que 4. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

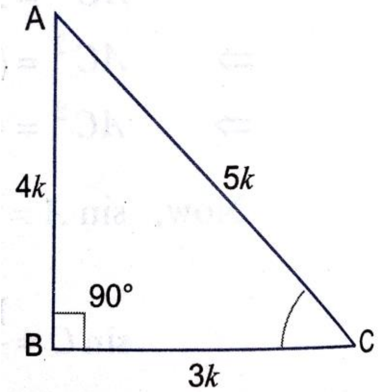

Fig. 10.9

Sol. Let us consider a right triangle ABC in which $\angle B = 90°$.

Now, $cot\,A = \frac{Base}{Perpendicular} = \frac{AB}{BC} = \frac{4}{3}$

Let AB = 4k and BC = 3k
∴ By Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad AC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$

$AC^2 = 25k^2$

$\therefore \quad AC = 5k$

Therefore, $\tan A = \frac{Perpendicular}{Base} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

And, $\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{Base}{Hypotenuse} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Now, LHS $= \frac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$RHS = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} - \frac{7}{25}$$

Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

Que 5. Write all the other trigonometric ratios of $\angle A$ in terms of sec A.

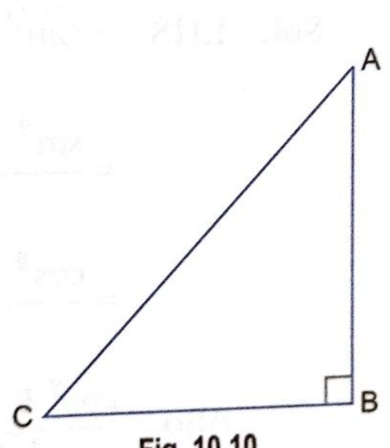

Fig. 10.10

Sol. Let us consider a right-angled ΔABC, in which $\angle B = 90°$.

For $\angle A$, we have

Base = AB, Perpendicular = BC and Hypotenuse = AC

$\therefore \quad sec\ A = \frac{Hypotenuse}{Base} = \frac{AC}{AB}$

$\Rightarrow \quad \frac{sec\ A}{1} = \frac{AC}{AB} \quad \Rightarrow \quad$ AC = AB sec A

Let AB = K and AC = k sec A

$\therefore$ By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 \quad \Rightarrow \quad k^2\, sec^2\, A = k^2 + BC^2$$

$$\therefore \quad BC^2 = k^2\, sec^2\, A - k^2 \quad \Rightarrow \quad BC = k\sqrt{sec^2 A - 1}$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{k\sqrt{sec^2 A - 1}}{k\, sec\, A} = \frac{\sqrt{sec^2 A - 1}}{sec\, A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k\, sec\, A} = \frac{1}{sec\, A}$$

$$\tan A = \frac{BC}{AB} = \frac{k\sqrt{sec^2 A-1}}{k} = \sqrt{sec^2 A - 1}$$

$$cot\, A = \frac{1}{\tan A} = \frac{1}{\sqrt{sec^2 A-1}}$$

$$cosec\, A = \frac{AC}{BC} = \frac{k\, sec\, A}{k\sqrt{sec^2 A-1}} = \frac{sec\, A}{\sqrt{sec^2 A-1}}.$$

Que 6. Prove that: $\left(\frac{1+\tan^2 A}{1+cot^2 A}\right) = \left(\frac{1-\tan A}{1-cot\, A}\right)^2 = \tan^2 A.$

Sol. LHS = $\left(\frac{1+\tan^2 A}{1+cot^2 A}\right) = \frac{sec^2 A}{cosec^2 A}$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$RHS = \left(\frac{1-\tan A}{1-cot\, A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A-1}\times \tan A\right)^2 = (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

Que 7. Prove that: $\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$.

Sol. LHS $= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2} - \frac{\sin^2 B}{\cos^2 B}$

$$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B} = \frac{(1-\cos^2 A)\cos^2 A\,(1-\cos^2 B)}{\cos^2 A \cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A \cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$$

Also $\frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A \cos^2 B}$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = RHS.$$

HOTS (Higher Order Thinking Skills)

Que 1. Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\ \mathrm{cosec}\theta=1+\tan\theta+\cot\theta$.

Sol. LHS = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin\theta\times\sin\theta}{\cos\theta\ (\sin\theta-\cos\theta)}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta\{-(\sin\theta-\cos\theta)\}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}=\frac{\sin^3\theta-\cos^3\theta}{\cos\theta(\sin\theta-\cos\theta)\sin\theta}$

$=\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\cos\theta\ \sin\theta\ (\sin\theta-\cos\theta)}=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta}.\frac{1}{\cos\theta}+1$ …..(i)

$=\sec\theta\ \mathrm{cosec}\theta\ +1$ ……(ii)

For second part

Now from (i), we have

LHS $=\frac{1}{\sin\theta\cos\theta}+1$ [Putting $1=\sin^2\theta+\cos^2\theta$]

$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1=\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\cos\theta\sin\theta}+1$

$=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1=\tan\theta+\cot\theta+1$

Que 2. If tan A = n tan B and sin A = m sin B, prove that $\cos^2 A=\frac{m^2-1}{n^2-1}$.

Sol. We have to find $\cos^2$ A in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now, tan A = n tan B

⇒ $\tan B=\frac{1}{n}\tan A$ ⇒ $\cot B=\frac{n}{\tan A}$

And $\sin A=m\sin B$

⇒ $\sin B=\frac{1}{m}\sin A$ ⇒ $\mathrm{cosec}\ B=\frac{m}{\sin A}$

Substituting the value of cot B and cosec B in $\mathrm{cosec}^2 B-\cot^2 B=1$, we get

$\frac{m^2}{\sin^2 A}-\frac{n^2}{\tan^2 A}=1$ ⇒ $\frac{m^2}{\sin^2 A}-\frac{n^2\cos^2 A}{\sin^2 A}=1$

$\Rightarrow \quad \frac{m^2-n^2\cos^2 A}{\sin^2 A} = 1 \qquad \Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \quad \frac{m^2-n^2\cos^2 A}{\sin^2 A} = 1 \qquad \Rightarrow \quad m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow \quad m^2 - n^2\cos^2 A = 1 - \cos^2 A \qquad \Rightarrow \quad m^2 - 1 = n^2\cos^2 A - \cos^2 A$

$\Rightarrow \quad m^2 - 1 = (n^2-1)\cos^2 A \qquad \Rightarrow \quad \frac{m^2-1}{n^2-1} = \cos^2 A.$

Que 3. Prove the following identity, where the angle involved is acute angle for which the expressions are defined.

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \boldsymbol{cosec\, A + cot\, A},$$

Using the identity cosec^2 A = 1 + cot^2 A.

Sol. LHS = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\sin A}{\frac{\cos A + \sin A - 1}{\sin A}} = \frac{cot\, A - 1 + cosec\, A}{cot\, A + 1 - cosec\, A}$

$= \frac{(cot\, A + cosec\, A) - (cosec^2 - cot^2 A)}{cot\, A - cosec\, A + 1} \qquad [\because\ cosec^2 A - cot^2 A = 1]$

$= \frac{(cot\, A + cosec\, A) - [(cosec\, A + cot\, A)(cosec\, A - cot\, A)]}{cot\, A - cosec\, A + 1}$

$= \frac{(cosec\, A + cot\, A)(1 - cosec\, A + cot\, A)}{(cot\, A - cosec\, A + 1)}$

= cosec A + cot A = RHS.

Que 4. If x sin^3 $\boldsymbol{\theta + y cos^3\theta = sin\theta cos\theta}$ and sin$\boldsymbol{\theta}$ = y cos$\boldsymbol{\theta}$, prove x^2 + y^2 = 1.

Sol. We have, $\quad x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \quad (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta\,(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta \qquad [\because\ x\sin\theta = y\cos\theta]$

$\Rightarrow \quad x\sin\theta\,(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta \qquad \Rightarrow \quad x = \cos\vartheta$

Now, we have x sin θ = y cos θ

$\Rightarrow \quad \cos\theta\sin\theta = y\, cos\theta \qquad [\because\ x = \cos\theta]$

$\Rightarrow$ y = sinθ

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$.

Que 5. If tan$\boldsymbol{\theta}$ + sin$\boldsymbol{\theta}$ = m and tan$\boldsymbol{\theta}$ – sin$\boldsymbol{\theta}$ = n, show that (m^2 – n^2) = 4 $\boldsymbol{\sqrt{mn}}$.

Sol. we have given tanθ + sinθ = m, and tanθ – sinθ = n, then

LHS = (m^2 – n^2) = (tanθ + sinθ)^2– (tanθ – sinθ)^2

= tan$^2\theta$ + sin$^2\theta$ + 2 tanθ sinθ – tan$^2\theta$ – sin$^2\theta$ + 2 tanθ sinθ

= 4 tanθ sinθ = $4\sqrt{\tan^2\theta \sin^2\theta}$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1 - \cos^2\theta)} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta - \sin\theta)(\tan\theta + \sin\theta)} = 4\sqrt{mn} = RHS$$

Value Based Questions

Que 1. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 10 m and angle made by the top with ground level is 60 °.
(i) Calculate the distance covered by the artist in climbing to the top of the pole.
(ii) Which mathematical concept is used in this problem?
(iii) What is its value?

Sol. (i) Clearly distance covered by the artist is equal to the length of the rope AC.
Let AB be the vertical pole of height 10 m.
It is given that $\angle ACB = 60°$
Thus, in right angled ΔABC.

$$\sin 60° = \frac{AB}{AC} \qquad \Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{10}{AC}$$

$$AC = \frac{10 \times 2}{\sqrt{3}} = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} m.$$

Hence, distance covered by artist is $\frac{20\sqrt{3}}{3} m.$
(ii) Trigonometric ratios of an acute angle of right angled triangle.
(iii) Single mindedness help us to gain success in life.

Que 2. A tree is broken by the wind. The top struck the ground at an angle of 45° and at a distance of 30 m from the root.
(i) Find whole height of the tree.
(ii) Which mathematical concept is used in this problem?
(iii) Which value is being emphasised here?

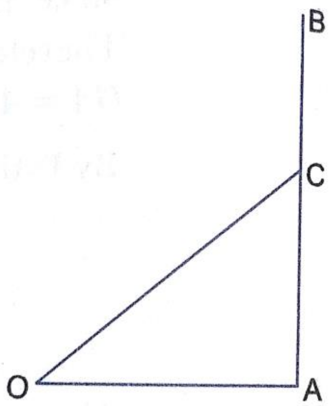

Fig. 6

Sol. (i) Let AB be the tree broken at C, such that the broken part CB takes the position CO and strikes the ground at O.
It is given that OA = 30 m and $\angle AOC = 45°$

Let AC = x and CB = y, then CO y

In ΔOAC, we have,

$$\tan 45° = \frac{AC}{OA} \quad \Rightarrow \quad 1 = \frac{x}{30} \quad \Rightarrow x = 30$$

Again in ΔOAC, we have

$$\cos 45° = \frac{OA}{OC} \quad \Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{30}{y} \quad \Rightarrow \; y = 30\sqrt{2}$$

Height of the tree = (x + y)

$$= 30 + 30\sqrt{2} = 30(1 + \sqrt{2})$$

$$= 30(1 + 1.414) = 30 \times 2.414 = \; 72.42\, m$$

(ii) Trigonometric ratios of an acute angle of right angled triangle.

(iii) Decreasing tree leads to deforestation which ultimately give birth to various problems.

Que 3. A person standing on the bank of a river observes that the angle of elevation of the top of a building of an organisation working for conservation of wild life, standing on the opposite bank is 60°. When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the building and the width of the river.
(a) Why do we need to conserve wild life?
(b) Suggest some steps that can be taken to conserve wild life.

Sol. Let AB be the building of height h metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that BC = x m. Let D be the new position of the man. It is given that CD = 40 m and the angles of elevation of the top of the building at C and D are 60° and 30° respectively, i.e., ∠ACB 60° and ∠ADB 30°.

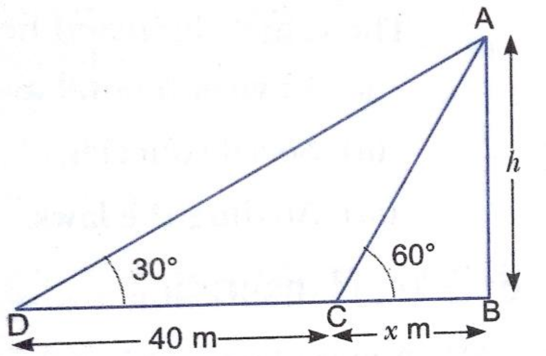

Fig. 7

In ΔACB, we have

$$\tan 60° = \frac{AB}{BC} \quad \Rightarrow \quad \tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots(i)$$

In ΔADB, we have

$$\tan 30° = \frac{AB}{BD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x + 40}$$

$$\Rightarrow \quad \sqrt{3}h = x + 40 \qquad \ldots(ii)$$

Substituting x = $\frac{h}{\sqrt{3}}$ in equation (ii), we get

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 40 \quad \Rightarrow \quad \sqrt{3}h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \frac{3h-h}{\sqrt{3}} = 40 \quad \Rightarrow \quad \frac{2h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad h = \frac{40\times\sqrt{3}}{2} \quad \Rightarrow \quad h = 20\sqrt{3} = 20 \times 1.732 = 34.64\ m$$

Substituting h in equation (i), we get x = $\frac{20\sqrt{3}}{\sqrt{3}} = 20\ metres.$

Hence, the height of the building is 34.64 m and width of the river is 20 m.
(a) Wild life is a part of our environment and conservation of each of its element is important for ecological balance.
(b) Ban on hunting, providing wild animals a healthy environment.

Que 4. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?

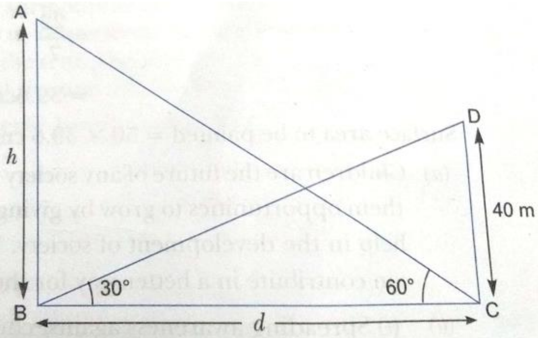

Fig. 8

Sol. Let the height of the chimney AB be h. Height of tower CD = 40 m.
The distance between the tower and chimney be d.
In ΔBCD

$$\tan 30° = \frac{CD}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{d} \quad \Rightarrow \quad d = 40\sqrt{3}$$

In ΔBCD

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{40\sqrt{3}} \quad \Rightarrow \quad h = 40\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad h = 120\ m.$$

The height of the chimney is 120 m which is more than the minimum requirement to meet the pollution norms.
The values discussed here are:
(i) Environmental awareness
(ii) Social concern
(iii) Abiding the laws.