Very Short Answer Type Questions

[1 Marks]

Que 1. What is the capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom? (Fig. 13.4).

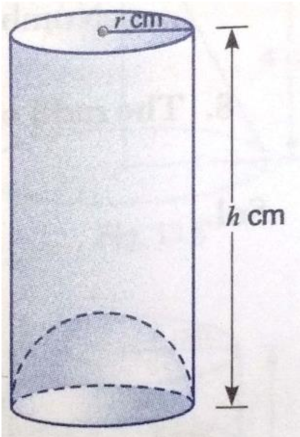

Fig. 13.4

Sol. Capacity of the given vessel

= capacity of cylinder – capacity of hemisphere

$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}(3h - 2r)$

Que 2. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. What is the total surface area of the combined solid?

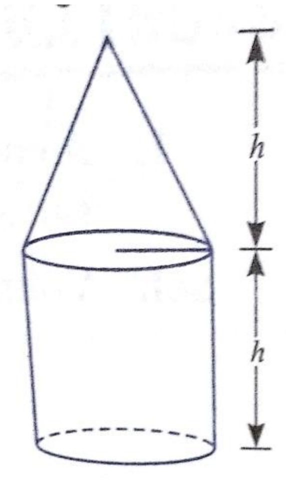

Fig. 13.5

Sol. The total surface area of the combined solid in Fig. 13.5

= curved surface area of cone + curved surface area of cylinder + Area of the base.

$= \pi rl + 2\pi rh + \pi r^2 = \pi r\,(l + 2h + r) = \pi r\left(\sqrt{r^2 + h^2} + 2h + r\right)$

Que 3. Two identical solid hemispheres of equal base radius cm are struck together along their bases. What will be the total surface area of the combination?

Sol. The resultant solid will be a sphere of radius r whose total surface area is 4 πr^2.

Que 4. A solid ball is exactly fitted inside the cubical box of side a. What is the volume of the ball?

Sol. Diameter of the solid ball = edge of the cube = a

$\therefore$ Volume of the ball $= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3}\times\frac{1}{8}\pi a^3 = \frac{1}{6}\pi a^3$

Que 5. If two cubes of edge 5 cm each are joined end to end, find the surface are of the resulting cuboid

Sol. Total length (l) = 5 + 5 = 10 cm
Breadth (b) = 5 cm, Height (h) = 5 cm
Surface Area = 2 (lb + bh + lh)
= 2(10 × 5 + 5 × 5 + 5 × 10)
= 2 × 125 = 250 cm^2

Que 6. A solid piece of iron in the form of a cuboid of dimension 49 cm × 33 cm × 24 cm is melted to form a solid sphere. Find the radius of sphere.

Sol. $49 \times 33 \times 24 = \frac{4}{3}\pi r^3$

$$\Rightarrow \quad r^3 = \frac{49\times33\times24\times3\times7}{4\times22}$$

$$r = 21\ cm$$

Que 7. A mason constructors a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Find the number of bricks used to construct the wall.

Sol. Space occupied with bricks = $\frac{7}{8}\times$ volume of the wall

$\therefore$ Number of bricks $= \dfrac{\frac{7}{8}\times270\times300\times350}{22.5\times11.25\times8.75} = 11.200$

Que 8. The radii of the ends of a frustum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.

Sol.

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$= \sqrt{40^2 + (20-11)^2} = \sqrt{1600 + 81}$$
$$= \sqrt{1681} = 41\ cm$$

Short Answer Type Questions – I

[2 marks]

Que 1. A cone, hemisphere and a cylinder stand on equal bases and have the same height. What is the ration of their volumes?

Sol. Volume of a cone: Volume of a hemisphere: Volume of a cylinder

$$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$$

$$= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \qquad (\because\ r = h)$$

$$= 1 : 2 : 3$$

Que 2. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?

Sol. Let edge of the cube be 'a'.

Then, diameter of the sphere that will fit inside the given cube = a

∴ Volume of the cube: Volume of the sphere

$$= a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = a^3 : \frac{4}{3} \times \frac{1}{8}\pi a^3 = a^3 : \frac{1}{6}\pi a^3 = 6 : \pi$$

Que 3. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, find the height of the frustum.

Sol. Let r and R be radii of the circular ends of the frustum of the cone.

Then, $R - r = 4$, $l = 5$

We know, $l^2 = (R - r)^2 + h^2$

$\Rightarrow 5^2 = 4^2 + h^2 \quad or \quad h^2 = 25 - 16 = 9$

$\Rightarrow h = 3$ cm

Que 4. If the slant height of the frustum of a cone is 10 cm and the perimeters of its circular base are 18 cm and 28 cm respectively. What is the curved surface rea of the frustum?

Sol. Let r and R be the radii of the two circular ends of the frustum of the cone

Then, $2\pi r = 18$ and $2\pi R = 28$

$$\Rightarrow \quad r = \frac{18}{2\pi} \quad \text{and} \quad R = \frac{28}{2\pi}$$

$$\Rightarrow \quad r = \frac{9}{\pi} \quad \text{and} \quad R = \frac{14}{\pi}$$

Now, curved surface area of the frustum = $\pi(r + R)l$

$$= \pi\left(\frac{9}{\pi} + \frac{14}{\pi}\right) \times 10 = 23 \times 10 = 230 \text{ cm}^2$$

Que 5. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

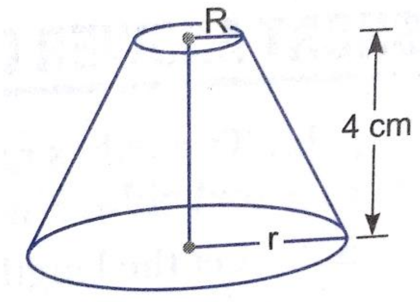

Fig. 13.6

Sol. We have, slant height, l = 4 cm

Let R and r be the radii of two circular ends respectively. Therefore, we have

$2\pi R = 18 \quad \Rightarrow \quad \pi R = 9$

$2\pi R = 6 \quad \Rightarrow \quad \pi r = 3$

∴ Curved surface area of the frustum = $(\pi R + \pi r)\, l$

$= (9 + 3) \times 4 = 12 \times 4 = 48 \text{ cm}^2$

Que 6. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

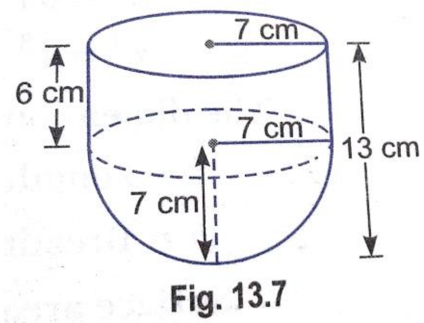

Fig. 13.7

Sol. Here, radius of hemisphere = radius of cylinder = r cm = 7 cm

And height of cylinder, h = (13 – 7) cm = 6 cm

Now, inner surface area of the vessel

= Curved surface area of the cylindrical part + Curved surface area of hemispherical part

$$= 2 \times \frac{22}{7} \times 7 \times (6 + 7) = 2 \times 22 \times 13 = 572\ cm^2$$

Que 7. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of

the solid in terms of π.

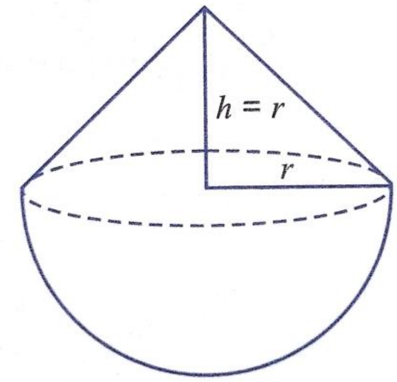

Fig. 13.8

Sol. We have,

Height of cone is equal to its radius

i.e., h = r = 1 cm (Given)

Also, radius of hemisphere = r = 1 cm

Now, Volume of the solid

= Volume of the cone + Volume of the hemisphere

$$= \frac{1}{2}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 \times r + \frac{2}{3}\pi r^3 \qquad [\because \quad h = r]$$

$$= \frac{\pi r^3}{3} + \frac{2}{3}\pi r^3 = \pi r^3 = \pi(1)^3 = \pi cm^3$$

Que 8. If the total surface area of a solid hemisphere is 462 cm², find its volume. $\left[\mathbf{Take}\ \pi = \frac{22}{7}\right]$

Sol. Given, total surface area of solid hemisphere = 462 cm²

$\Rightarrow \qquad 3\pi r^2 = 462\ cm^2$

$$3 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = 49 \Rightarrow r = 7\ cm$$

Volume of solid hemisphere = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.66\ cm^3$$

Short Answer Type Questions – II

[3 marks]

Que 1. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

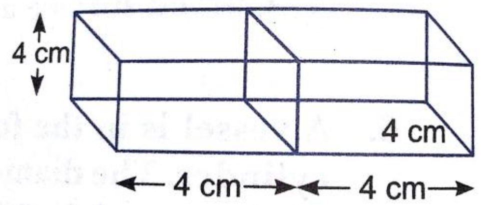

Fig. 13.9

Sol. Let the length of each edge of the cube of volume 64 cm^3 be x cm.

Then, Volume = 64 cm^3

$\Rightarrow \quad x^3 = 64$

$\Rightarrow \quad x^3 = 4^3 \qquad \Rightarrow \quad x = 4\,cm$

The dimensions of cuboid so formed are

l = Length = (4 + 4) cm = 8 cm

B = Breadth = 4 cm and h = Height = 4 cm

∴ Surface area of the cuboid = 2 (lb + bh + lh)

= 2 (8 × 4 + 4 × 4 + 8 × 4)

= 2 (32 + 16 + 32)

= 160 Cm^2

Que 2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. The greatest diameter that a hemisphere can have = 7 cm = l

Radius of the hemisphere (R) = $\frac{7}{2}$ cm

∴ Surface area of the solid after surmounted hemisphere

$$= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2$$

$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5$$

$$= 332.5\ cm^2$$

Que 3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

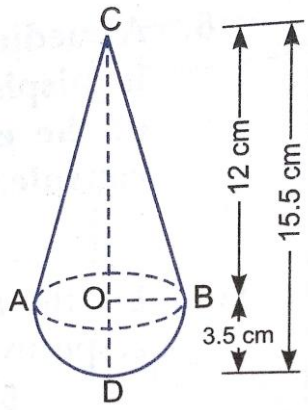

Fig. 13.10

Sol. We have,

Cd = 15.5 cm and OB = OD = 3.5 cm

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, r = OB = 3.5 cm

h = OC = CD – OD = (15.5 – 3.5) cm = 12 cm

$$l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2}$$

$$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\ cm$$

Also, radius of the hemisphere, r = 3.5 cm

∴ Total surface area of the toy

= Surface area of cone + Surface area of hemisphere

$$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times 3.5(12.5 + 2 \times 3.5)$$

$$= \frac{22}{7} \times 3.5 \times 19.5 = 214.5\ cm^2$$

Que 4. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. Here, we have

Edge of the cube = l = Diameter of the hemisphere

Therefore, radius of the hemisphere = $\frac{l}{2}$

∴ Surface area of the remaining solid after cutting out the hemispherical

Depression = $6l^2 - \pi\left(\frac{1}{2}\right)^2 + 2\pi\left(\frac{1}{2}\right)^2$

$$= 6l^2 + \pi \times \frac{l^2}{4} = \frac{l^2}{4}(24 + \pi)$$

Que 5. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas).

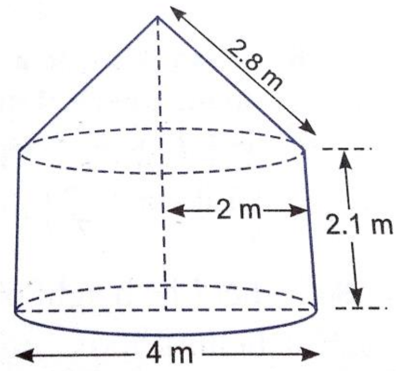

Fig. 13.11

Sol. We have,

Radius of cylindrical base = $\frac{4}{2} = 2m$

Height of cylindrical portion = 2.1 m

∴ Curved surface area of cylindrical portion = 2πrh

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 = 26.4\ m^2$$

Radius of conical base = 2 m

Slant height of conical portion = 2.8 m

∴ Curved surface area of conical portion = πrl

$$= \frac{22}{7} \times 2 \times 2.8 = 17.6\ m^2$$

Now, total area of the canvas = (26.4 + 17.6) m^2 = 44 m^2

∴ Total cost of the canvas used = ₹ 500 × 44 = ₹ 22,000

Que 6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (Fig. 13.12). The length of the entire capsule is 5 mm. Find its surface area.

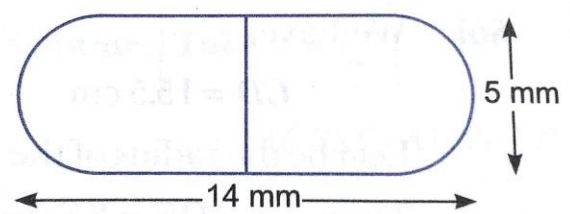

Fig. 13.12

Sol. Let the radius and height of the cylinder be r cm and h cm respectively. Then,

$$r = \frac{5}{2} mm = 2.5\ mm$$

And $h = \left(14 - 2 \times \frac{5}{2}\right) mm = 9\ mm$

Also, radius of hemisphere r = $\frac{5}{2} mm$

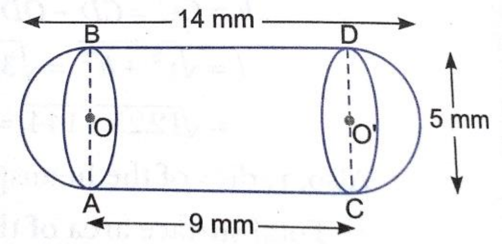

Fig. 13.13

Now, surface area of the capsule

= Curved surface of cylinder + Surface area of two hemispheres

= $2\pi rh + 2 \times 2\pi r^2 = 2\pi r\ (h + 2r)$

$$= 2 \times \frac{22}{7} \times \frac{5}{2} \times \left(9 + 2 \times \frac{5}{2}\right) = 2 \times \frac{22}{7} \times \frac{5}{2} \times 14 = 220\ mm^2$$

Que 7. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.14. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

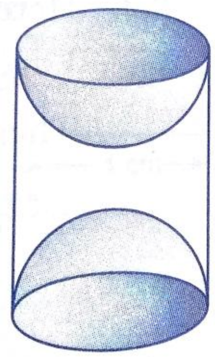
Fig. 13.14

Sol. We have, r = 3.5 cm and h = 10 cm

Total surface area of the article

= Curved surface area of cylinder + 2 × Curved surface area of hemisphere

= $2\pi rh + 2 \times 2\pi r^2 = 2\pi r\ (h + 2r)$

$$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5) = 2 \times \frac{22}{7} \times 3.5 \times 17 = 374\ cm^2$$

Que 8. Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (Fig. 13.15). The height of the cylinder is 1.45 m

and its radius is 30 cm. Find the total surface area of the bird-bath. $\left(\textbf{Take}\pi = \frac{22}{7}\right)$

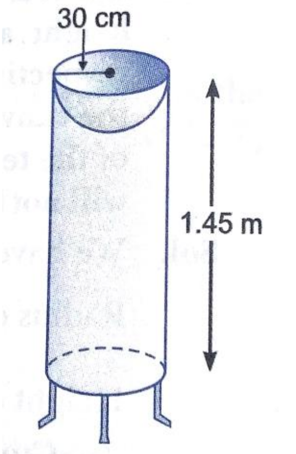

Fig. 13.15

Sol. Let h be height of the cylinder, and r be the common radius of the cylinder and hemisphere.
Then, the total surface area of the bird-bath

= Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 30\,(145 + 30)\ cm^2 = 33{,}000\ cm^2 = 3.3\ m^2$

Que 9. A juice seller was serving his customers using glasses as shown in Fig. 13.16. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14).

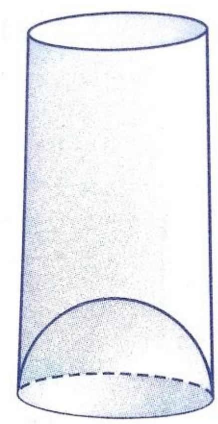
Fig. 13.16

Sol. Since, the inner diameter of the glass = 5 cm and height = 10 cm.

the apparent capacity of the glass = $\pi r^2 h$

$= (3.14 \times 2.5 \times 2.5 \times 10)\ cm^2 = 196.25\ cm^3$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3$

$= 32.71\ cm^3$

So, the actual capacity of the glass

= Apparent capacity of glass – Volume of the hemisphere

= (196.25 – 32.71 cm^3 = 163.54 cm^3

Que 10. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

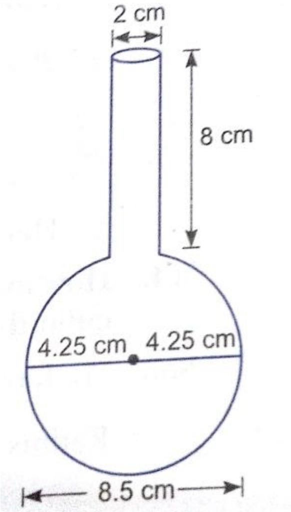

Fig. 13.17

Sol. We have,

Radius of cylindrical neck = 1 cm and height of cylindrical neck = 8 cm

Radius of spherical part = 4.25 cm

Now, Volume of spherical vessel = $\pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi(1)^2 \times 8 + \frac{4}{3} \times \pi \times (4.25)^3$

$= 3.14 \times \left[8 + \frac{4}{3} \times (4.25)^3\right]$

$= 3.14 \times [8 + 102.354]$

$= 3.14 \times 110.354 = 346.514\ cm^3$

$\therefore$ The answer found by the child is incorrect.

Hence, the correct answer is 346.51 cm^3.

Que 11. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. We have,

Radius of sphere = 4.2 cm, Radius of cylinder = 6 cm

Let h cm be the height of cylinder.

Now, since sphere is melted and recast into cylinder

∴ Volume of sphere = Volume of cylinder

i.e., $\frac{4}{3}\pi {r_1}^3 = \pi {r_2}^2 h$ $\Rightarrow \frac{4}{3} \times \pi \times (4.2)^3 = \pi \times (6)^2 \times h$

$$\Rightarrow \ h = \frac{\frac{4}{3} \times \pi \times (4.2)^3}{\pi \times (6)^2} = \frac{\frac{4}{3} \times 4.2 \times 4.2 \times 4.2}{36} \quad \Rightarrow \ h = 2.744\ cm$$

Hence, height of the cylinder is 2.744 cm.

Que 12. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Let r be the radius of resulting sphere.

We have,

Volume of resulting sphere = Sum of the volumes of three gives spheres

$$\Rightarrow \ \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3$$

$$\Rightarrow \ \frac{4}{3}\pi r^3 = \frac{4}{3}\pi[(16)^3 + (8)^3 + (10)^3] \quad \Rightarrow \ r^3 = (6)^3 + (8)^3 + (10)^3$$

$$\Rightarrow \ r^3 = 216 + 512 + 1000 \quad \Rightarrow \ r^3 = 1728 = (12)^3$$

$\therefore \ r = 12\ cm$

Hence, the radius of the resulting sphere is 12 cm.

Que 13. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Sol. Here, radius of cylindrical well = $\frac{7}{2}$ m

Depth of cylindrical well = 20 m

Let H metre be the required height of the platform.

Now, the volume of the platform = Volume of the earth dugout from the cylindrical well.

$$i.e., 22 \times 14 \times H = \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 770\ \text{m}^3$$

$$\Rightarrow \quad H = \frac{770}{22 \times 14} = \frac{5}{2} = 2.5\ m$$

∴ Height of the platform = 2.5 m

Que 14. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimension 5.5 cm × 3.5 cm?

Sol. We have,

Radius of coin = $\frac{1.75}{2} = 0.875\ cm$

And thickness i.e., height = 2 mm = $\frac{2}{10}$ cm = 0.2 cm

The shape of a coin will be like the shape of cylinder

∴ Volume of the coin = $\pi r^2 h = \frac{22}{7} \times 0.875 \times 0.875 \times 0.2$

Now, Volume of the cuboid = 5.5 × 10 × 3.5

∴ Number of coins required to form a cuboid = $\frac{Volume\ of\ the\ cuboid}{Volume\ of\ the\ coin}$

$$= \frac{5.5 \times 10 \times 3.5}{\frac{22}{7} \times 0.875 \times 0.875 \times 0.2} = 400$$

Que 15. A copper rod of a diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Sol. The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \times 8\text{cm}^3 = 2\pi\text{cm}^3$

The length of the new wire of the same volume = 18 m = 1800 cm
If r is the radius (in cm) of cross-section of the wire, its volume = $\pi \times r^2 \times 1800\ \text{cm}^3$
Therefore, $\pi \times r^2 \times 1800 = 2\pi$

$$i.e., r^2 = \frac{1}{900} \qquad i.e., r^2 = \frac{1}{30}$$

So, the diameter of the cross section, i.e., the thickness of the wire is $\frac{1}{15}$ cm.
i.e., 0.67 mm (approx.).

Que 16. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameter of its to circular ends are 4 cm and 2 cm. Find the capacity of the glass.

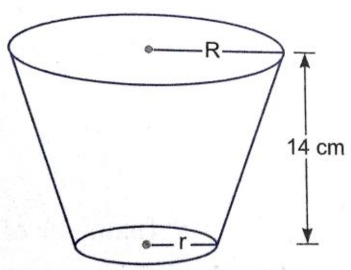

Fig. 13.18

Sol. We have, R = 2 cm, r = 1 cm, h = 14 cm

∴ Capacity of the glass = Volume of the frustum

$$= \frac{1}{2} \pi h (R^2 + r^2 + R)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \times [(2)^2 + (1)^2 + (2 \times 1)]$$

$$= \frac{44}{3} \times (4 + 1 + 2) = \frac{44}{3} \times 7 = \frac{308}{3} = 102\frac{2}{3}\ cm^3$$

Long Answer Type Questions

[4 MARKS]

Que 1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

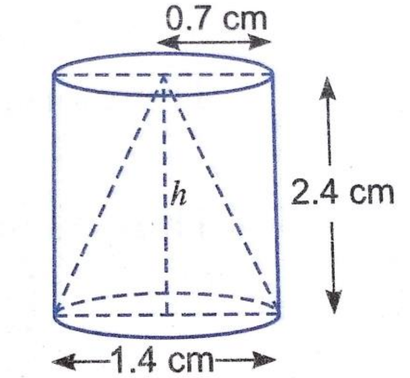

Fig. 13.23

Sol. We have,

Radius of the cylinder = $\frac{1.4}{2} = 0.7\ cm$

Height of the cylinder = 2.4 cm

Also. Radius of the cone = 0.7 cm
and height of the cone = 2.4 cm

Now, slant height of the cone = $\sqrt{(0.7)^2 + (2.4)^2}$

$\Rightarrow \quad l = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5\ cm$

$\therefore$ Total surface area of the remaining solid
= Curved surface area of cylinder + Curved surface area of the cone + Area of upper circular base of cylinder

$= 2\pi rh + \pi rl + \pi r^2 = \pi r\,(2h + l + r)$

$= \frac{22}{7} \times 0.7 \times [2 \times 2.4 + 2.5 + 0.7] = 22 \times 0.1 \times (4.8 + 2.5 + 0.7)$

$= 2.2 \times 8.0 = 17.6\ cm^2 \approx 18\ cm^2$

Que 2. The decorative block shown in figure is made of two solids – a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. $\left(\textbf{Take}\,\pi = \frac{22}{7}\right)$.

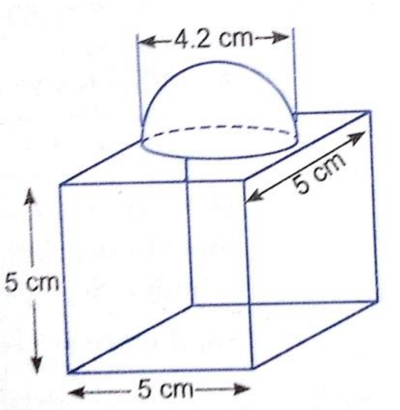

Fig. 13.24

Sol. The total surface area of the cube = $6 \times (\text{edge})^2$

$= 6 \times 5 \times 5\ cm^2 = 150\ cm^2$

∴ Total surface area of the block

= Total surface area of cube – Base area of hemisphere + Curved surface area of hemisphere

$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) cm^2$

$= \left(150 + \frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right)\ cm^2 = (150 + 13.86)\ cm^2\ = 163.86\ cm^2$

Que 3. Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (Fig. 13.25). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. $\left(\text{Take} \pi = \frac{22}{7}\right)$

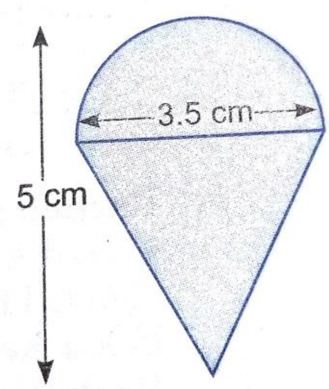

Fig. 13.25

Sol. Total surface area of the top

= Curved surface area of hemisphere + Curved surface area of cone.

Now, the curved surface area of hemisphere = $2\pi r^2$

$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ cm^2$

Also, the height of the cone = Height of the top – Height (radius) of the hemispherical part

$= \left(5 - \frac{3.5}{2}\right)\ cm\ = 3.25\ cm$

So, the slant height of the cone (l)

$$= \sqrt{r^2 + h^2} = \sqrt{\frac{3.5}{2}^2 + (3.25)^2} cm = 3.7\ cm\ (approx)$$

Therefore, curved surface area of cone = $\pi rl = \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) cm^2$

Thus, the surface area of the top

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) cm^2 + \left(\frac{22}{7} \times \frac{3.5}{2} 3.7\right) cm^2$$

$$= \frac{22}{7} \times \frac{3.5}{2} (3.5 + 3.7)\ cm^2 = \frac{22}{7} \times \frac{3.5}{2} \times 7.2\ cm^2$$

$$= 39.6\ cm^2 \text{ (approx.)}.$$

Que 4. A wooden toy rocket is in the shape of a mounted on a cylinder, in Fig. 13.26. The height of the entire rocket is 26 cm, while the of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π = 3.14)

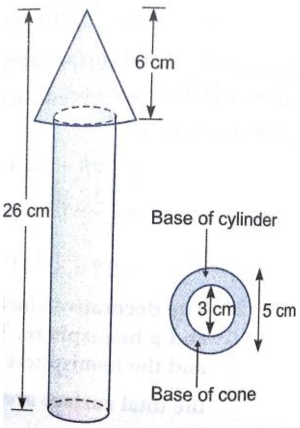

Fig. 13.26

Sol. Denote radius, slant height and height of cone by r, l and h, respectively, and radius and height of cylinder by r' and h', respectively. Then r = 2.5 cm, h = 6 cm, r' = 1.5 cm, h' = 26 – 6 = 20 cm and

$$l = \sqrt{r^2 + h^2} = \sqrt{(2.5)^2 + (6)^2} = 6.5\ cm$$

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange

= Curved surface area of the cone + Base area of the cone – Base area of the cylinder

$$= \pi rl + \pi r^2 - \pi(r')^2$$

$$= \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2] cm^2$$

$$= \pi[20.25] cm^2 = 3.14 \times 20.25\ cm^2 = 63.585\ cm^2$$

Now, the area to be painted yellow

= Curved surface area of the cylinder + Area of one base of the cylinder

$= 2\pi r' h' + \pi(r')^2 = \pi r'(2h' + r')$

$= (3.14 \times 1.5)(2 \times 20 + 1.5)\ cm^2 = 4.71 \times 41.5\ cm^2 = 195.465\ cm^2$

Que 5. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

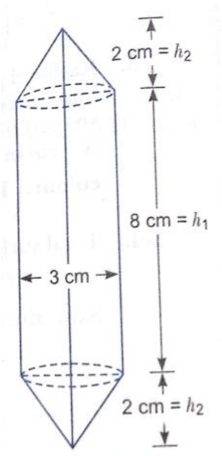

Fig. 13..27

Sol. Here, radius of cylindrical portion = $\frac{3}{2}\ cm$

Height of each cone = 2 cm

Height of cylindrical portion = 12 – 2 – 2 = 8 cm

Volume of the air contained in the model

= Volume of the cylindrical portion of the model + Volume of two conical ends.

$= \pi r^2 h^1 + 2 \times \frac{1}{3}\pi r^2 h^2 = \pi r^2 \left(h^1 + \frac{2}{3}h^2\right)$

$= \pi \times \left(\frac{3}{2}\right)^2 \times \left(8 + \frac{2}{3} \times 2\right) = \frac{22}{7} \times \frac{9}{4} \times \frac{28}{3} = 66\ \text{cm}^3$

Que 6. A gulab jamun, contains sugar syrup about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (Fig. 13.28).

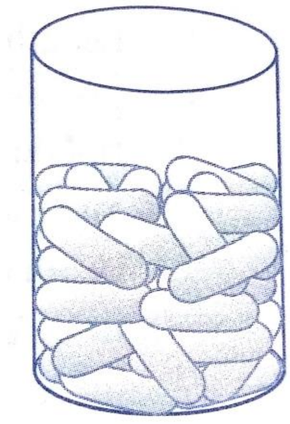

Fig. 13.28

Sol. We have,

Radius of cylindrical portion and hemispherical portion of a gulab jamun

$$= \frac{2.8}{2} = 1.4\ cm$$

Length of cylindrical portion = 5 – 1.4 – 1.4 = 2.2 cm

Volume of one gulab jamun

= Volume of the cylindrical portion + Volume of the hemispherical ends

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 \ = \pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{4}{3} r\right) \ = \frac{22}{7} \times (1.4)^2 \left(22 + \frac{4}{3} \times 1.4\right)$

$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(\frac{6.6+5.6}{3}\right) \ = \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$

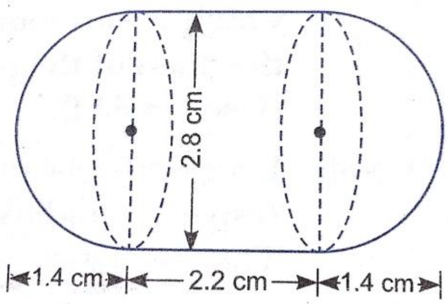

Fig. 13.29

∴ Volume of 45 gulab jamuns = $45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$

∴ Quantity of syrup in 45 gulab jamuns = 30% of their Volume

$$= \frac{30}{100} \times 45 \times \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

= 338.184 cm^3 = 338 cm^3 (approx.)

Que 7. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the

volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)

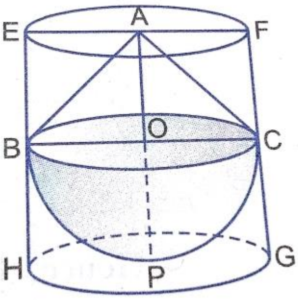

Fig. 13.30

Sol. Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see Fig. 13.30).

The radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2} \times 4\ cm\ =\ 2\ cm$

So, Volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \left[\frac{2}{3} \times 3.14 \times (2)^3\ + \frac{1}{2} \times 3.14 \times (2)^2 \times 2\right]\ cm^3\ = 25.12\ cm^3$$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = HP = BO = 2 cm and its height is
EH = AO + OP = (2 + 2) cm = 4 cm
So, the required Volume

= Volume of the right circular cylinder – Volume of the toy

= $(3.14 \times 2^2 \times 4 - 25.12)\ cm^3\ = 25.12\ cm^3$

Hence, the required difference of the two volumes = 25.12 cm^3.

Que 8. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (Fig. 13.31).

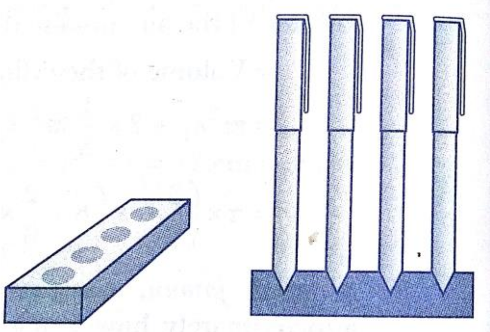
Fig. 13.31

Sol. We have,

Length of cuboid = l = 15 cm

Breadth of cuboid = b = 10 cm
Height of cuboid = h = 3.5 cm
And radius of conical depression = 0.5 cm
Depth of conical depression = 1.4 cm
Now, Volume of wood in the entire pen stand

= Volume of cuboid – 4 × Volume of a conical depression

$= lbh - 4 \times \frac{1}{3}\pi r^2 h \;= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= (525 - 1.47)\ cm^3 \;= 523.53\ cm^2$

Que 9. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. $(\boldsymbol{Use\ \pi = 3.14})$.

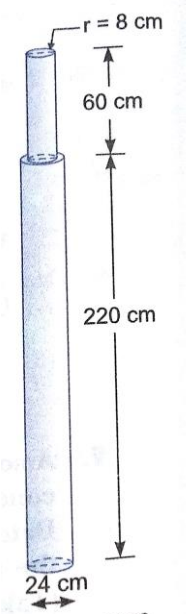

Fig. 13.32

Sol. Let r_1 and h_1 be the radius and height of longer cylinder, respectively, and r_2, h_2 be the respective radius and height of smaller cylinder mounted on the longer cylinder.
Then we have,

$r_1 = 12$ cm, $h_1 = 220$ cm
$r_2 = 8$ cm, $h_2 = 60$cm

Now, Volume of solid iron pole

= Volume of the longer cylinder + Volume of smaller cylinder
$= \pi {r_1}^2 h_1 \;+\; \pi {r_2}^2$
$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$
$= 3.14 \times 144 \times 220 + 3.14 \times 64 \times 60$
= 99475.2 + 12057.6 = 111532.8 cm_3

Hence, the mass of the pole = (111532.8 x 8) grams

$$= \frac{111532.8 \times 8}{1000}\ kg \;=\; 892.2624\ kg$$

Que 10. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

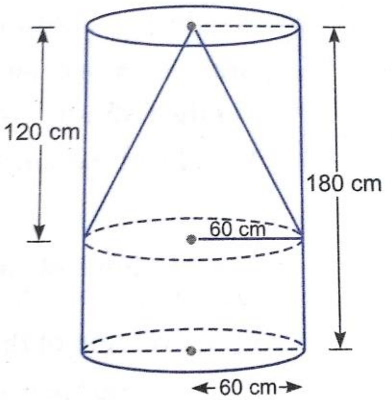

Fig. 13.33

Sol. We have,

Radius of cylinder = Radius of cone = Radius of hemisphere = 60 cm

Height of cone = 120 cm

∴ Height of cylindrical vessel = 120 + 60 = 180 cm

Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times (60)^2 \times 180$

Now, volume of the solid

= Volume of cone + Volume of hemisphere

$= \frac{1}{3} \times \frac{22}{7} \times (60)^2 \times 120 + \frac{2}{3} \times \frac{22}{7} \times (60)^3$

Volume of water left in the cylinder

= Volume of the cylinder – Volume of the solid

$= \frac{22}{7} \times (60)^2 \times 180 - \frac{1}{3} \times \frac{22}{7}(60)^2 \times 120 - \frac{2}{3} \times \frac{22}{7} \times (60)^3$

$= \frac{22}{7}(60)^2 \left[180 - \frac{1}{3} \times 120 - \frac{2}{3} \times 60\right]$

$= \frac{22}{7} \times 3600[180 - 40 - 40] = \frac{22 \times 3600 \times 100}{7}$

$= \frac{7920000}{7} = \frac{792}{700} m^3 = 1.131\ m^3$ (approx.)

HOTS (Higher Order Thinking Skills)

Que 1. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to from an embankment. Find the height of the embankment.

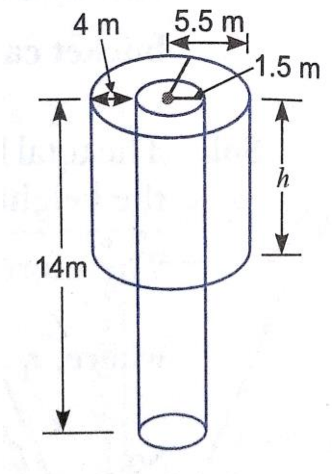

Fig. 13.40

Sol. Here, radius of the well $= \frac{3}{5} = 1.5\ m$

Depth of the well = 14 m
Width of the embankment = 4 m
∴ Radius of the embankment = 1.5 + 4 = 5.5 m
Let h be the height of the embankment.
∴ Volume of the embankment
= Volume of the well (Cylinder)

$\Rightarrow\ \pi[(5.5)^2 - (1.5)^2] \times h = \pi(1.5)^2 \times 14$

$\Rightarrow\ (30.25 - 2.25) \times h = 2.25 \times 14$

$\Rightarrow\ h = \frac{2.25\times14}{28} = 1.125\ m$

Que 2. A hollow cone is cut by a plane parallel to the base and upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

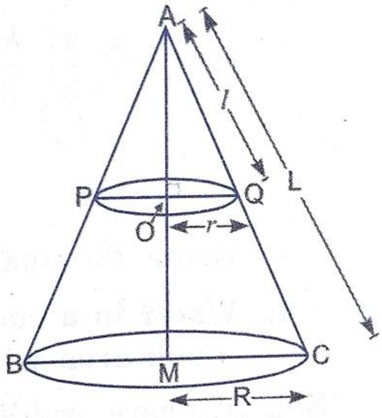

Fig. 13.41

Sol. In Fig. 13.41, the smaller cone APQ has been cut off through the plane PQ||BC. Let r and R be the radii of the smaller and larger cone and I and L their slant heights respectively.

Here, in the adjoining figure

OQ = r, MC = R, AQ = I, AC = L.

Now, $\Delta AOQ \sim \Delta AMC$

$$\Rightarrow \quad \frac{OQ}{MC} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{I}{L} \quad \Rightarrow \quad r = \frac{RI}{L} \quad \ldots(i)$$

Since, curved surface area of the remainder = $\frac{8}{9}$ of the curved surface area of the whole cone, therefore, we get,

CSA of smaller cone = $\frac{1}{9}$ of the CSA of the whole cone

$$\therefore \quad \pi rl = \frac{1}{9}\pi RL$$

$$\Rightarrow \quad \pi\left(\frac{RI}{L}\right) I = \frac{1}{9}(\pi RL) \quad \text{[Using (i)]}$$

$$\Rightarrow \quad I^2 = \frac{L^2}{9} \quad \Rightarrow \quad \frac{I}{L} = \frac{1}{3}$$

Now again in similar triangles, AOQ and AMC, we have

$$\frac{AO}{AM} = \frac{AQ}{AC} \quad \Rightarrow \quad \frac{AO}{AM} = \frac{I}{L} = \frac{1}{3} \quad \Rightarrow \quad AO = \frac{AM}{3}$$

$$\Rightarrow \quad OM = AM - OA = AM - \frac{AM}{3} = \frac{2}{3}AM$$

$$\therefore \quad \frac{AO}{OM} = \frac{AM/3}{2AM/3} = \frac{1}{2}$$

Hence, the required ratio of their heights = 1: 2

Que 3. The height of the cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the given cone, at what height above the base is the section made?

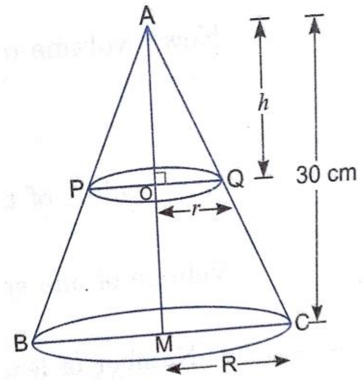

Fig. 13.42

Sol. Let the small cone APQ be cut off at the top by the plane PQ||BC.
Let r and h be the radius and height of the smaller cone, respectively and also let the radius of larger cone = R.

Now, $\Delta AOQ \sim \Delta AMC$

$$\Rightarrow \quad \frac{AO}{AM} = \frac{OQ}{MC}$$

$$\Rightarrow \quad \frac{h}{30} = \frac{r}{R} \qquad \therefore \; r = \frac{hR}{30} \qquad \ldots(i)$$

Since, it is given that volume of the smaller cone = $\frac{1}{27}$ (Volume of larger cone)

$$\frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2.30\right)$$

$$\Rightarrow \quad \frac{\pi}{3}\left(\frac{hR}{30}\right)^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2.30 \qquad \text{[From (i)]}$$

$$\Rightarrow \quad h^3 = \frac{30\times30\times30}{27} = 1000 \qquad \therefore \; h = 10\ cm$$

Hence, the smaller cone has been cut off at a height of (30 – 10) cm = 20 cm from the base.

Que 4. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. We have, width of the canal = 6 m

Depth of the canal = 1.5 m

Now, length of water flowing per hour = 10 km

∴ Length of water flowing in half hour = 5 km = 5,000 m

∴ Volume of water flow in 30 minutes = 1.5 × 6 × 5,000 = 45,000 m^3

Here, standing water needed is 8 cm = 0.08 m

∴ Area irrigated in 30 minutes = $\frac{Volume}{Height} = \frac{45{,}000}{0.08}$

$= 562500 \text{ m}^2 \qquad [1\ hectare = 10000\ m^2]$

$= 56.25$ hectares

Que 5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots in the vessel.

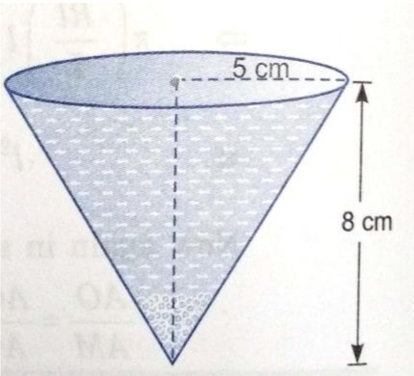

Fig. 13.43

Sol. We have,

Height of conical vessel = h = 8 cm

and its radius = r = 5 cm

Now, volume of cone = Volume of water in the cone

$$= \frac{1}{2}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 = \frac{4{,}400}{21} \text{ cm}^3$$

Now, Volume of water flows out = Volume of lead shots

$$= \frac{1}{4} \times Volume\ of\ water\ in\ the\ cone = \frac{1}{4} \times \frac{4{,}400}{21} = \frac{1{,}100}{21} cm^3$$

Now, radius of the lead shots = 0.5 cm = $\frac{5}{10} cm = \frac{1}{2} cm$

Volume of one spherical lead shot = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{11}{21} \text{cm}^3$

∴ Number of lead shots dropped in the vessel = $\frac{Volume\ of\ water\ flows\ out}{Volume\ of\ one\ lead\ shot}$

$$= \frac{\frac{1{,}100}{21}}{\frac{11}{21}} = \frac{1{,}100}{21} \times \frac{21}{11} = 100$$

Value Based Questions

Que 1. A manufacture involved ten children in colouring playing top (lattu) which is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area they had to paint if 50 playing tops were given to them. $\left(\boldsymbol{Take}\ \pi = \frac{22}{7}\right)$

(a) How is child labour an abuse for the society?

(b) What steps can be taken to abolish child labour?

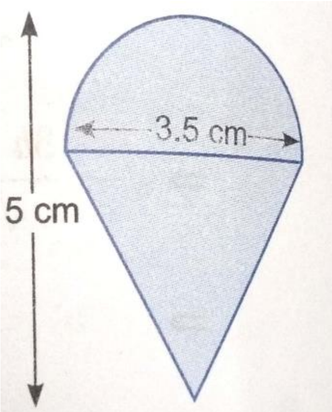

Fig. 9

Sol. This top is exactly like the object in Fig. 9.

∴ Total surface area of the top = Curved surface area of hemisphere + Curved surface area of cone

Now, the curved surface area of hemisphere = $2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ cm^2$$

Also, the height of the cone

= Height of the top – Height (radius) of the hemisphere part

$$= \left(5 - \frac{3.5}{2}\right)\ cm = 3.25\ cm$$

So, the slant height of the cone (l) = $\sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2} cm = \ 3.7\ cm\ (approx)$$

Therefore, curved surface area of cone πrl = $\left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right)$ cm²

Thus, the surface area of the top = $\left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ cm^2\ +\ \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right)\ cm^2$

$$= \frac{22}{7} \times \frac{3.5}{2}(3.5 + 3.7)\ cm^2\ = \frac{22}{7} \times \frac{3.5}{2} \times 7.2\ cm^2$$

= 39.6 cm² (approx.)

Surface area to be painted = $50 \times 39.6\ cm^2 = 1980\ cm^2$

(a) Children are the future of any society or country and they possess various talents. So, providing them opportunities to grow be giving proper education instead of involving them in work will help in the development of society. With education and proper nurture of their talent, they can contribute in a better way for the development of society and the country.

(b) (i) Spreading awareness against child labour in the society.
(ii) Abolishing the use of products involving child labour.
(iii) Providing free education at elementary level to poor children.
(iv) Enforcing the law to abolish child labour.

Que 2. A child prepares a poster on 'Save Energy' on a square sheet whose each side measures 60 cm. At each corner of the sheet, she draws a quadrant of radius 17.5 cm in which she shows the ways to save energy. At the centre, she draws a circle of diameter 21 cm and writes a slogan in it. Find the area of the remaining sheet.
(a) Write down the four ways by which energy can be saved.
(b) Write a slogan on 'Save Energy'.
(c) Why do we need to save energy?

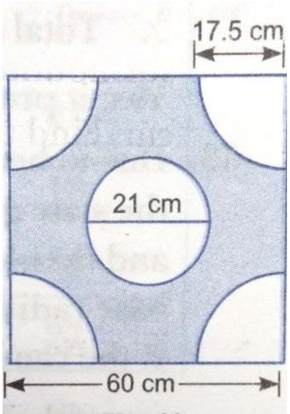

Fig. 10

Sol. Area of the square = 60 x 60 = 3600 cm^2

Area of the remaining sheet = $3600 - \pi\left(\frac{21}{2}\right)^2 - 4 \times \frac{\pi}{4} \times (17.5)^2$

$$= 3600 - \pi\left(\frac{441}{4} + \frac{1225}{4}\right)$$

$$= 3600 - \frac{22}{7} \times \frac{1666}{4} = 3600 - 1309 = 2291\ cm^2$$

(a) (i) Saving electricity by using CFLs, switching off appliances when not in use.
(ii) Saving water by using it efficiently.
(iii) Saving petroleum resources by using public transport.
(iv) Using solar energy.

(b) ‘Save energy, Save Environment’ or any other given by students.

(c) We should save energy to save our environment so that we can give a better tomorrow to the forth coming generations.

Que 3. A teacher brings clay in the classroom to teach the topic' mensuration'. She forms a cylinder of radius 6 cm and height 8 cm with the clay. Then she moulds that cylinder into a sphere. Find the radius of the sphere formed.
Do teaching aids enhance teaching learning process? Justify your answer.

Sol. Volume of the cylinder formed = Volume of the sphere

$\Rightarrow \quad \pi(6)^2 \times 8 = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{6^2 \times 8 \times 3}{4} = r^3$

$\Rightarrow \quad r^3 = 6^3$ or r = 6 cm

Yes, teaching aids make the learning practical, interesting, easy to learn and leave long lasting impact.

Que 4. A night camp was organised for Class X students for two days and their accommodation was planned in tents. Each tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas). Is camping helpful to students in their development? Justify your answer.

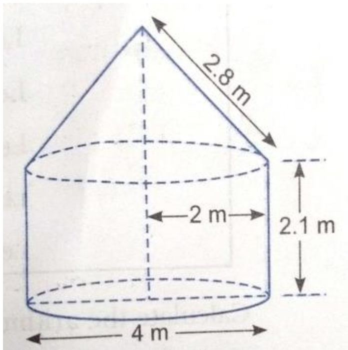

Fig. 11

Sol. We have, Radius of cylindrical base = $\frac{4}{2} = 2\ m$

Height of cylindrical portion = 2.1 m

∴ Curved surface area of cylindrical portion = 2 πrh

$= 2 \times \frac{22}{7} \times 2 \times 2.1$

$= 26.4\ m^2$

Radius of conical base (r) = 2 m

Slant height of conical portion (l) = 2.8 m

∴ Curved surface area of conical = πrl

$= \frac{22}{7} \times 2 \times 2.8$

= 17.6 m^2

Now, total area of the canvas = (26.4 + 17.6) m^2 = 44 m^2

∴ Total cost of the canvas used = ₹ 500 × 44 = ₹ 22,000

Yes, it provides the feeling of self-confidence, sharing, caring and other social values.

Que 5. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem? $\left(Use\ \pi = \frac{22}{7}\right)$

Sol. Slant height of conical part = $\sqrt{(2.8)^2 + (2.1)^2} = 3.5\ m$

Area of canvas/tent = $2\pi rh + \pi rl$

$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5\ m^2$$

$$= \frac{22}{7} \times 2.8 \times 3.5(2 + 1)$$

$$= 3 \times \frac{22}{7} \times 2.8 \times 3.5$$

$$= 92.4\ m^2$$

Cost of 1500 tents = 1500 × 92.5 × 120 = ₹ 1,66,32,000

Share of each school = $\frac{1}{50} \times 16632000 = ₹\ 3,32,640$

Values: Helping the needy people