RD Sharma Solutions Class 12 Maths Chapter 28 Ex 28.1

Straight Line in Space Ex 28.1 Q1

Vector equation of a line

is $\vec{r} = \vec{a} + \lambda\vec{b}$

The Cartesian equation of a line is

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{x - x_3}{a_3}$$

Using the above formula,

Vector equation of the line,

$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

The Cartesian equation of the line

$$\frac{x - 5}{3} = \frac{y - 2}{2} = \frac{z + 4}{-8}$$

Straight Line in Space Ex 28.1 Q2

The direction ratios of the line are

$(3 + 1, 4 - 0, 6 - 2) = (4, 4, 4)$

Since the line passes through $(-1, 0, 2)$

The vector equation of the line,

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

$\therefore$ The vector equation of the line,

$\vec{r} = (-\vec{i} + 0\vec{j} + 2\vec{k}) + \lambda(4\vec{i} + 4\vec{j} + 4\vec{k})$

Straight Line in Space Ex 28.1 Q3

We know that, vector equation of line passing through a fixed point $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar}$$

Here, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

So, equation of required line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, so

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (5 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 5 + 2\lambda,\ y = -2 - \lambda,\ z = 4 + 3\lambda$$

$$\Rightarrow \quad \frac{x-5}{2} = \lambda,\ \frac{y+2}{-0} = \lambda,\ \frac{z-4}{3} = \lambda$$

Cortesian form of equation of the line is,

$$\frac{x-5}{2} = \frac{y+2}{-0} = \frac{z-4}{3}$$

Straight Line in Space Ex 28.1 Q4

We know that, equation of line passing through a vector $\vec{a}$ and parallel to a vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar,}$$

Here, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

Required equation of line is,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$$

On equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$\Rightarrow \quad 2 + 3\lambda = x,\ -3 + 4\lambda = y,\ 4 - 5\lambda = z$$

$$\Rightarrow \quad \frac{x-2}{3} = \lambda,\ \frac{y+3}{4} = \lambda,\ \frac{z-4}{-5} = z$$

So, cortesian form of equation of the line is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

Straight Line in Space Ex 28.1 Q5

$ABCD$ is a parallelogram.

$\Rightarrow$ AC and BD bisect each other at point O (say).

Position vector of point $O = \dfrac{\vec{a}+\vec{c}}{2}$

$$= \frac{\left(4\hat{i}+5\hat{j}-10\hat{k}\right)+\left(-\hat{i}+2\hat{j}+\hat{k}\right)}{2}$$

$$= \frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}$$

Let position vector of point O and B are represented by $\vec{o}$ and $\vec{b}$.

Equation of the line BD is the line passing through O and B is given by

$$\vec{r} = \vec{a} + \lambda\left(\vec{b}-\vec{a}\right) \qquad \left[\begin{array}{l}\text{Since equation of the line passing through} \\ \text{two points } \vec{a} \text{ and } \vec{b}\end{array}\right]$$

$$\vec{r} = \vec{b} + \lambda\left(\vec{o}-\vec{b}\right)$$

$$= \left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(\frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}-2\hat{i}-3\hat{j}+4\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}-9\hat{k}-4\hat{i}+6\hat{j}-8\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(-\hat{i}+13\hat{j}-17\hat{k}\right)$$

Put $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right) = (2-\lambda)\hat{i}+(-3+13\lambda)\hat{j}+(4-17\lambda)\hat{k}$$

Equation the coefficients of $\hat{i},\hat{j},\hat{k}$, so

$\Rightarrow$ $x = 2-\lambda,\ y = -3-13\lambda,\ z = 4-17\lambda$

$\Rightarrow$ $\dfrac{x-2}{-1} = \lambda,\ \dfrac{y+3}{13} = \lambda,\ \dfrac{z-4}{-17} = \lambda$

So equation of the line BD in cortesian form,

$$\frac{x-2}{-1} = \frac{y+3}{13} = \frac{z-4}{-17}$$

Straight Line in Space Ex 28.1 Q6

We know that, equation of line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = A(1, 2, -1)$
$(x_2, y_2, z_2) = B(2, 1, 1)$

Using equation (i), equation of line AB,

$$\frac{x-1}{2-1} = \frac{y-2}{1-2} = \frac{z+1}{1+1}$$

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2} = \lambda \text{ (say)}$$

$$x = \lambda + 1,\ y = -\lambda + 2,\ z = 2\lambda - 1$$

Vector form of equation of line AB is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (-\lambda + 2)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

Straight Line in Space Ex 28.1 Q7

We know that vector equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, required vector equation of line is,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Now,

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (1 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

Equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$\Rightarrow \quad x = 1 + \lambda,\ y = 2 - 2\lambda,\ z = 3 + 3\lambda$

$\Rightarrow \quad x - 1 = \lambda,\ \frac{y-2}{2} = \lambda,\ \frac{z-3}{3} = \lambda$

So, required equation of line is cortesian form,

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$$

Straight Line in Space Ex 28.1 Q8

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (2, -1, 1)$ and

Given line $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$ is prallel to required line.

$\Rightarrow \quad a = 2\mu,\ b = 7\mu,\ c = -3\mu$

So, equation of required line using equation (i),

$$\frac{x-2}{2\mu} = \frac{y+1}{7\mu} = \frac{z-1}{-3\mu}$$

$\Rightarrow \quad \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3} = \lambda$ (say)

$\Rightarrow \quad x = 2\lambda + 2,\ y = 7\lambda - 1,\ z = -3\lambda + 1$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (2\lambda + 2)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} - 3\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q9

The Cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} \qquad \ldots(1)$$

The given line passes through the point (5, −4, 6). The position vector of this point is $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector, $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

This is the required equation of the given line in vector form.

Straight Line in Space Ex 28.1 Q10

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, -1, 2)$ and

Given line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$ is prallel to required line, so

$$\Rightarrow \qquad a = \mu,\ b = 2\mu,\ c = -2\mu$$

So, equation of required line using equation (i) is,

$$\frac{x-1}{\mu} = \frac{y+1}{2\mu} = \frac{z-2}{-2\mu}$$

$$\Rightarrow \qquad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{-2} = \lambda \text{ (say)}$$

$$x = \lambda + 1,\ y = 2\lambda - 1,\ z = -2\lambda + 2$$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (2\lambda - 1)\hat{j} + (-2\lambda + 2)\hat{k}$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q11

Given, line is,

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

$$\Rightarrow \qquad \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$$x = -2\lambda + 4,\ y = 6\lambda,\ z = -3\lambda + 1$$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (-2\lambda + 4)\hat{i} + (6\lambda)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(4\hat{i} + \hat{k}\right) + \lambda\left(-2\hat{i} + 6\hat{j} - 3\hat{k}\right)$$

Direction ratios of the line are $= -2, 6, -3$

Direction cosines of the line are,

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}},\ \frac{b}{\sqrt{a^2 + b^2 + c^2}},\ \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\Rightarrow \qquad \frac{-2}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}},\ \frac{6}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}},\ \frac{-3}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}}$$

$$\Rightarrow \qquad \frac{-2}{7},\ \frac{6}{7},\ \frac{-3}{7}$$

Straight Line in Space Ex 28.1 Q12

$x = ay + b,$

$z = cy + d$

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} = \lambda(say)$$

So DR's of line are (a, 1, c)

From above equation, we can write

$x = a\lambda + b$

$y = \lambda$

$z = c\lambda + d$

So vector equation of line is

$$x\hat{i} + y\hat{j} + z\hat{k} = (b\hat{i} + d\hat{k}) + \lambda\left(a\hat{i} + \hat{j} + c\hat{k}\right)$$

Straight Line in Space Ex 28.1 Q13

We know that, equation of a line passing through $\bar{a}$ and parallel to vector $\bar{b}$ is,

$$\vec{r} = \bar{a} + \lambda\bar{b} \qquad ---(i)$$

Here, $\bar{a} = \hat{i} - 2\hat{j} - 3\hat{k}$

and, $\bar{b}$ = line joining $\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

$= \left(2\hat{i} + \hat{j} + 2\hat{k}\right) - \left(\hat{i} - \hat{j} + 4\hat{k}\right)$

$= 2\hat{i} - \hat{i} + \hat{j} + \hat{j} + 2\hat{k} - 4\hat{k}$

$= \hat{i} + 2\hat{j} - 2\hat{k}$

Equation of the line is

$$\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

For cortesion form of equation put $x\hat{i} + y\hat{j} + z\hat{k}$,

$$x\hat{i} + y\hat{j} + z\hat{k} = (1+\lambda)\hat{i} + (-2+2\lambda)\hat{j} + (-3-2\lambda)\hat{k}$$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 1 + \lambda,\ y = -2 + 2\lambda,\ z = -3 - 2\lambda$$

$$\Rightarrow \quad \frac{x-1}{1} = \lambda,\ \frac{y+2}{2} = \lambda,\ \frac{z+3}{-2} = \lambda$$

So, $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z+3}{-2}$

Straight Line in Space Ex 28.1 Q14

Distance of point P from $Q = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$PQ = \sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$$

$\Rightarrow \quad (5)^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2$

$\Rightarrow \quad 25 = 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2$

$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0$

$\Rightarrow \quad 17\lambda(\lambda - 2) = 0$

$\Rightarrow \quad \lambda = 0 \text{ or } 2$

So, points on the line are $(3(0) - 2,\ 2(0) - 1,\ 2(0) + 3)$

$(3(2) - 2,\ 2(2) - 1,\ 2(2) + 3)$

$= (-2, -1, 3),\ (4, 3, 7)$

Straight Line in Space Ex 28.1 Q15

Let the given points are A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, so

$\vec{a} = -2\hat{i} + 3\hat{j},\ \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{c} = 7\hat{i} - \hat{k}$

We know that, equation of a line passing through $\vec{a}$ and $\vec{b}$ are,

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$= (-2\hat{i} + 3\hat{j}) + \lambda((\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j}))$

$= (-2\hat{i} + 3\hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j})$

$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} + 3\hat{k}) \quad --- (i)$

If A, B, C are collinear then $\vec{c}$ must satisfy equation (i),

$7\hat{i} - \hat{k} = (-2 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + (3\lambda)\hat{k}$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$-2 + 3\lambda = 7 \quad \Rightarrow \lambda = 3$

$3 - \lambda = 0 \quad \Rightarrow \lambda = 3$

$3\lambda = -1 \quad \Rightarrow \lambda = -\frac{1}{3}$

Since, value of λ are not equal, so,

Given points are not collinear.

Straight Line in Space Ex 28.1 Q16

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, 3)$ and

Given line $\frac{-x - 2}{1} = \frac{y + 3}{7} = \frac{2z - 6}{3}$

$$\Rightarrow \quad \frac{x + 2}{-1} = \frac{y + 3}{7} = \frac{z - 3}{\frac{3}{2}}$$

It parallel to the required line, so

$$a = \mu,\ b = 7\mu,\ c = \frac{3}{2}\mu$$

So, equation of required line using equation (i) is,

$$\frac{x - 1}{-\mu} = \frac{y - 2}{7\mu} = \frac{z - 3}{\frac{3}{2}\mu}$$

$$\Rightarrow \quad \frac{x - 1}{-1} = \frac{y - 2}{7} = \frac{z - 3}{\frac{3}{2}}$$

Straight Line in Space Ex 28.1 Q17

Given equation of line is,

$$3x + 1 = 6y - 2 = 1 - z$$

Dividing all by 6,

$$\frac{3x+1}{6} = \frac{6y-2}{6} = \frac{1-z}{6}$$

$$\Rightarrow \quad \frac{3x}{6} + \frac{1}{6} = \frac{6y}{6} - \frac{2}{6} = \frac{1}{6} - \frac{z}{6}$$

$$\Rightarrow \quad \frac{1}{2}x + \frac{1}{6} = y - \frac{1}{3} = -\frac{z}{6} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{2}\left(x + \frac{1}{3}\right) = 1\left(y - \frac{1}{3}\right) = +\frac{1}{6}(z-1)$$

$$\Rightarrow \quad \frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda \ (\text{say}) \qquad ---(\text{i})$$

Comparing it with equation of line passing through (x_1, y_1, z_1) and direction ratios a, b, c,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \quad (x_1, y_1, z_1) = \left(-\frac{1}{3}, \frac{1}{3}, 1\right)$$

$$a = 2,\ b = 1,\ -6$$

So, direction ratios of the line are $= 2, 1, -6$

From equation (i),

$$x = \left(2\lambda - \frac{1}{3}\right),\ y = \left(\lambda + \frac{1}{3}\right),\ z = (-6\lambda + 1)$$

So, vector equation of the given line is,

$$x\hat{i} + y\hat{j} + z\hat{k} = \left(2\lambda - \frac{1}{3}\right)\hat{i} + \left(\lambda + \frac{1}{3}\right)\hat{j} + (-6\lambda + 1)\hat{k}$$

$$\vec{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right)$$