Very Short Answer Type Questions

[1 Marks]

Que 1. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, find the sun's elevation.

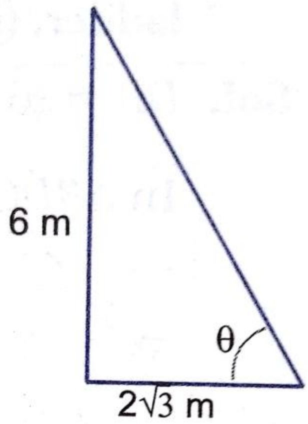


Fig. 11.5

Sol. $\tan\theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{300}$

$\Rightarrow \theta = 60°$

Que 2. An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

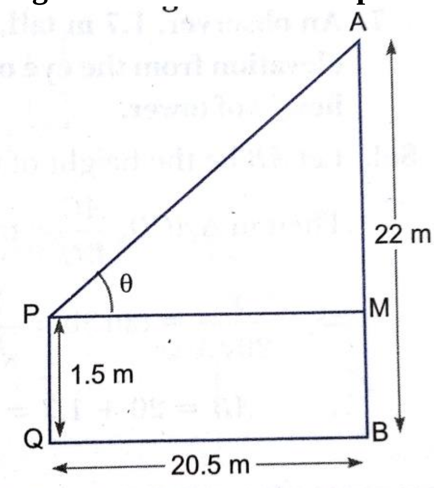


Fig. 11.6

Sol. PQ = MB = 1.5m

AM = AB – MB = 22 – 1.5 = 20.5m

Now in ΔAPM

$$\tan\theta = \frac{AM}{PM}$$

$$\Rightarrow \quad = \frac{20.5}{20.5} = 1$$

$\Rightarrow \qquad \theta = 45°$

Que 3. A ladder 15 m long leans against a wall making an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

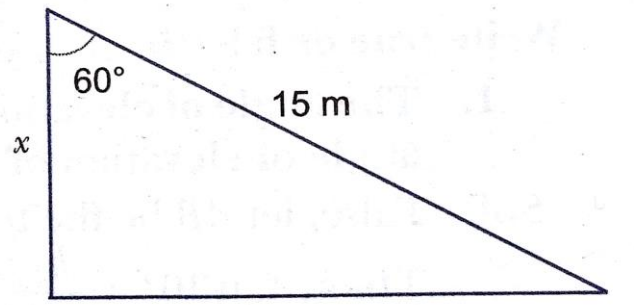


Fig. 11.7

Sol. $\cos 60° = \frac{x}{15}$

$\Rightarrow \frac{1}{2} = \frac{x}{15} \qquad \Rightarrow x = \frac{15}{2}\ m$

Que 4. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1, then the angle of elevation of the Sun is 30°. Is it true or false?

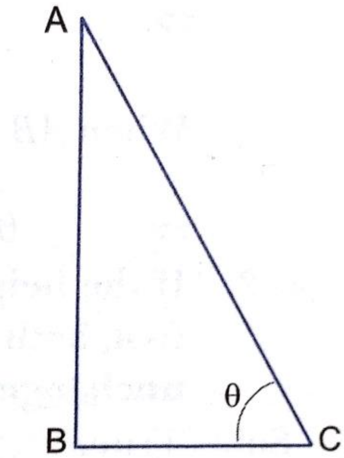


Fig. 11.8

Sol. False,

Given: $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$

Then, $\tan\theta = \frac{AB}{BC} = \sqrt{3}$

$\Rightarrow \qquad \theta = 60°$

Que 5. If the angle of elevation of a tower from a distance of 100 m from its foot is 60°, then what will be the height of the tower?

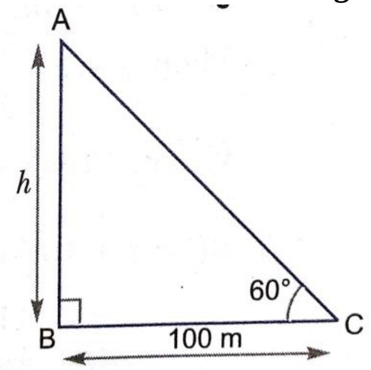


Fig. 11.9

Sol. Let h be the height of the tower.

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{100}$$

$$h = 100\sqrt{3}\ m.$$

Que 6. In Fig. 11.10, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m, find the length of the ladder. (Use $\sqrt{3} = 1.73$)

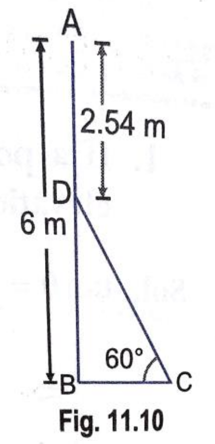


Fig. 11.10

Sol. DB = (6 – 2.54) m = 3.46 m

In ΔBDC, $\sin 60° = \frac{BD}{CD}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.46}{CD} \quad \Rightarrow \quad CD = \frac{3.46 \times 2}{1.73} = 4$$

$\therefore$ DC = 4 m

Que 7. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

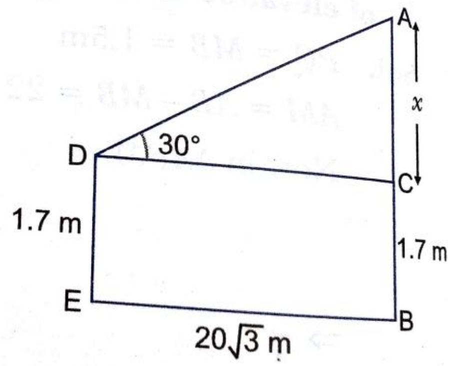


Fig. 11.11

Sol. Let AB be the height of tower and DE be the height of observer.

Then in $\Delta ACD, \frac{AC}{DC} = \tan 30°$.

$\Rightarrow \quad \frac{x}{20\sqrt{3}} = \tan 30° \; = \frac{1}{\sqrt{3}} \Rightarrow \; x \; = \; 20\, m\ (\boldsymbol{Fig. 11.11})$

$\therefore \qquad AB \; = \; 20 + 1.7 = 21.7\, m$

Short Answer Type Questions – I

[2 marks]

Write true or false in each of the following and justify your answer (Q. 1 to 4).

Que 1. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.

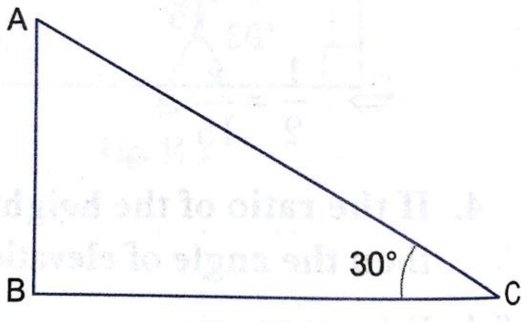


Fig. 11.12

Sol. False, let AB be the tower of height h **(Fig. 11.12)**

Then, tan 30° = $\frac{h}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC}$ …(i)

When AB = 2h, $\tan\theta = \frac{2h}{BC} = 2 \times \frac{1}{\sqrt{3}}$ [$Using\ (i)$]

$\Rightarrow \quad \theta \neq 60°$

Que 2. If the height of a tower and the distance of the point of observation from its foot, both are increased by 10%, then the angle of elevation of its top remains unchanged.

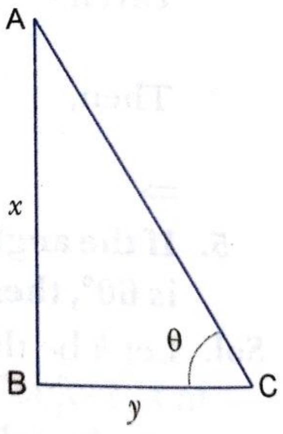


Fig. 11.13

Sol. True,

Let AB = x, BC = y **(Fig. 11.13)** (AB be the tower)

Then, $\tan\theta = \frac{x}{y}$

When, $AB = x + 10\%\, of\ x = x + \frac{1}{10}x = \frac{11}{10}x$

$BC = y + 10\%\, of\ y = y + \frac{1}{10}y = \frac{11}{10}\ y$

Then, $\frac{AB}{BC} = \frac{\frac{11}{10}x}{\frac{11}{10}y} = \frac{x}{y} = \tan\theta$

Que 3. If a man standing on a platform, 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

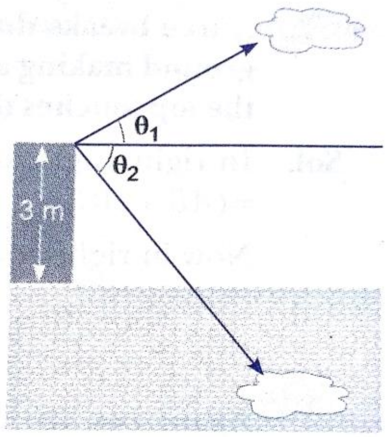


Fig. 11.14

Sol. False, $\theta_1 = \theta_2$ **(Fig. 11.14)**

Que 4. Find the angle of elevation of the sun when the shadow of a pole h m high is $\sqrt{3}$ h m long.

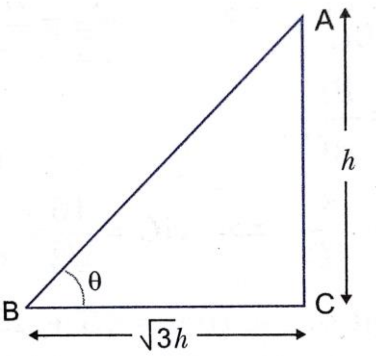


Fig. 11.15

Sol. In ΔABC

$$\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}\,h}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad \theta = 30°$$

Que 5. The height of a tower is 12 m. What is the length of its shadow when sun's altitude is 45°?

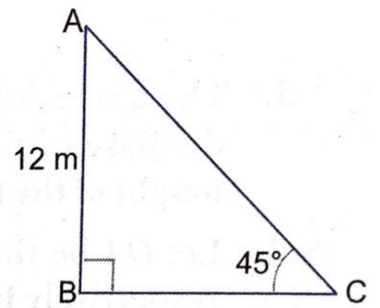


Fig. 11.16

Sol. Let AB be the tower **[Fig. 11.16].**

Then, $\angle C = 45°$, $AB = 12\,m$

$$\tan 45° = \frac{AB}{BC} = \frac{12}{BC} \quad \Rightarrow \quad 1 = \frac{12}{BC} \quad \Rightarrow \quad BC = m$$

$\therefore$ The length of the shadow is 12 m.

Que 6. A circus artist is climbing a 20 m long pore, which is tight stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 ° [Fig. 11.17].

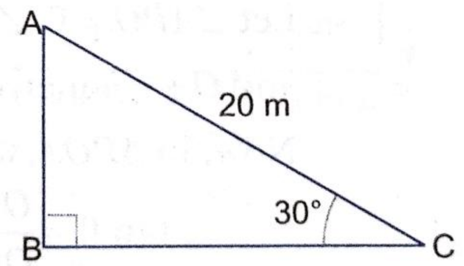


Fig. 11.17

Sol. Let AB be the vertical pole and AC be the long rope tied to point C.

In rightΔABC. We have

$$\sin 30° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{1}{2} = \frac{AB}{20} \quad \Rightarrow \quad \frac{20}{2} = AB \quad \Rightarrow$$

$$AB = 10\,m$$

Therefore, height of the pole is 10 m.

Short Answer Type Questions – II

[3 marks]

Que 1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

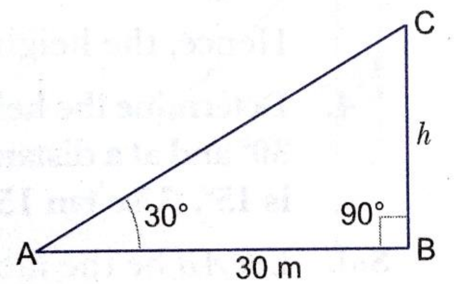


Fig. 11.18

Sol. Let, BC be the tower whose height is h metres and A be the point at a distance of 30 m from the foot of the tower. The angle of elevation of the top of the tower from point A is given to be 30°. Now, in right angleΔCBA, we have,

$$\tan 30° = \frac{BC}{AB} = \frac{h}{30} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \qquad h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\, m$$

Hence, the height of the tower is $10\sqrt{3}$ m.

Que 2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

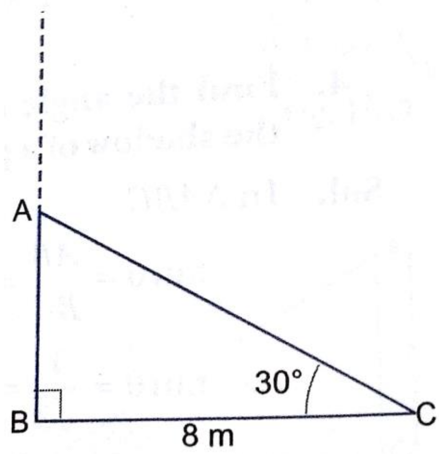


Fig. 11.19

Sol. In right angle $\Delta ABC, AC$ is the broken part of the tree (**Fig. 11.19**). So, the total height of tree = (AB + AC)

Now in right angle ΔABC.

$$\tan 30° = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{8} \quad \Rightarrow \quad AB = \frac{8}{\sqrt{3}}$$

Again, $\cos 30° = \frac{BC}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{AC} \quad \Rightarrow \quad AC = \frac{16}{\sqrt{3}}$$

Hence, the height of the tree = AB + AC

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\, m$$

Que 3. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

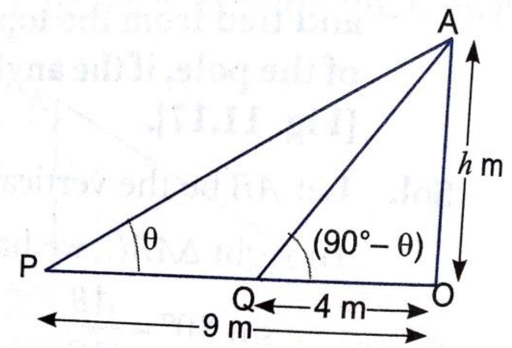


Fig. 11.20

Sol. Let OA be the tower of height h metre and P, Q be the two points at distance of 9 m and 4 m respectively from the base of the tower.

Now, we have OP = 9 m, OQ = 4 m.

Let $\angle APO = \theta, \angle AQO = (90° - \theta)$

and OA = h metre (**Fig. 11.20**)

Now, in ΔPOA, we have

$$\tan\theta = \frac{OA}{OP} = \frac{h}{9} \quad \Rightarrow \quad \tan\theta = \frac{h}{9} \quad \ldots(i)$$

Again, in ΔAQO, we have

$$\tan(90° - \theta) = \frac{OA}{OQ} = \frac{h}{4} \Rightarrow \quad cot\,\theta = \frac{h}{4} \quad \ldots(ii)$$

Multiplying (i) and (ii), we have

$$\tan\theta \times \cot\theta = \frac{h}{9} \times \frac{h}{4} \quad \Rightarrow \quad 1 = \frac{h^2}{36} \quad \Rightarrow \quad h^2 = 36$$

$\therefore \qquad h = \underline{+6}$

Height cannot be negative
Hence, the height of the tower is 6 metre.

Que 4. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15°. (Use tan 15° = 0.27)

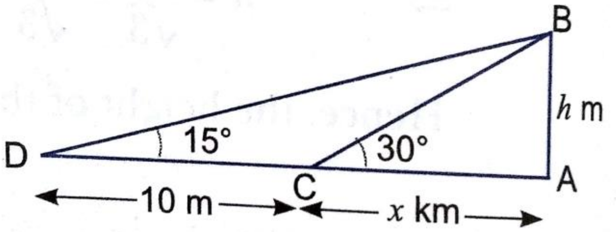


Fig. 11.21

Sol. Let AB be the mountain of height h kilometres. Let C be a point at a distance of x km, from the base of the mountain such that angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.
In ΔACB (**Fig. 11.21**), we have

$$\tan 30° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$\Rightarrow \qquad x = \sqrt{3}h$

In ΔADB, we have

$$\tan 15° = \frac{AB}{AD} \quad \Rightarrow \quad 0.27 = \frac{h}{x+10}$$

$\Rightarrow \qquad (0.27)\,(x+10) = h$
Substituting x = $\sqrt{3}h$ in equation (i), we get

$$0.27\,(\sqrt{3}h + 10) = h$$

$\Rightarrow \quad 0.27 \times \sqrt{3}h + 0.27 \times 10 = h$

$\Rightarrow \quad 2.7 = h - 0.27 \times \sqrt{3}h \quad \Rightarrow \quad 27 = h(1 - 0.27 \times \sqrt{3})$

$\Rightarrow \quad 27 = h\,(1 - 0.46) \quad \Rightarrow \quad h = \dfrac{2.7}{0.54} = 5$

Hence, the height of the mountain is 5 km.

Que 5. The shadow of a tower standing on a level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

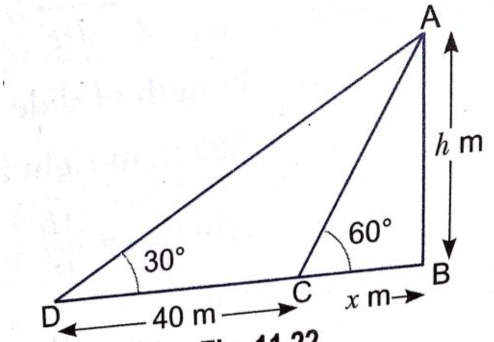


Fig. 11.22

Sol. In **Fig. 11.22**, AB is the tower and BC is the length of the shadow when the sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m. According to the question, DB is 40 m longer than BC.

So, BD = (40+x) m

Now, we have two right triangles ABC and ABD.

In $\Delta ABC,$ $\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow$ $x\sqrt{3} = h$ …(i)

In $\Delta ABD,$ $\tan 30° = \frac{AB}{BD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x+40}$ …(ii)

Using (i) in (i), we get $(x\sqrt{3})\ \sqrt{3} = x + 40$ $i.e.,$ $3x = x + 40$

i.e., x = 20

So, h = $20\sqrt{3}$ [From (i)]

Therefore, the height of the tower is $20\sqrt{3}\ m$.

Que 6. From a point P on the ground, the angles of elevation of the top of a 10m tall building is 30°. A flag is hosted at the top of the building and angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (you may take $\sqrt{3} = 1.732$).

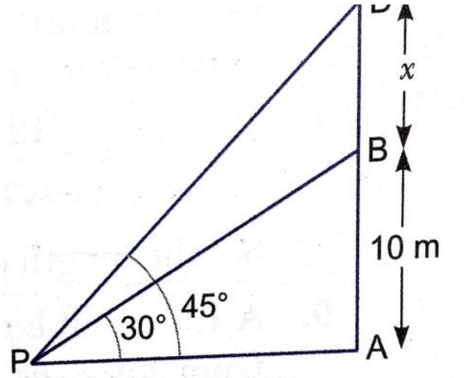


Fig. 11.23

Sol. In **Fig. 11.23,** AB denotes the height of the building, BD the flagstaff and P the given point. Note that there are two right triangles PAB and PAD. We are required to find the length of the flagstaff, i.e., BD and the distance of the building from the point P, i.e., PA. Since, we know the height of the building AB, we will first consider the right ΔPAB.

We have, $\tan 30° = \frac{AB}{AP} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow \qquad AP = 10\sqrt{3}$

i.e., the distance of the building from P is $10\sqrt{3}\,m = 10 \times 1.732 = 17.32\,m.$
Next, let us suppose DB = x m. Then, AD = (10+x) m.
Now, in right $\Delta PAD,$

$$\tan 45° = \frac{AD}{AP} = \frac{10+x}{10\sqrt{3}} \quad \Rightarrow \quad 1 = \frac{10+x}{10\sqrt{3}} \quad \Rightarrow \quad 10\sqrt{3} = 10 + x$$

i.e., $\qquad x = 10(\sqrt{3} - 1) = 7.32\,m.$

Que 7. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

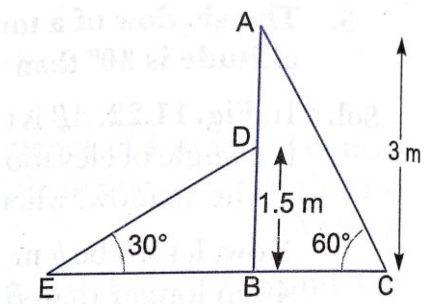


Fig. 11.24

Sol. Let AC be a steep slide for elder children and DE be a slide for younger children. Then AB = 3 m and DB = 1.5 m (**Fig. 11.24**).
Now, in right angleΔDBE, we have

$$\sin 30° = \frac{BD}{DE} = \frac{1.5}{DE}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{DE} \qquad \therefore \quad DE = 2 \times 15 = 3\,m$$

$\therefore$ Length of slide for younger children = 3 m
Again, in right ΔABC, we have

$$\sin 60° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow \quad AC = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}\,m$$

So, the length of slide for elder children is $2\sqrt{3}\,m$.

Que 8. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the sting with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

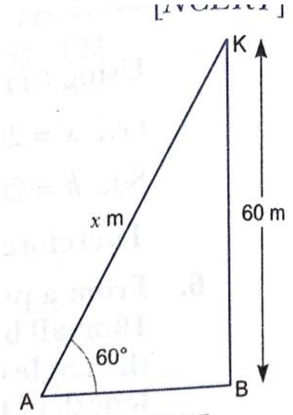


Fig. 11.25

Sol. Let AB be the horizontal ground and k be the position of the kite and its height from the ground is 60 m and length of string AK be x m. **(Fig. 11.25)**

$$\angle KAB = 60°$$

Now, in right angle ΔABK, we have

$$Sin\ 60° = \frac{BK}{AK} = \frac{60}{x} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{x} \quad \Rightarrow \quad \sqrt{3}\,x = 120$$

$$\therefore \quad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}\,m$$

So, the length of string is $40\sqrt{3}\,m$.

Long Answer Type Questions

[4 MARKS]

Que 1. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.

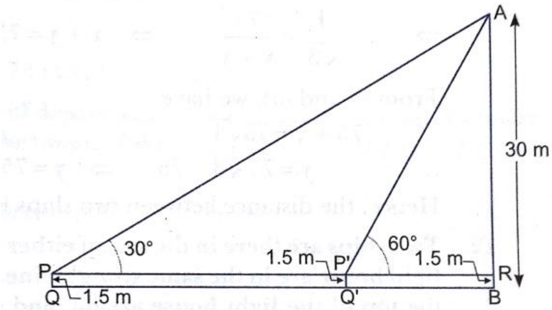


Fig. 11.26

Sol. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground (**Fig. 11.34**).

Here, $\angle BDA = 45° \, and \, \angle ADC = 60°$

Now, in ΔADC, we have

$$\tan 60° = \frac{AC}{AD} \quad \Rightarrow \quad \sqrt{3} = \frac{x+20}{AD}$$

$\Rightarrow \quad AD = \frac{x+20}{\sqrt{3}}$ …(i)

Again, in $\Delta ADB, we\ have\ \tan 45° = \frac{AB}{AD}$

$\Rightarrow \quad 1 = \frac{20}{AD} \quad \Rightarrow \quad AD = 20\ m$ …(ii)

Putting the value of AD in equation (i), we have

$$20 = \frac{x+20}{\sqrt{3}} \quad \Rightarrow \quad 20\sqrt{3} = x + 20$$

$\Rightarrow \quad x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) = 20(1.732 - 1) = 20 \times 0.732 = 14.64\ m$

Hence, the height of tower is 14.64 m.

Que 2. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the bottom of the pedestal is 45°. Find the height of the pedestal.

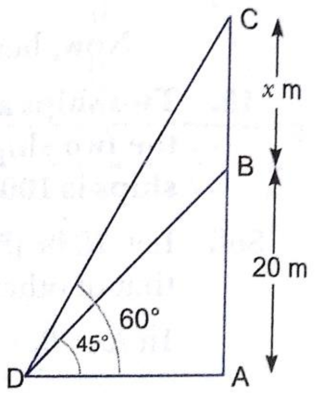


Fig. 11.34

Sol. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground (**Fig. 11.34**).
Here, $\angle BDA = 45° \; and \; \angle ADC = 60°$
Now, in ΔADC, we have

$$\tan 60° = \frac{AC}{AD} \quad \Rightarrow \quad \sqrt{3} = \frac{x+20}{AD}$$

$$\Rightarrow \quad AD = \frac{x+20}{\sqrt{3}} \qquad \ldots(i)$$

Again, in ΔADB, we have $\tan 45° = \frac{AB}{AD}$

$$\Rightarrow \quad 1 = \frac{20}{AD} \quad \Rightarrow \quad AD = 20\, m \qquad \ldots(ii)$$

Putting the value of AD in equation (i), we have

$$20 = \frac{x+20}{\sqrt{3}} \quad \Rightarrow \quad 20\sqrt{3} = x + 20$$

$$\Rightarrow \quad x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) = 20(1.732 - 1) = 20 \times 0.732 = 14.64\, m$$

Hence, the height of tower is 14.64 m.

Que 2. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the bottom of the pedestal is 45°. Find the height of the pedestal.

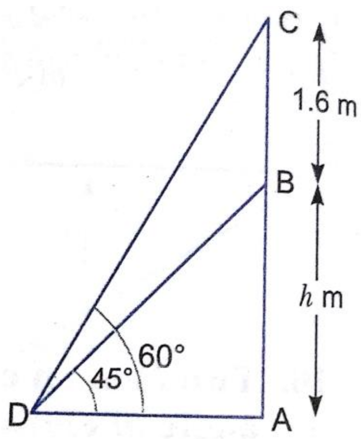


Fig. 11.35

Sol. Let AB be the pedestal of height h metres and BC be the statue of height 1.6 m **(Fig. 11.35).**
Let D be any point on the ground such that,

$$\angle BDA = 45° \; and \; \angle CDA = 60°$$

Now, in $\Delta BDA, we \; have$

$$\tan 45° = \frac{AB}{DA} = \frac{h}{DA} \qquad \Rightarrow \quad 1 = \frac{h}{DA}$$

$\therefore \quad DA = h \qquad \qquad \ldots(i)$

Again in $\Delta ADC, we \; have$

$$\tan \; 60° = \frac{AC}{AD} = \frac{AB+BC}{AD}$$

$\Rightarrow \qquad \sqrt{3} = \frac{h+16}{h} \qquad$ [From equation (i)]

$\Rightarrow \qquad \sqrt{3}h = h + 16 \qquad \Rightarrow \quad \left(\sqrt{3} - 1\right) h = 1.6$

$\therefore \qquad h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1.6\left(\sqrt{3}+1\right)}{3-1} = \frac{1.6\left(\sqrt{3}+1\right)}{2} = \; 0.8 \times \left(\sqrt{3} + 1\right) m$

Hence, height of the pedestal is 0.8 $(\sqrt{3} + 1) \; m.$

Que 3. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

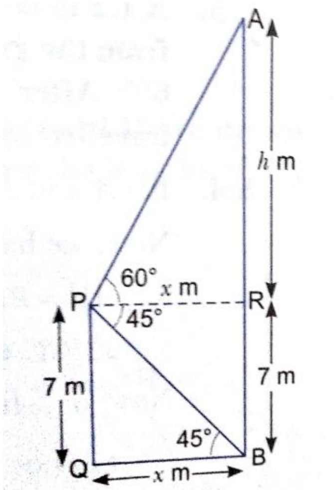


Fig. 11.36

Sol. Let PQ be the building of height 7 metres and AB be the cable tower. Now it is given that the angle of elevation of the top A of the tower observed from the top P of building is 60° and the angle of depression of the base B of the tower observed from P is 45° **(Fig. 11.36).**
So, $\angle APR = 60° \ and \ \angle QBP = 45°$
Let QB = x m, AR = h m then, PR = x m
Now, in ΔAPR, we have

$$\tan 60° = \frac{AR}{PR} \qquad \Rightarrow \qquad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \qquad \sqrt{3x} = h \qquad \Rightarrow \qquad h = \sqrt{3x} \qquad \ldots\text{(i)}$$

Again, in ΔPBQ, we have

$$\tan 45° = \frac{PQ}{QB} \qquad \Rightarrow \qquad 1 = \frac{7}{x} \qquad \Rightarrow \qquad x = 7 \ldots\text{(ii)}$$

Putting the value of x in equation (i), we have

$$h = \sqrt{3} \times 7 = 7\sqrt{3}$$

i.e., AR = 7 $\sqrt{3}$ meters
So, the height of tower = AB = AR + RB = 7 $\sqrt{3}$ $+ 7 = 7\ (\sqrt{3} + 3)\ m.$

Que 4. At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$. On walking 240 m nearer to the tower, the tangent of the angle of elevation becomes $\frac{3}{4}$. Find the height of the tower.

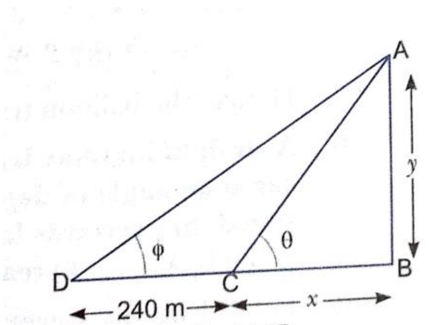


Fig. 11.37

Sol. In the **Fig. 11.37,** Let AB be the tower, C and D be the positions of observation from where given that

$$\tan\phi = \frac{5}{12} \qquad \ldots(i)$$

And $$\tan\phi = \frac{3}{4} \qquad \ldots(ii)$$

Let BC = x m. AB = y m
Now in right-angled triangle ABC

$$\tan\phi = \frac{y}{x} \qquad \ldots(iii)$$

From (ii) and (iii), we get $\frac{3}{4} = \frac{y}{x}$

$$\Rightarrow \quad x = \frac{4}{3}y \qquad \ldots(iv)$$

Also in right – angled triangle ABD, we get

$$\tan\phi = \frac{y}{x+240} \qquad \ldots(v)$$

From (i) and (v), we get

$$\frac{5}{12} = \frac{y}{x+240} \quad \Rightarrow \quad 12y = 5x + 1200 \qquad \ldots(vi)$$

$$\Rightarrow \quad 12y = 5 \times \frac{4}{3}y + 1200 \quad (Using\ (iv))$$

$$\Rightarrow \quad 12y - \frac{20}{3}y = 1200 \quad \Rightarrow \quad \frac{36y - 20y}{3} = 1200$$

$$\Rightarrow \quad 16y = 3600 \quad \Rightarrow \quad y = \frac{3600}{16} = 225$$

Hence, the height of the tower is 225 metres.

Que 5. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (Fig. 11.38). Find the distance travelled by the balloon during the interval.

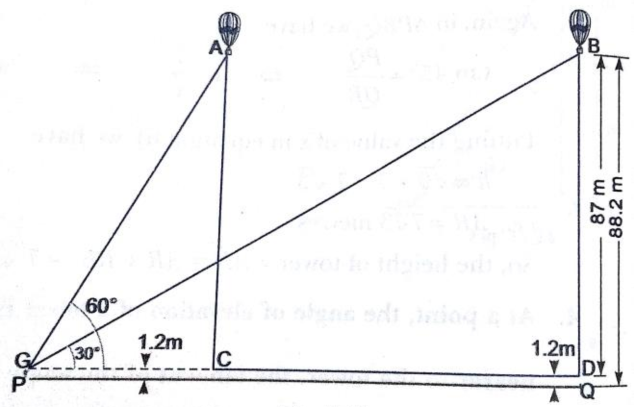


Fig. 11.38

Sol. Let A and be two positions of the balloon and G be the point of observation. (Eyes of the girl)

Now, we have

$$AC = BD = BQ - DQ = 88.2\,m - 1.2\,m = 87\,m$$

$$\angle AGC = 60°,\ \angle BGD = 30°$$

Now, in $\Delta AGC, we\ have$

$$\tan 60° = \frac{AC}{GC} \qquad \Rightarrow \qquad \sqrt{3} = \frac{87}{GC}$$

$$\Rightarrow \quad GC = \frac{87}{\sqrt{3}} = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times \sqrt{3}}{3}$$

$$\Rightarrow \quad GC = 29 \times \sqrt{3} \qquad \ldots\text{(i)}$$

Again, in $\Delta BGD, we\ have$

$$\tan 30° = \frac{BD}{GD} \qquad \Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{87}{GD}$$

$$GD = 87 \times \sqrt{3} \qquad \ldots\text{(ii)}$$

From (i) and (ii), we have

$$CD = 87 \times \sqrt{3} - 29 \times \sqrt{3}$$

$$= \sqrt{3}(87 - 29) = 58\sqrt{3}$$

Hence, the balloon travels $58\sqrt{3}$ metres.

Que 6. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

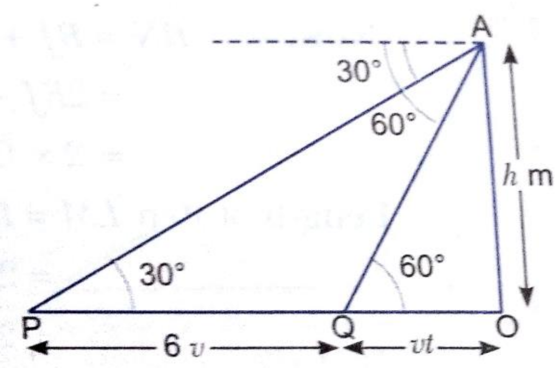


Fig. 11.39

Sol. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.
After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6v (∵ Distance = speed × time)

And let the car take t seconds to reach the tower OA from Q **(Fig. 11.39).** Then OQ = vt metres.
Now, in ΔAQO we have

$$\tan\ 60° = \frac{OA}{QO}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{vt} \quad \Rightarrow \quad h = \sqrt{3}\,vt \quad \ldots\text{(i)}$$

Now, in $\Delta APO, we\ have$

$$\tan 30° = \frac{OA}{PO}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \quad \Rightarrow \quad \sqrt{3}h = 6v + vt \quad \ldots\text{(ii)}$$

Now, substituting the value of h from (i) into (ii), we have

$$\sqrt{3} \times \sqrt{3}\,vt = 6v + vt$$

$$\Rightarrow\ 3vt = 6v + vt \quad \Rightarrow\ 2vt = 6v \quad \Rightarrow\ t = \frac{6v}{2v} = 3$$

Hence, the car will reach the tower from Q in 3 seconds.

Que 7. In Fig. 11.40, ABDC is a trapezium in which AB || CD. Line segments RN and LM are drawn parallel to AB such that AJ = JK = KP. If AB = 0.5 m and AP = BQ = 1.8 m, find the lengths of AC, BD, RN and LM.

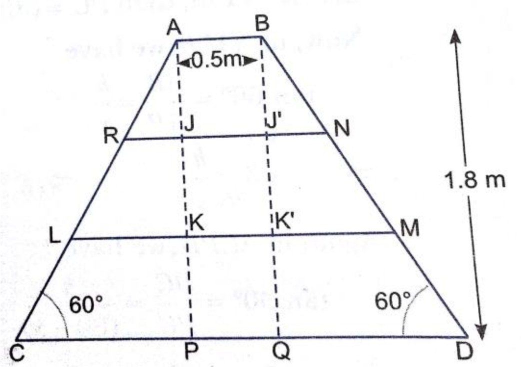


Fig. 11.40

Sol. we have,

AP = 1.8m

∴ AJ = JK = KP = 0.6 m

⇒ AK = 2AJ = 1.2 m

In ΔARJ and $\Delta BNJ'$, we have

AJ = BJ', ∠ARJ = ∠BNJ' = 60°

and ∠AJR = ∠BJ' N = 90°

∴ $\Delta ARJ \cong \Delta BNJ'$

⇒ RJ = NJ' (By AAS congruence criterion)

Similarly, $\Delta ALK \cong \Delta BMK'$ ⇒ LK = MK'

In ΔARJ, $\tan 60° = \frac{AJ}{RJ}$

$$\Rightarrow \quad \sqrt{3} = \frac{0.6}{RJ} \Rightarrow RJ = \frac{0.6}{\sqrt{3}} = \frac{0.6\sqrt{3}}{3} = 0.2 \times 1.732 = 0.3464\, m$$

In ΔALK,

$$\tan 60° = \frac{AK}{LK} \Rightarrow \sqrt{3} = \frac{1.2}{LK}$$

$$\Rightarrow \quad LK = \frac{1.2}{\sqrt{3}} = \frac{1.2\times\sqrt{3}}{3} = 0.4 \times 1.732\, m = 0.6928\, m$$

In ΔACP, $\sin 60° = \frac{AP}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{1.8}{AC} \Rightarrow AC = \frac{3.6}{\sqrt{3}} = \frac{3.6 \times \sqrt{3}}{3} = 1.2 \times 1.732 = 2.0784\, m$$

Since $\Delta ACP \cong \Delta BDQ$

So, BD = AC = 2.0784 m

Now, RN = RJ + JJ' +J, N

= 2RJ + AB $[\because RJ = J'N \text{ and } JJ' = AB]$

= 2 ×0.3464 + 0.5 = 1.1928 m

Length of step LM = LK + KK' + K' M

= 2LK + AB $[\because LK = K'M \text{ and } KK' = AB]$

= 2× 0.3464 + 0.5 = 1.1928 m

Length of step LM = LK + KK' + K' M

= 2LK + AB $[\because\ LK = K'M \text{ and } KK' = AB]$

= 2 × 0.6928 + 0.5 = 1.8856 m

Thus, length of each leg = 2.0784 m = 2.1 m
Length of step RN = 1.1928 m = 1.2 m
and, length of step LM = 1.8856 m = 1.9 m

Que 8. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60 ° and 30 °, respectively. Find the height of the poles the distances of the point from the poles.

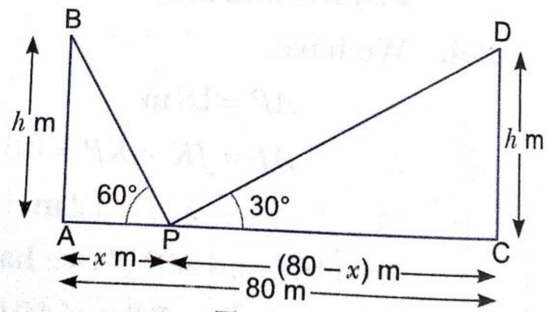


Fig. 11.41

Sol. Let AB and CD be two poles of equal height h metre and let P be any point between the poles, such that

$$\angle APB = 60° \text{ and } \angle DPC = 30°.$$

The distance between two poles is 80 m. (Given)
Let AP = x m, then PC = (80 – x) m.
Now, in ΔAPB, we have

$$\tan 60° = \frac{AB}{AP} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \qquad \ldots(i)$$

Again in ΔCPD, we have

$$\tan 30° = \frac{DC}{PC} = \frac{h}{(80-x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x} \quad \Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}} \quad \Rightarrow \quad mx = 80 - x \quad \Rightarrow \quad 4x = 80 \quad \Rightarrow \quad x = \frac{80}{4} = 20\,m$$

Now, putting the value of x in equation (i), we have

h = $\sqrt{3} \times 20 = 20\sqrt{3}$

Hence, the height of the pole is $20\sqrt{3}m$ and the distance of the point first pole is 20 m and that of the second pole is 60 m.

HOTS (Higher Order Thinking Skills)

Que 1. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

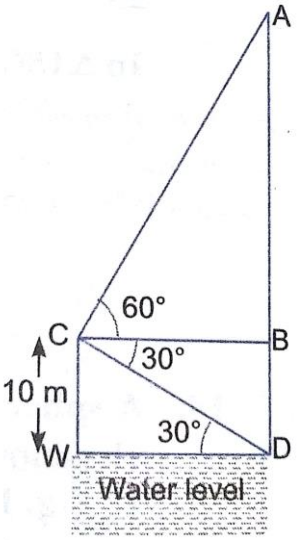


Fig. 11.51

Sol. In **Fig. 11.15,** let C represents the position of the man on the deck of the ship, A represents the top of hill and D its base.

Now in right-angled triangle CWD.

$\tan 30° = \frac{10}{WD} \quad \Rightarrow \quad WD = \frac{10}{\tan 30°}$

$\Rightarrow \quad WD = \frac{10}{\frac{1}{\sqrt{3}}} = 10\sqrt{3} = 17.3\ m$

Also, in right-angled triangle ABC, we have,

$\tan 60° = \frac{AB}{BC} or \frac{AB}{WD}$ [From fig. BC = WD]

$\Rightarrow \quad \sqrt{3} = \frac{AB}{10\sqrt{3}} \quad \Rightarrow \quad AB = 10\sqrt{3} \times \sqrt{3} = 30\ m$

Now, AD = AB + BD = 30 m + 10 m = 40 m.

Therefore, the distance of the hill from the ship = 17.3 m and height of the hill = 40 m

Que 2. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

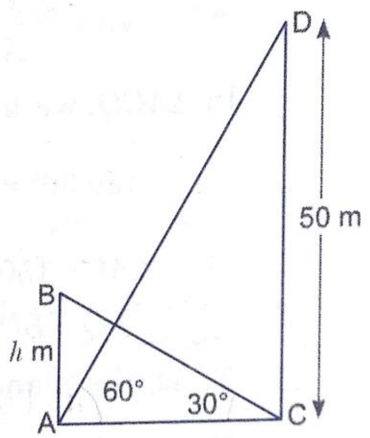


Fig. 11.52

Sol. Let AB be the building of height h m and CD be the tower of height 50 m. We have,

$\angle ACB = 30° \; and \; \angle DAC = 60°$

Now, in $\Delta ACD,$ we have

$$\tan 60° = \frac{DC}{AC} \qquad \Rightarrow \quad \sqrt{3} = \frac{50}{AC}$$

$$\Rightarrow \quad AC = \frac{50}{\sqrt{3}} = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$$

$$\Rightarrow \quad AC = \frac{50\sqrt{3}}{3} \qquad \ldots.(i)$$

Now in $\Delta ABC,$ we have

$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AC} \quad \Rightarrow \quad AC = \sqrt{3}h$$

$$\therefore \quad h = \frac{AC}{\sqrt{3}} = \frac{\frac{50\sqrt{3}}{3}}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}\ m \qquad \text{(From equation (i))}$$

Hence, the height of the building is $16\frac{2}{3}\ m.$

Que 3. From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is h (1 + tan θ cotϕ).

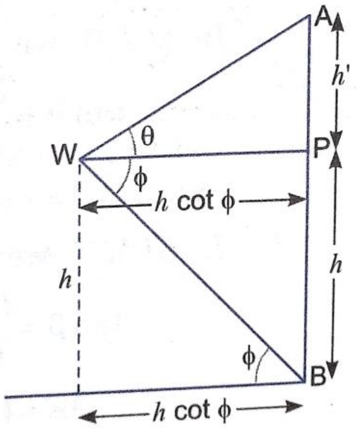


Fig. 11.53

Sol. Let W be the window and AB be the house on the opposite side.

Then, WP is the width of the street **(Fig. 11.53).**

Let AP = h' m

In ΔBPW, $\tan\phi = \frac{PB}{WP}$

$\Rightarrow \quad \frac{h}{WP} = \tan\phi \quad \Rightarrow \quad WP = h\cot\phi$ …(i)

Now, in ΔAWP, $\tan\theta = \frac{AP}{WP} = \frac{h\prime}{WP}$

$\Rightarrow \quad h' = WP\tan\theta \quad \Rightarrow \quad h' = h\cot\phi\tan\theta$

∴ Height of house = h' + h

= h cot ϕ tan θ + h (1 + tan θ cotϕ)

Que 4. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $1500\sqrt{3}$ m find the speed of the jet plane.

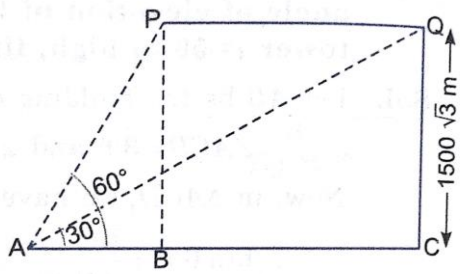


Fig. 11.54

Sol. Let P and Q be the two position of the plane and jet A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30°, respectively.

Then, $\angle PAB = 60°, \angle QAB = 30°$

It is also given that PB = $1500\sqrt{3}$ metres

In ΔABP, we have

$$\tan 60° = \frac{BP}{AB} m$$

$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{AB} \quad \Rightarrow \quad AB = 1500$

In ΔACQ, we have

$$\tan 30° = \frac{CQ}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$$

$\Rightarrow \quad AC = 1500 \times 3 = 4500\ m$

∴ PQ = BC = AC – AB = 4500 – 1500 = 3000 m

Thus, the plane travels 3000 m in 15 seconds.

Hence, the speed of plane = $\frac{3000}{15} = 200 = 200\ m/s$

$$= 200 \times \frac{3600}{1000} km/h = 200 \times \frac{18}{5} km/h = 720\ km/h.$$

Que 5. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h\ (\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}$.

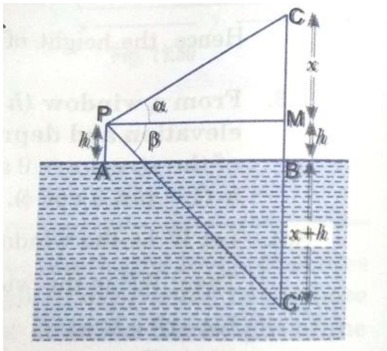


Fig. 11.55

Sol. Let AB be the surface of the lake and let P be a point of observation **(Fig. 11.55)** such that AP = h metres. Let C be the position of the cloud and C" be its reflection in the lake.

Then, CB = C" B. Let PM be perpendicular from P on CB. Then $\angle CPM = \alpha$ and $\angle MPC" = \beta$. Let CM = x.

Then, CB = CM + MB = CM + PA = x + h.

In ΔCPM, we have

$$\tan\alpha = \frac{C"M}{PM} \quad \Rightarrow \quad \tan\alpha = \frac{x}{AB} \quad [\because\ PM = AB]$$

$\Rightarrow$ AB = x cot α ….(i)

In $\Delta PMC"$, we have

$\tan\beta = \frac{C'M}{PM} \qquad \Rightarrow \qquad \tan\beta = \frac{x+2h}{AB} \quad [\because\ C'M = C'B + BM = x + h + h]$

$\Rightarrow \quad AB = (x + 2h)\cot\beta$ ….(ii)

From (i) and (ii), we have

$$x\cot\alpha = (x+2h)\cot\beta \quad \Rightarrow \quad x(\cot\alpha - \cot\beta) = 2h\cot\beta$$

$$\Rightarrow \quad x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta} \quad \Rightarrow \quad x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, the height CB of the cloud is given by

$$CB = x + h \quad \Rightarrow \quad CB = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$$

$$\Rightarrow \quad CB = \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha} = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$

Value Based Questions

Que 1. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 10 m and angle made by the top with ground level is 60 °.
(i) Calculate the distance covered by the artist in climbing to the top of the pole.
(ii) Which mathematical concept is used in this problem?
(iii) What is its value?

Sol. (i) Clearly distance covered by the artist is equal to the length of the rope AC.
Let AB be the vertical pole of height 10 m.
It is given that $\angle ACB = 60°$
Thus, in right angled ΔABC.

$$\sin 60° = \frac{AB}{AC} \qquad \Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{10}{AC}$$

$$AC = \frac{10\times 2}{\sqrt{3}} = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}m.$$

Hence, distance covered by artist is $\frac{20\sqrt{3}}{3}m.$
(ii) Trigonometric ratios of an acute angle of right angled triangle.
(iii) Single mindedness help us to gain success in life.

Que 2. A tree is broken by the wind. The top struck the ground at an angle of 45° and at a distance of 30 m from the root.
(i) Find whole height of the tree.
(ii) Which mathematical concept is used in this problem?
(iii) Which value is being emphasised here?

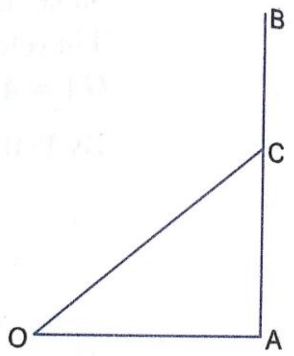


Fig. 6

Sol. (i) Let AB be the tree broken at C, such that the broken part CB takes the position CO and strikes the ground at O.
It is given that OA = 30 m and $\angle AOC = 45°$
Let AC = x and CB = y, then CO y
In ΔOAC, we have,

$$\tan 45° = \frac{AC}{OA} \quad \Rightarrow \quad 1 = \frac{x}{30} \quad \Rightarrow x = 30$$

Again in ΔOAC, we have

$$\cos 45° = \frac{OA}{OC} \quad \Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{30}{y} \quad \Rightarrow \; y = 30\sqrt{2}$$

Height of the tree = (x + y)

$$= 30 + 30\sqrt{2} = 30\left(1+\sqrt{2}\right)$$

$$= 30(1 + 1.414) = 30 \times 2.414 = \; 72.42\, m$$

(ii) Trigonometric ratios of an acute angle of right angled triangle.

(iii) Decreasing tree leads to deforestation which ultimately give birth to various problems.

Que 3. A person standing on the bank of a river observes that the angle of elevation of the top of a building of an organisation working for conservation of wild life, standing on the opposite bank is 60°. When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the building and the width of the river.
(a) Why do we need to conserve wild life?
(b) Suggest some steps that can be taken to conserve wild life.

Sol. Let AB be the building of height h metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that BC = x m. Let D be the new position of the man. It is given that CD = 40 m and the angles of elevation of the top of the building at C and D are 60° and 30° respectively, i.e., ∠ACB 60° and ∠ADB 30°.

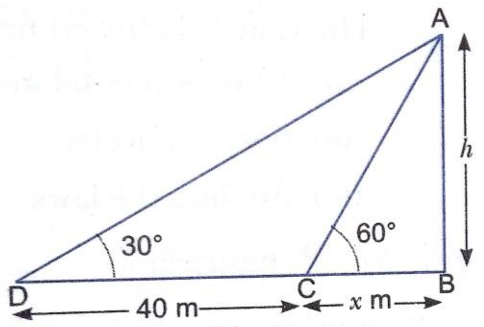


Fig. 7

In ΔACB, we have

$$\tan 60° = \frac{AB}{BC} \quad \Rightarrow \quad \tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots\text{(i)}$$

In ΔADB, we have

$$\tan 30° = \frac{AB}{BD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$\Rightarrow \quad \sqrt{3}h = x + 40 \qquad \ldots\text{(ii)}$$

Substituting x = $\frac{h}{\sqrt{3}}$ in equation (ii), we get

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 40 \quad \Rightarrow \quad \sqrt{3}h - \frac{h}{\sqrt{3}} = 40$$

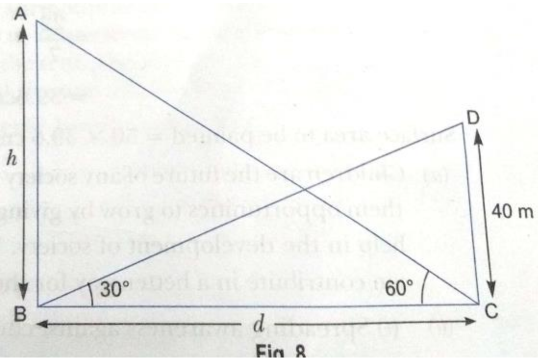
$$\Rightarrow \quad \frac{3h-h}{\sqrt{3}} = 40 \quad \Rightarrow \quad \frac{2h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad h = \frac{40\times\sqrt{3}}{2} \quad \Rightarrow \quad h = 20\sqrt{3} = 20 \times 1.732 = 34.64\, m$$

Substituting h in equation (i), we get x = $\frac{20\sqrt{3}}{\sqrt{3}} = 20\ metres.$

Hence, the height of the building is 34.64 m and width of the river is 20 m.
(a) Wild life is a part of our environment and conservation of each of its element is important for ecological balance.
(b) Ban on hunting, providing wild animals a healthy environment.

Que 4. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?



Sol. Let the height of the chimney AB be h. Height of tower CD = 40 m.
The distance between the tower and chimney be d.
In ΔBCD

$$\tan 30° = \frac{CD}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{d} \quad \Rightarrow \quad d = 40\sqrt{3}$$

In ΔBCD

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{40\sqrt{3}} \quad \Rightarrow \quad h = 40\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad h = 120\, m.$$

The height of the chimney is 120 m which is more than the minimum requirement to meet the pollution norms.

The values discussed here are:
(i) Environmental awareness
(ii) Social concern
(iii) Abiding the laws.