Very Short Answer Type Questions

[1 Marks]

Que 1. Find the area of a square inscribed in a circle of diameter p cm.

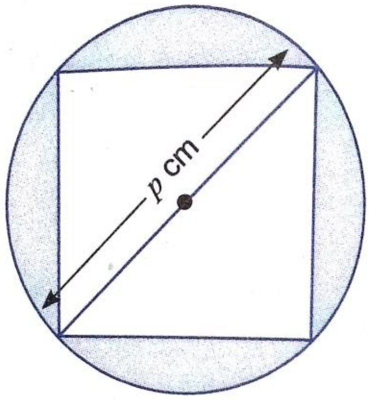

Fig. 12.3

Sol. Diagonal of the square = p cm

$\therefore \quad p^2 = side^2 + side^2$

$\Rightarrow \quad p^2 = 2\, side^2$

$or \quad side^2 = \frac{p^2}{2} cm^2$ = Area of the square

Que 2. Find the area of the circle inscribed in a square of side a cm.

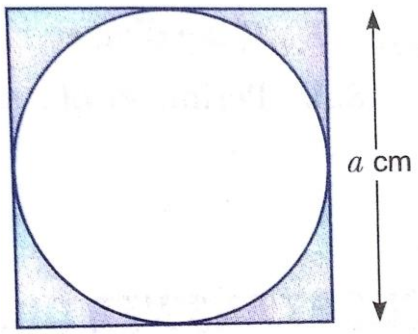

Fig. 12.4

Sol. Diameter of the circle = a

$\Rightarrow$ Radius = $\frac{a}{2}$ $\quad \Rightarrow \quad Area = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4} cm^2$

Que 3. Find the area of a sector of a circle whose radius is r and length of the arc is l.

Sol. Area of a sector of a circle with radius r

$$= \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times 2\pi r \frac{r}{2} = \frac{1}{2} lr \text{sq. units} \qquad \left(\because\ l = \frac{2\pi r\theta}{360}\right)$$

Que 4. Find the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal.

Sol. Given, 2r = a $\Rightarrow \frac{r}{a} = \frac{1}{2}$

$$\frac{Area\ of\ circle}{Area\ of\ equilateral\ triangle} = \frac{\pi r^2}{\frac{\sqrt{3}}{4}a^2} = \frac{4\pi}{\sqrt{3}}\left(\frac{r}{a}\right)^2 = \frac{4\pi}{\sqrt{3}} \times \frac{1}{4} = \frac{\pi}{\sqrt{3}}$$

Que 5. A square inscribed in a circle of diameter d and another square is circumscribing the circle. Show that the area of the outer square is four times the area of the inner square.

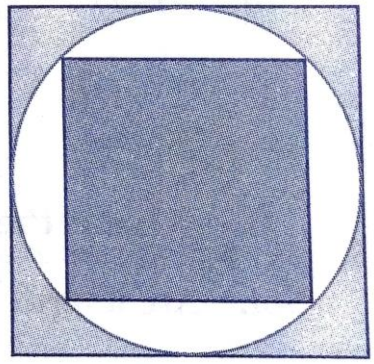

Fig. 12.5

Sol. Side of outer square = d [Fig. 12.5]

∴ Its area = d^2

Diagonal of inner square = d

∴ Side = $\frac{d}{\sqrt{2}}$ $\Rightarrow$ $Area = \frac{d^2}{2}$

∴ Area of outer square = 2 × Area of inner square.

Que 6. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

Sol. Given, $2\pi r = \pi r^2$

$\Rightarrow 2r = r^2$

$\Rightarrow r(r-2) = 0$ or $r = 2$

i.e., d = units

Que 7. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.

Sol. Number of revolutions $= \frac{11 \times 1000}{2 \times \frac{22}{7} \times 0.25} = 7000.$

Que 8. If the perimeter of a semi-circle protractor is 36 cm, find its diameter.

Sol. Perimeter of a semicircular protractor = Perimeter of a semicircle

= (2r + πr) cm.

2r + πr = 36

$r = \left(2 + \frac{22}{7}\right) = 36 \quad \Rightarrow \quad r = 7\ cm$

Diameter = 2r = 2 × 7 = 14 cm.

Que 9. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

Sol. Perimeter of a semicircle = $\pi r + 2r$

$$= \frac{22}{7} \times 7 + 2 \times 7 \ = \ 22 + 14 \ = \ 36\ cm$$

Short Answer Type Questions – I

[2 marks]

Que 1. If a square is inscribed in a circle, what is the ratio of the areas of the circle and the square?

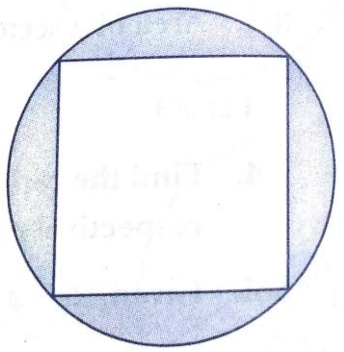

Fig. 12.6

Sol. Let radius of the circle be r units.

Then, diagonal of the square = 2r

$\Rightarrow$ Side of the square $= \frac{2r}{\sqrt{2}} = \sqrt{2}r$

$\therefore \quad \frac{Area\ of\ the\ circle}{Area\ of\ the\ square} = \frac{\pi r^2}{(\sqrt{2}r)^2} = \frac{\pi r^2}{2r^2} = \pi : 2$

Que 2. What is the area of the largest triangle that is inscribed in a semi-circle of radius r unit?

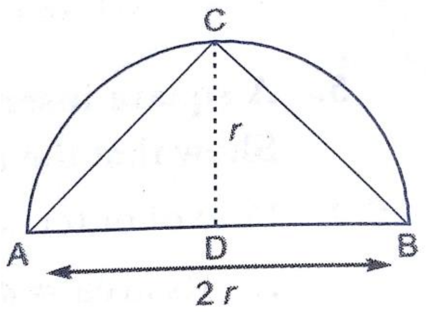

Fig. 12.7

Sol. Area of largest $\Delta ABC = \frac{1}{2} \times AB \times CD$

$$= \frac{1}{2} \times 2r \times r = r^2\ sq.units$$

Que 3. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

Sol. Arc length of a circle of radius r $= \frac{\theta}{360^\circ} \times 2\pi r$

$$\Rightarrow \; 5\pi = \frac{\theta}{360^\circ} \times 2\pi \times 10 \quad \text{or} \frac{\theta}{360^\circ} = \frac{5\pi}{20\pi} = \frac{1}{4} \quad \Rightarrow \quad \theta = \frac{360^\circ}{4} = 90^\circ$$

Que 4. What is the area of the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm (a>b)?

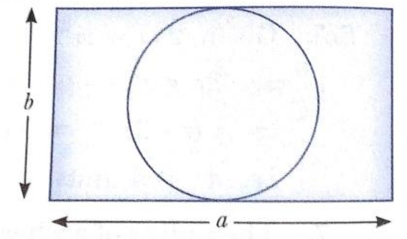

Fig. 12.8

Sol. Diameter of the largest circle that can be inscribed in the given rectangle = b cm

$$\therefore \quad Radius = \frac{b}{2}\, cm$$

$\Rightarrow$ Area of the required circle $= \pi\left(\frac{b}{2}\right)^2 = \frac{\pi b^2}{4} cm^2$

Que 5. Different between the circumference and radius of a circle is 37 cm. Find the area of circle.

Sol. Given $2\pi r - r = 37$

or $r\,(2\pi - 1) = 37$

$$r = \frac{37}{2\pi - 1} = \frac{37}{2\times\frac{22}{7} - 1} = \frac{37\times 7}{37} = 7$$

So area of circle = πr^2

$$= \frac{22}{7} \times 7 \times 7 = 154\; cm^2$$

Que 6. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. Let r be the radius of required circle. Then, we have

$$\pi r^2 = \pi(8)^2 + \pi(6)^2$$

$$\Rightarrow \quad \pi r^2 = 64\pi + 36\pi \quad \Rightarrow \quad \pi r^2 = 100\pi$$

$$\therefore \quad r^2 = \frac{100\pi}{x} = 100 \quad \Rightarrow \quad r = 10\; cm$$

Hence, radius of required circle is 10 cm.

Que 7. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. Let R be the radius of required circle. Then, we have

$$2\pi R = 2\pi(19) + 2\pi(9)$$

$$\Rightarrow \quad 2\pi R = 2\pi(19 + 9) \quad \Rightarrow \quad R = \frac{2\pi \times 28}{2\pi} = 28 \quad \Rightarrow \quad R = 28 \text{ cm}$$

Hence, the radius of required circle is 28 cm.

Que 8. Find the area of a circle whose circumference is 22 cm.

Sol. Let r be the radius of the circle. Then,

Circumference = 22 cm

$$\Rightarrow \quad 2\pi r = 22 \quad \Rightarrow \quad r = \frac{22}{2\pi} = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{cm}$$

$\therefore$ Area of the circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^2 = 38.5 \text{ cm}^2$

Que 9. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹50 per m.

Sol. Area of circular playground = 22176 m^2

$$\pi r^2 = 22176$$

$$\Rightarrow \quad \frac{22}{7} r^2 = 22176 \quad \Rightarrow \quad r^2 = \frac{22176 \times 7}{22}$$

$$\Rightarrow \quad r = 84 \text{ m}$$

$\therefore$ Circumference of a circle = $2\pi r = 2 \times \frac{22}{7} \times 84 = 44 \times 12 = 528$ m

$\therefore$ Cost of fencing this ground = 528 × 50 = ₹ 26400.

Que 10. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. We know that

Area of a sector $= \frac{\theta}{360} \times \pi r^2$

Here, $\theta = 60° \; and \; r = 6 \; cm$

$\therefore$ Area of the sector $= \frac{60}{360} \times \pi(6)^2$

$$= 6\pi = \frac{6 \times 22}{7} = \frac{132}{7} \text{cm}^2 = 18\frac{6}{7} \text{cm}^2$$

Short Answer Type Questions – II

[3 marks]

Que 1. If the perimeter of a semicircular protractor is 66 cm, find the diameter of the protractor (Take $\pi = 22/7$).

Sol. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

$\Rightarrow \quad \pi r + 2r = 66$ [∴ Perimeter of a semicircle $=\pi r + 2r$]

$\Rightarrow \quad r\left(\frac{22}{7} + 2\right) = 66 \qquad \Rightarrow \frac{36}{7}r = 66$

$\Rightarrow \quad r = \frac{66\times7}{36} = \frac{77}{6} = cm$

∴ Diameter of the protractor $= 2r = 2 \times \frac{77}{6} = \frac{77}{3} = 25\frac{2}{3\,cm}$

Que 2. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r cm. Then

Diameter = 2r cm and circumference = 2πr cm

According to question,

∴ Circumference = Diameter + 16.8

$\Rightarrow \quad 2\pi r = 2r + 16.8$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 2r + 16.8 \qquad \Rightarrow \qquad 44r = 14r + 16.8 \times 7$

$\Rightarrow \quad 44r = 14r = 117.6$ or $30r = 117.6$

$\Rightarrow \quad r = \frac{117.6}{30} = 3.92$

Hence, radius = 3.92 cm.

Que 3. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

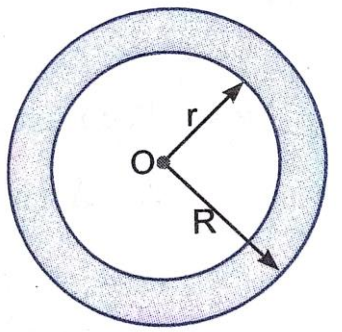

Fig. 12.9

Sol. Let the outer and inner radii of the ring be R m and r m respectively. Then,

$2\pi R = 396$ and $2\pi r = 352$

$\Rightarrow \; 2 \times \frac{22}{7} \times R = 396$ and $\Rightarrow \; 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow \; R = 396 \times \frac{7}{22} \times \frac{1}{2}$ and $r = 352 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow \; R = 63\,m$ and $r = 56\,m$

Hence, width of the track = (R – r) m = (63 – 56) m = 7 m

Que 4. The inner circumference of a circular track [Fig. 12.10] is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹ 2 per metre.

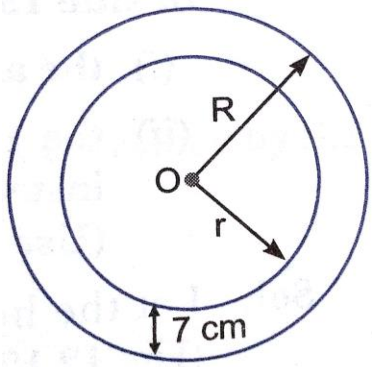

Fig. 12.10

Sol. Let the inner and outer radii of the circular track be r m and R m respectively. Then,

Inner circumference = $2\pi r = 220$ m

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220 \;\Rightarrow\; r = \frac{220 \times 7}{2 \times 22} = 35\,m$

Since the track is 7 m wide everywhere. Therefore,

R = Outer radius = r + 7 = (35 + 7) m = 42 m

∴ Outer circumference = $2\pi R = 2 \times \frac{22}{7} \times 42 \text{ m} = 264 \text{ m}$

Rate of fencing = ₹ 2 per metre

∴ Total cost of fencing = (Circumference × Rate) = ₹ (264 × 2) = ₹ 528

Que 5. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Sol. The diameter of a wheel = 80 cm.

So, radius of the wheel = 40 cm.

Now, distance travelled in one complete revolution of wheel = 2π × 40 = 80π

Since, speed of the car is 66 Km/h

So, distance travelled in 10 minutes = $\frac{66 \times 100000 \times 10}{60}$

$= 11 \times 100000\ cm = 1100000\ cm.$

So, Number of complete revolutions in 10 minutes

$$= \frac{1100000}{80\pi} = \frac{110000}{8 \times \frac{22}{7}}$$

$$= \frac{110000 \times 7}{8 \times 22} = \frac{70000}{16} = 4375$$

Que 6. An umbrella has 8 ribs which are equally spaced (Fig. 12.11). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

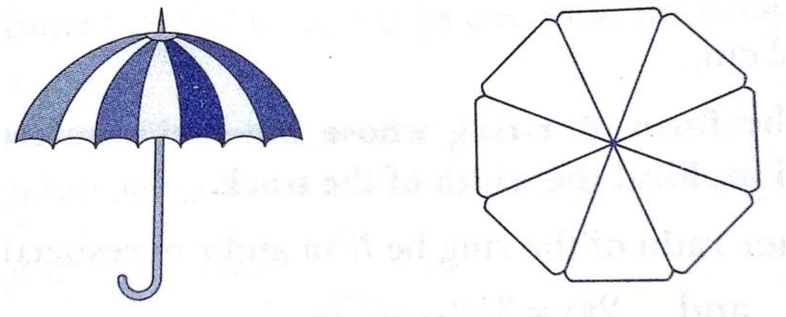

Fig. 12.11

Sol. We have, r = 45 cm

∴ Area between two consecutive ribs $= \frac{1}{8} \times \pi r^2$

$$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 = \frac{11 \times 45 \times 45}{4 \times 7}$$

$$= \frac{22275}{28} = 795.54\ cm^2$$

Que 7. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (Fig. 12.12). Find

(i) the area of that part of the field in which the horse can graze;

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m.

(Use π = 3.14)

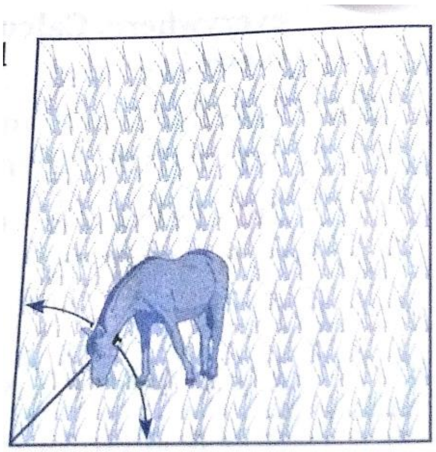

Fig. 12.12

Sol. Let the horse be tied at point O and the length of the rope is OH (Fig. 12.12).

Thus, (i) the area of the part of the field in which the horse can graze

= Area of the quadrant of a circle (OAHB)

$$= \frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times 5 \times 5 = \frac{78.5}{4} = 19.625\ m^2$$

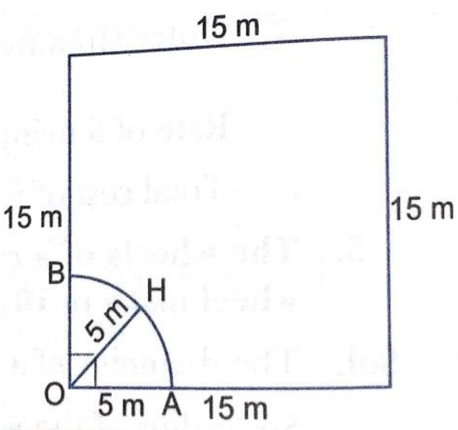

Fig. 12.13

(ii) Now, r = 10 m and

$$\therefore \text{Required area} = \frac{\pi r^2}{4}$$

$$= \frac{3.14 \times (10)^2}{4} = \frac{3.14 \times 100}{4}$$

$$= \frac{314}{4} = 78.5\ m^2$$

Increase in the grazing area

$= (78.5 – 19.625)\ m^2$

$= 58.875\ m^2$

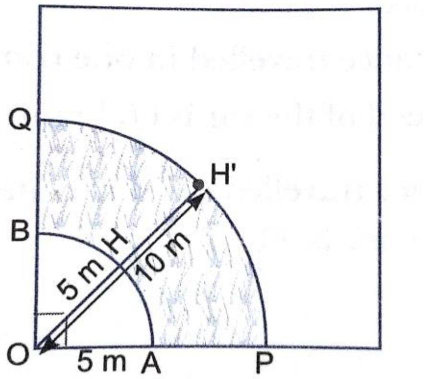

Fig. 12.14

Que 8. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115 °. Find the total area cleaned at each sweep of the blades.

Sol. We have, r = 25 cm and $\theta = 115°$

∴ Total area cleaned at each sweep of the blades

= 2 × (Area of the sector having radius 25 cm and angle $\theta = 115°$).

$$= 2 \times \frac{\theta}{360°} \times \pi r^2 = 2 \times \frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25$$

$$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} = \frac{158125}{126} = 1254.96\ cm^2$$

Que 9. In Fig. 12.15, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown. Find the area of the shaded region.

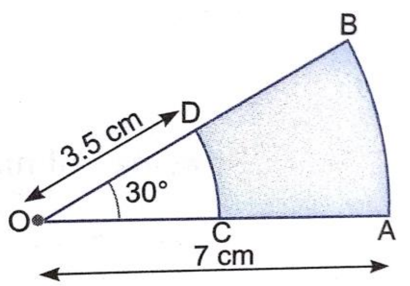

Fig. 12.15

Sol. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30° in a circle of radius 7 cm.

$$\Rightarrow \quad A_1 = \left\{\frac{30}{360} \times \frac{22}{7} \times (7)^2\right\}\ cm^2 \qquad \left[Using: A = \frac{\theta}{360} \times \pi r^2\right]$$

$\Rightarrow \;\; A_1 = \left\{\frac{1}{12} \times \frac{22}{7} \times 7 \times 7\right\} \; cm^2 \;\; \Rightarrow \;\; A_1 = \frac{77}{6} \; cm^2$

A_2 = Area of a sector of angle 30° in a circle of radius 3.5 cm.

$\Rightarrow \;\; A_2 = \left\{\frac{30}{360} \times \frac{22}{7} \times (3.5)^2\right\} \; cm^2 \;\; \Rightarrow \;\; A_2 = \left\{\frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\} \; cm^2 \; = \; \frac{77}{24} cm^2$

∴ Area of the shaded region = $A_1 - A_2 = \left(\frac{77}{6} - \frac{77}{24}\right) \; cm^2$

$$= \frac{77}{24} \times (4 - 1) \; cm^2 = \frac{77}{8} cm^2 = 9.625 \; cm^2$$

Que 10. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 AM and 9.35 AM.

Sol. We have,
Angle described by the minute hand in one minute = 6 °
∴ Angle described by the minute hand in 35 minutes = (6 × 35)° = 210°
∴ Area swept by the minute hand in 35 minutes = Area of a sector of a circle of radius 10 cm

$$= \left\{\frac{210°}{360°} \times \frac{22}{7} \times (10)^2\right\} \; cm^2 \; = \; 183.3 \; cm^2$$

Que 11. Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use$\pi = 3.14$)

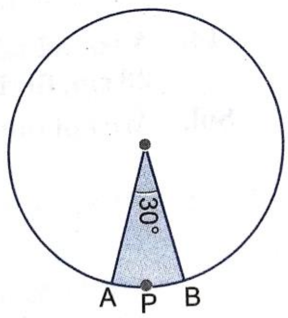

Fig. 12.16

Sol. Area of the sector OAPB = $\frac{\theta}{360°} \times \pi r^2$

$$= \frac{30°}{360°} \times 3.14 \times 4 \times 4 \; cm^2$$

$$= \frac{12.56}{3} cm^2 \; = \; 4.19 \; cm^2 \; (approx.)$$

Area of the corresponding major sector = πr^2 – Area of sector OAPB

= (3.14 × 4 × 4 – 4.19) cm^2 = (50.24 – 4.19) cm^2

= 46.05 cm^2 = 46.4 cm^2 (approx.)

Que 12. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use$\pi =$ 3.14 and$\sqrt{3} = 1.73$)

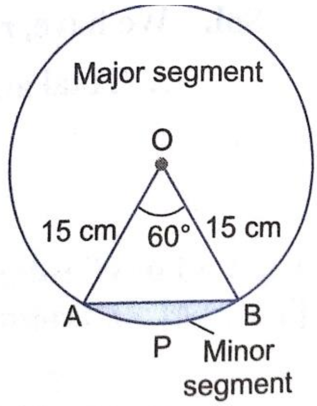

Fig. 12.17

Sol. We have, r = 15 cm and θ = 60°

Given segment is APB

∴ Area of minor segment $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

$$= \frac{60}{360} \times 3.14 \times 15 \times 15 - \frac{1}{2} \times 15 \times 15 \times \sin 60°$$

$$= \frac{1}{6} \times 3.14 \times 15 \times 15 - \frac{1}{2} \times 15 \times 15 \times \frac{\sqrt{3}}{2}$$

$$= 225 \left(\frac{3.14}{6} - \frac{1.73}{4}\right) = 225 \left(\frac{6.28 - 5.19}{12}\right)$$

$$= \frac{225 \times 1.09}{12} = \frac{245.25}{12} = 20.44\ cm^2$$

Now, area of major segment = Area of circle – Area of minor segment

$= \pi(15)^2 - 20.44 = 3.14 \times 225 - 20.44$

$= 706.5 - 20.44 = 686.06\ cm^2$

Que 13. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use$\pi = 3.14$ and $\sqrt{3} = 1.73$)

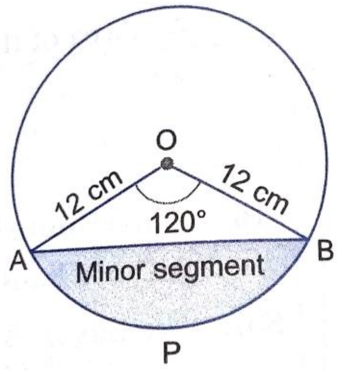

Fig. 12.18

Sol. We have, r = 12 cm and $\theta = 120°$

Given segment is APB

Now, area of the corresponding segment of circle

= Area of the minor segment

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

$= \frac{120}{360} \times 3.14 \times (12)^2 - \frac{1}{2} \times (12)^2 \sin 120°$

$\left[\because \sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{3} \times 3.14 \times 144 - \frac{1}{2} \times 144 \times \frac{\sqrt{3}}{2} = 144 \left[\frac{3.14}{3} - \frac{\sqrt{3}}{4}\right] = 144 \left[\frac{12.56 - 3\sqrt{3}}{12}\right]$

$= 12(12.56 - 3 \times 1.73) = 12(12.56 - 5.19) = 12 \times 7.37 = 88.44\ cm^2$

Que 14. A round table cover has six equal designs as shown in Fig. 12.19. If the radius of the cover is 28 cm, find the cost of making the design at the rate of ₹ 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)

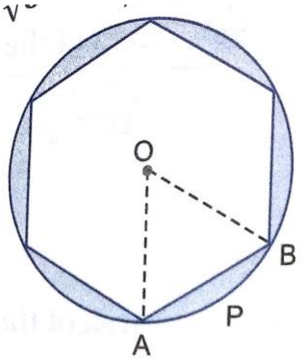

Fig. 12.19

Sol. Area of one design = Area of the sector OAPB – Area of ΔAOB

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \sin 60°$$

$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \frac{\sqrt{3}}{2}$$

$$= 28 \times 28 \left(\frac{11}{21} - \frac{1.7}{4}\right) = 28 \times 28 \times \frac{8.3}{84} = 77.47\ cm^2$$

∴ Area of 6 such designs = 77.47 × 6 = 464.8 cm^2

Hence, cost of making such designs = 464.8 × 0.35 = ₹ 162.69

Que 15. Find the area of the shaded region in Fig. 12.20, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

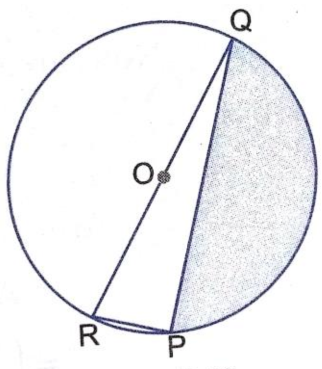

Fig. 12.20

Sol. Here, ROQ is the diameter if given circle, therefore $\angle RPQ = 90°$

Now, in right angled triangle $\Delta PRQ, we\ have$

$RQ^2 = RP^2 + PQ^2$ (by Pythagoras Theorem)

⇒ $RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$

∴ $RQ = \sqrt{625} = 25\ cm$

Therefore, radius r = $\frac{25}{2}\ cm$

Now, area of shaded region

= Area of the semi-circle – Area of ΔRPQ

$= \frac{1}{2}\pi r^2 - \frac{1}{2} PQ \times RP = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - \frac{1}{2} \times 24 \times 7$

$= \left(\frac{6875}{28} - 84\right) cm^2 = \left(\frac{6875-2352}{28}\right) cm^2 = \frac{4523}{28}\ cm^2 = 161.54\ cm^2$

Long Answer Type Questions

[4 MARKS]

Que 1. PQRS is a diameter of a circle of radius 6 cm. The length PQ, QR and RS are equal. Semicircles are drawn on PQ and QS as diameters is shown in Fig. 12.36. Find the perimeter and area of the shaded region.

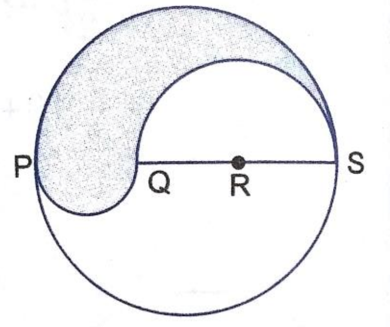

Fig. 12.36

Sol. We have,

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore \; PQ = QR = RS = \frac{12}{3} = 4\ cm$

$QS = QR + RS = (4 + 4)\ cm = 8cm$

Hence, required perimeter

= Arc of semicircle of radius 6 cm + Arc of semicircle of radius 4 cm

+ ARC of semi-circle of radius 2 cm

= $(\pi \times 6 + \pi \times 4 + \pi \times 2)\ cm = 12\pi\text{cm} = 12 \times \frac{22}{7} = \frac{264}{7} = 37.71$ cm.

Required area = Area of semicircle with PS as diameter + Area of semicircle with PQ as diameter – Area of semi-circle with QS as diameter

$$= \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times (2)^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$$

$$= \frac{1}{2} \times \frac{22}{7}(6^2 + 2^2 - 4^2) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7} cm^2$$

$$= 37.71\ cm^2$$

Que 2. Figure 12.37 depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

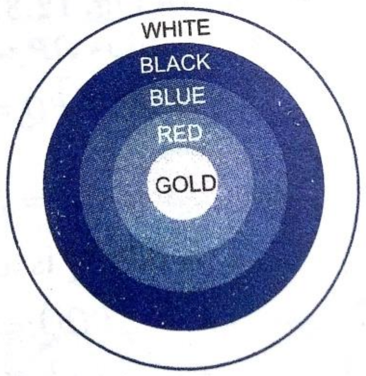

Fig. 12.37

Sol. The area of Gold region $= \pi(10.5)^2 = \frac{22}{7} \times 110.25$

$$= \frac{2425.2}{7} cm^2 = 346.5\ cm^2$$

The area of Red region $= \pi[(21)^2 - (10.5)^2]$

$$= \pi[441 - 110.25]$$

$$= \frac{22}{7} \times 330.75 = \frac{7276.5}{7} cm^2 = 1039.5\ cm^2$$

The area of Blue region $= [(31.5)^2 - (21)^2] = \pi[992.25 - 441]$

$$= \frac{22}{7} \times 551.25 = \frac{12127.5}{7} = 1732.5\ cm^2$$

The area of Black region $= \pi[(42)^2 - (31.5)^2]$

$$= \frac{22}{7}[1764 - 992.25] = \frac{16978.5}{7} = 2425.5\ cm^2$$

And the area of White region $= \pi[(52.5)^2 - (42)^2]$

$$= \frac{22}{7}[2756.25 - 1764]$$

$$= \frac{22}{7} \times 992.25 = \frac{21829.5}{7} = 3118.5\ cm^2$$

Que 3. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distance travelled by their tips in 2 days.

Sol. In 2 days, the short hand will complete 4 rounds.

$\therefore$ Distance moved by its tip = 4 (Circumference of a circle of radius 4 cm)

$$= 4 \times \left(2 \times \frac{22}{7} \times 4\right)\ cm = \frac{704}{7} cm$$

In 2 days, the long hand will complete 48 rounds.

$\therefore$ Distance moved by its tip = 48 (Circumference of a circle of radius 6 cm)

$$= 48 \times \left(2 \times \frac{22}{7} \times 6\right)\ cm = \frac{12672}{7} cm$$

Hence, Sum of the distance moved by the tips of two hands of the clock

$= \frac{704}{7} + \frac{12672}{7} = \frac{13376}{7}$

= 1910.86 cm

Que 4. Fig. 12.38, depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:
(i) the distance around the track along its inner edge.
(ii) the area of the track.

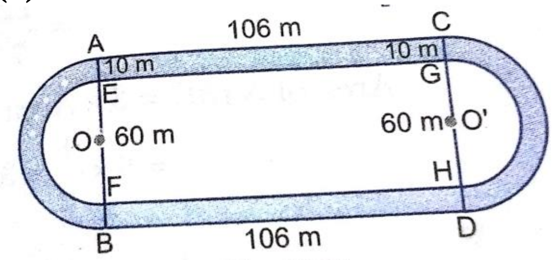

Fig. 12.38

Sol. Here, we have

OE = O'G = 30 m
AE = CG = 10 m
OA = O'C = (30 + 10) m = 40 m
AC = EG = FH = BD = 106 m

(i) The distance around the track along its inner edge

= EG + FH + 2 × (circumference of the semicircle of radius OE = 30 cm)

$= 106 + 106 + 2\left(\frac{1}{2} \times 2\pi \times 30\right) = 212 + 60\pi$

$= 212 + 60 \times \frac{22}{7} = \left(212 + \frac{1320}{7}\right) m = \left(\frac{1484+1320}{7}\right) m = \frac{2804}{7} m = 400\frac{4}{7} m$

(ii) Area of the track = Area of the shaded region

= Area of rectangle AEGC + Area of rectangle BFHD + 2 (Area of the semicircle of radius 40 m – Area of the semicircle with radius 30 m)

$= [(10 \times 106) + (10 \times 106)] + 2\left\{\frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2\right\}$

$= 1060 + 1060 + \frac{22}{7}[(40)^2 - (30)^2]$

$= 2120 + \frac{22}{7} \times 700 = 2120 + 2200 = 4320\ m^2$

Que 5. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.39). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

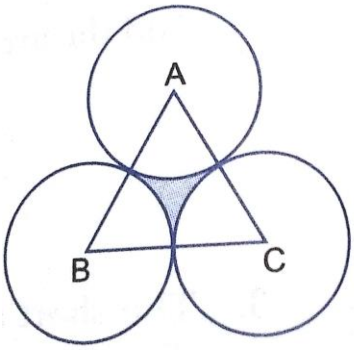

Fig. 12.39

Sol. Let each side of the equilateral triangle be x cm. Then,

Area of equilateral triangle ABC = 17320.5 cm^2 (Given)

$\Rightarrow \quad \frac{\sqrt{3}}{4}x^2 = 17320.5 \qquad \Rightarrow \quad \frac{1.73205}{4}x^2 = 17320.5$

$\Rightarrow \quad x^2 = \frac{4 \times 17320.5}{1.73205} \qquad \Rightarrow \quad x^2 = 40000$

$\therefore \quad x = 200\ cm$

Thus, radius of each circle = $\frac{200}{2} = 100\ cm$

Now, area of shaded region = Area of $\Delta ABC - 3 \times$ Area of a sector of angle 60° and radius 100 cm

$= 17320.5 - 3 \times \frac{60}{360} \times \pi \times (100)^2$

$= 17320.5 - \frac{1}{2} \times \pi \times 100 \times 100 = 17320.5 - 3.14 \times 5000$

= 17320.5 – 15700 = 1620.5 cm^2

Que 6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.40. Find the area of the design.

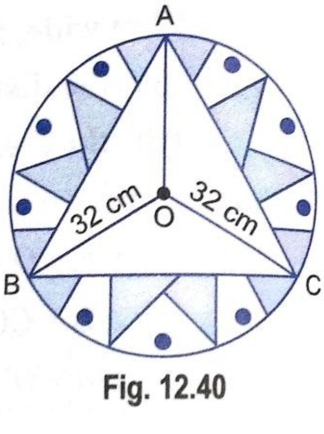

Fig. 12.40

Sol. Here, ΔABC is an equilateral triangle. Let O be the circumcenter of circumcircle.

Radius, r = 32 cm.

Now, area of circle = πr^2

$$= \frac{22}{7} \times 32 \times 32 = \frac{22528}{7}\ cm^2$$

Area of ΔABC = 3 × Area of ΔBOC

$$= 3 \times \frac{1}{2} \times 32 \times 32 \times \sin 120°$$

$$[\angle BOC = 2\angle BAC = 2 \times 60° = 120°]$$

$$= 3 \times 16 \times 32 \times \frac{\sqrt{3}}{2} \quad \left(\because \sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}\right)$$

$$= 3 \times 16 \times 16 \times \sqrt{3} = 768\sqrt{3}\ cm^2$$

∴ Area of the design = Area of the circle – Area of ΔABC

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\ cm^2$$

$$= (3218.28 - 1330.176)\ cm^2 = 1888.7\ cm^2$$

Que 7. In Fig. 12.41, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

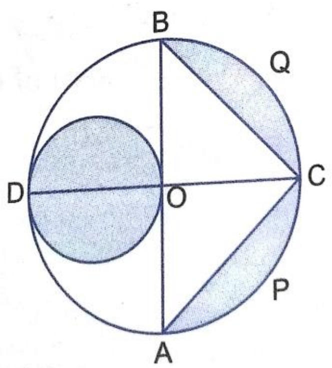

Fig. 12.41

Sol. Here, area of sector OBQC = $\frac{90°}{360°} \times \pi \times (7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} cm^2$$

And, Area of $\Delta OBC = \frac{1}{2} \times OC \times OB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}\ cm^2$

∴ Area of the segment BQC = Area of sector OBQC – Area of ΔOBC

$$= \frac{77}{2} - \frac{49}{2} = \frac{28}{2} = 14\ cm^2$$

Similarly, area of the segment APC = 14 cm^2

Now, the area of the circle with OD as diameter = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} cm^2$

Hence, the total area of the shaded region

$$= \left(14 + 14 + \frac{77}{2}\right)\ cm^2 = \left(28 + \frac{77}{2}\right)\ cm^2$$

$$= \left(\frac{56+77}{2}\right)\ cm^2 = \frac{133}{2} cm^2 = 66.5\ cm^2$$

Que 8. In Fig. 1242 ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

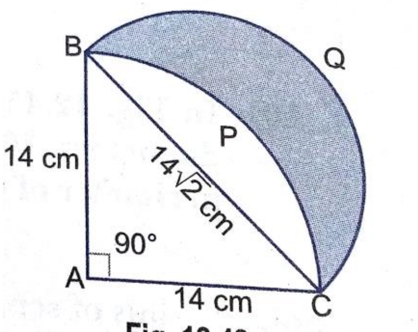

Fig. 12.42

Sol. In $\Delta ABC,$ we have

$BC = \sqrt{(AC)^2 + (AB)^2}$ (By Pythagoras Theorem)

$= \sqrt{(14)^2 + (14)^2} = \sqrt{196 + 196} = \sqrt{392} = 14\sqrt{2}$

Now, area of sector ABPC = $\frac{90°}{360°} \times \pi(14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154\ cm^2$

And, Area of segment BPC = Area of sector ABPC – Area of ΔABC

= (154 – 98) cm^2 = 56 cm^2

Now, we have radius of semi-circle BQC = $\frac{14\sqrt{2}}{2}\ cm = 7\sqrt{2}\ cm$

∴ Area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} = 154\ cm^2$

Hence, area of the shaded region

= Area of the semi-circle BQC – Area of the segment BPC

= (154 – 56) cm^2 = 98 cm^2

HOTS (Higher Order Thinking Skills)

Que 1. Two circles touch internally. The sum of their area is 116 πcm^2 and distance between their centres is 6 cm. Find the radii of the circles.

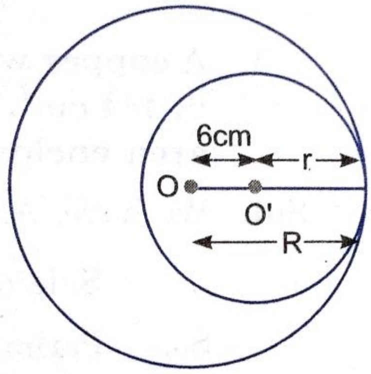

Fig. 12.49

Sol. Let R and r be the radii of the circles [Fig. 12.49].

Then, according to question,

$\Rightarrow \quad \pi R^2 + \pi r^2 = 116\pi$

$\Rightarrow \quad R^2 + r^2 = 116 \quad ....(i)$

Distance between the centres = 6 cm

$\Rightarrow \quad OO' = 6$ cm

$\Rightarrow \quad R - r = 6 \quad .....(ii)$

Now, $(R + r)^2 + (R - r)^2 = (R^2 + r^2)$

Using the equation (i) and (ii), we get

$(R + r)^2 + 36 = 2 \times 116$

$\Rightarrow \quad (R + r)^2 = (2 \times 116 - 36) = 196$

$\Rightarrow \quad R + r = 14 \quad ....(iii)$

Solving (ii) and (iii), we get R = 10 and r = 4.

Hence, radii of the gives circles are 10 cm and 4 cm respectively.

Que 2. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Sol. Let the radius of the wheel be r cm.

Distance covered by the wheel in one revolution = $\frac{Distance\ moved}{Number\ of\ revolutions} = \frac{11}{5000} km$

$= \frac{11}{5000} \times 1000 \times 100\ cm = 220\ cm$

$\therefore$ Circumference of the wheel = 220 cm

$\Rightarrow \quad 2\pi r = 220 \text{ cm} \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{2 \times 22} \quad \Rightarrow \quad r = 35$ cm

∴ Diameter = 2r cm = $(2 \times 35)\ cm = 70\ cm$

Hence, the diameter of the wheel is 70 cm.

Que 3. Find the area of the shaded design of Fig. 12.50, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter (use $\pi = 3.14$).

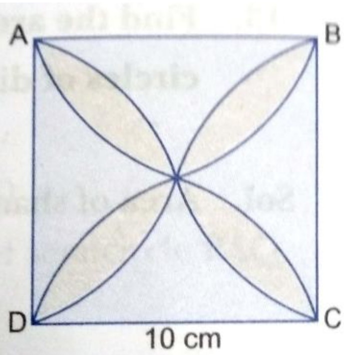

Fig. 12.50

Sol. Let us mark the four unshaded regions as I, II, III and IV (Fig. 12.50).

Area of 1 + Area of III

= Area of ABCD – Area of two semicircles of radius 5 cm each

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2\right)\ cm^2$

$= (100 - 3.14 \times 25)\ cm^2 = (100 - 78.5)\ cm^2 = 21.5\ cm^2$

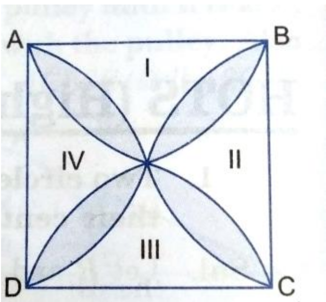

Fig. 12.51

Similarly, Area of II + Area of IV = 21.5 cm^2

So, Area of the shaded design

= Area of ABCD – Area of (I + II + III + IV)

= (100 – 2 × 21.5) cm^2

= (100 – 43) cm^2 = 57 cm^2

Que 4. A copper wire, when bent in the form of a square, enclosed an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it.

Sol. We have, Area of the square = a^2 = 484 cm^2

∴ Side of the square = $\sqrt{484}\ cm = 22\ cm$

So, Perimeter of the square = 4 (side) = (4 × 22) cm = 88 cm

Let r be the radius of the circle. Then, according to question,

Circumference of the circle = Perimeter of the square

$\Rightarrow \qquad 2\pi r = 88 \qquad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow \qquad r = \frac{88 \times 7}{2 \times 22} \qquad \Rightarrow \quad r = 14\ cm$

$\therefore$ Area of the circle = $\pi r^2 = \left\{\frac{22}{7} \times (14)^2\right\} \text{cm}^2 = 616\ \text{cm}^2$

Que 5. Two circles touch externally. The sum of their areas is 130 π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

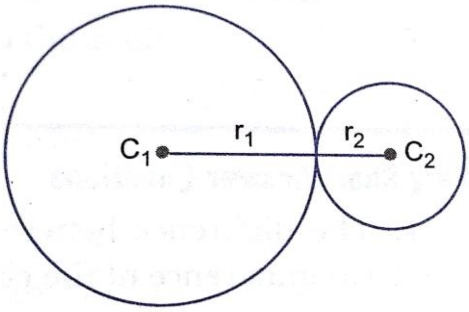

Fig. 12.52

Sol. If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

Let the radii of the two circles be r_1 cm and r_2 cm respectively {Fig. 12.52].

Let C_1 and C_2 be the centres of the given circles. Then,

$$C_1C_2 = r_1 + r_2$$

$\Rightarrow \qquad 14 = r^1 + r^2 \qquad [\because\ C_1C_2 = 14\ cm\ given]$

$\Rightarrow \qquad r_1 + r_2 = 14 \qquad \ldots(i)$

It is given that the sum of the areas of two circles is equal to 130 π cm^2.

$\therefore \qquad \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow \qquad r_1^2 + r_2^2 = 130 \qquad \ldots(ii)$

Now, $\qquad (r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \qquad 14^2 = 130 + 2r_1r_2$ [using (i) and (ii)]

$\Rightarrow \qquad 196 - 130 = 2r_1r_2$

$\Rightarrow \qquad r_1r_2 = 33 \qquad \ldots(iii)$

Now, $\qquad (r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$\Rightarrow \qquad (r_1 - r_2)^2 = 130 - 2 \times 33 \qquad$ [using (ii) and (iii)]

$\Rightarrow \qquad (r_1 - r_2)^2 = 64$

$\Rightarrow \qquad r_1 - r_2 = 8 \qquad \ldots(iv)$

Solving (i) and (iv), we get $r_1 = 11$ cm and $r_2 = 3$ cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

Value Based Questions

Que 1. A manufacture involved ten children in colouring playing top (lattu) which is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area they had to paint if 50 playing tops were given to them. $\left(\boldsymbol{Take}\ \pi = \frac{22}{7}\right)$

(a) How is child labour an abuse for the society?

(b) What steps can be taken to abolish child labour?

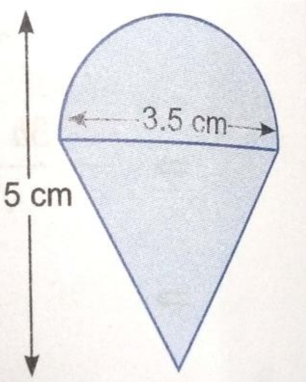

Fig. 9

Sol. This top is exactly like the object in Fig. 9.

∴ Total surface area of the top = Curved surface area of hemisphere + Curved surface area of cone

Now, the curved surface area of hemisphere = $2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ cm^2$$

Also, the height of the cone

= Height of the top – Height (radius) of the hemisphere part

$$= \left(5 - \frac{3.5}{2}\right)\ cm = 3.25\ cm$$

So, the slant height of the cone (l) = $\sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2} cm = 3.7\ cm\ (approx)$$

Therefore, curved surface area of cone πrl = $\left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right)$ cm²

Thus, the surface area of the top = $\left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ cm^2 + \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right)\ cm^2$

$$= \frac{22}{7} \times \frac{3.5}{2}(3.5 + 3.7)\ cm^2 = \frac{22}{7} \times \frac{3.5}{2} \times 7.2\ cm^2$$

= 39.6 cm^2 (approx.)

Surface area to be painted = $50 \times 39.6\ cm^2 = 1980\ cm^2$

(a) Children are the future of any society or country and they possess various talents. So, providing them opportunities to grow be giving proper education instead of involving them in work will help in the development of society. With education and proper nurture of their talent, they can contribute in a better way for the development of society and the country.

(b) (i) Spreading awareness against child labour in the society.
(ii) Abolishing the use of products involving child labour.
(iii) Providing free education at elementary level to poor children.
(iv) Enforcing the law to abolish child labour.

Que 2. A child prepares a poster on 'Save Energy' on a square sheet whose each side measures 60 cm. At each corner of the sheet, she draws a quadrant of radius 17.5 cm in which she shows the ways to save energy. At the centre, she draws a circle of diameter 21 cm and writes a slogan in it. Find the area of the remaining sheet.
(a) Write down the four ways by which energy can be saved.
(b) Write a slogan on 'Save Energy'.
(c) Why do we need to save energy?

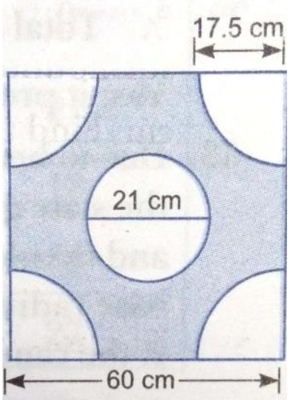

Fig. 10

Sol. Area of the square = 60 x 60 = $3600\ cm^2$

$$\text{Area of the remaining sheet} = 3600 - \pi\left(\frac{21}{2}\right)^2 - 4 \times \frac{\pi}{4} \times (17.5)^2$$

$$= 3600 - \pi\left(\frac{441}{4} + \frac{1225}{4}\right)$$

$$= 3600 - \frac{22}{7} \times \frac{1666}{4} = 3600 - 1309 = 2291\ cm^2$$

(a) (i) Saving electricity by using CFLs, switching off appliances when not in use.
(ii) Saving water by using it efficiently.
(iii) Saving petroleum resources by using public transport.
(iv) Using solar energy.

(b) 'Save energy, Save Environment' or any other given by students.
(c) We should save energy to save our environment so that we can give a better tomorrow to the forth coming generations.

Que 3. A teacher brings clay in the classroom to teach the topic' mensuration'. She forms a cylinder of radius 6 cm and height 8 cm with the clay. Then she moulds that cylinder into a sphere. Find the radius of the sphere formed.
Do teaching aids enhance teaching learning process? Justify your answer.

Sol. Volume of the cylinder formed = Volume of the sphere

$\Rightarrow \quad \pi(6)^2 \times 8 = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{6^2\times8\times3}{4} = r^3$

$\Rightarrow \quad r^3 = 6^3$ or r = 6 cm

Yes, teaching aids make the learning practical, interesting, easy to learn and leave long lasting impact.

Que 4. A night camp was organised for Class X students for two days and their accommodation was planned in tents. Each tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas). Is camping helpful to students in their development? Justify your answer.

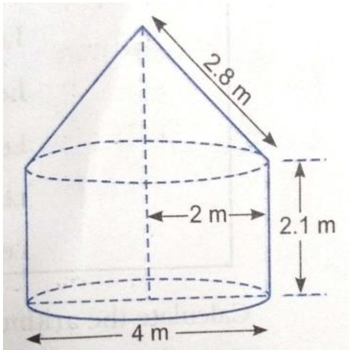

Fig. 11

Sol. We have, Radius of cylindrical base = $\frac{4}{2} = 2\,m$

Height of cylindrical portion = 2.1 m

∴ Curved surface area of cylindrical portion = 2 πrh

$= 2 \times \frac{22}{7} \times 2 \times 2.1$

$= 26.4\ m^2$

Radius of conical base (r) = 2 m

Slant height of conical portion (l) = 2.8 m

∴ Curved surface area of conical = πrl

$= \frac{22}{7} \times 2 \times 2.8$

$= 17.6\ m^2$

Now, total area of the canvas = (26.4 + 17.6) m^2 = 44 m^2

∴ Total cost of the canvas used = ₹ 500 × 44 = ₹ 22,000

Yes, it provides the feeling of self-confidence, sharing, caring and other social values.

Que 5. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem? $\left(Use\ \pi = \frac{22}{7}\right)$

Sol. Slant height of conical part = $\sqrt{(2.8)^2 + (2.1)^2} = 3.5\ m$

Area of canvas/tent = $2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5\ m^2$

$= \frac{22}{7} \times 2.8 \times 3.5(2 + 1)$

$= 3 \times \frac{22}{7} \times 2.8 \times 3.5$

$= 92.4\ m^2$

Cost of 1500 tents = 1500 × 92.5 × 120 = ₹ 1,66,32,000

Share of each school = $\frac{1}{50} \times 16632000 = ₹\ 3{,}32{,}640$

Values: Helping the needy people