RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.2

The Plane 29.2 Q1

Given, intercepts on the coordinate axes are 2, – 3 and 4

We know that,
The equation of a plane whose intercepts on the coordinate axes are a, b and c respectively, is given by

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(i)$$

Here, $a = 2$, $b = -3$, $c = 4$

So,
Equation of required plane is

$$\frac{x}{2}+\frac{y}{-3}+\frac{z}{4}=1$$

$$\frac{6x-4y+3z}{12}=1$$

$$6x-4y+3z=12$$

The Plane 29.2 Q2(i)

Reduce the equation $4x+3y-6z-12=0$ in intercept form:

$$4x+3y-6z-12=0$$
$$4x+3y-6z=12$$

Divide by 12,

$$\frac{4x}{12}+\frac{3y}{12}-\frac{6z}{12}=\frac{12}{12}$$
$$\frac{x}{3}+\frac{y}{4}-\frac{z}{2}=1$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{(-2)}=1 \qquad ---(i)$$

This is of the form,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a=3,\ b=4,\ c=-2$

Intercepts on the coordinate axes are $3, 4, -2$

The Plane 29.2 Q2(ii)

Reduce $2x+3y-z=6$ in the intercept form:

$$2x+3y-z=6$$

Divide by 6,

$$\frac{2x}{6}+\frac{3y}{6}-\frac{z}{6}=\frac{6}{6}$$
$$\frac{x}{3}+\frac{y}{2}-\frac{z}{6}=1$$

$$\frac{x}{3}+\frac{y}{2}+\frac{z}{(-6)}=1 \qquad ---(i)$$

We know intercept form of plane with a, b, c as intercepts on coordinate axes is,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a=3,\ b=2,\ c=-6$

So, intercepts on coordinate axes by the given plane are $3, 2, -6$

The Plane 29.2 Q2(iii)

We have to find intercepts on coordinate axes by plane $2x - y + z = 5$

$$2x - y + z = 5$$

Divide by 5,

$$\frac{2x}{5} - \frac{y}{5} + \frac{z}{5} = \frac{5}{5}$$

$$\frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{(-5)} + \frac{z}{5} = 1 \qquad ---(i)$$

We know that if a, b, c are intercepts on coordinate axes by the plane, then equation of such plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$$a = \frac{5}{2}, \ b = -5, \ c = 5$$

So, intercepts on coordinate axes by the plane are $\frac{5}{2}, -5, 5.$

The Plane 29.2 Q3

Here, it is given that the plane meets axes in A, B and C

Let, $A = (a,0,0)$, $B = (0,b,0)$, $C = (0,0,c)$

We have centroid of ▯ABC is (α, β, γ) we know that, centroid of ▯ABC is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(\alpha, \beta, \gamma) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

So,

$$\frac{a}{3} = \alpha \quad \Rightarrow \quad a = 3\alpha \quad ---(i)$$

$$\frac{b}{3} = \beta \quad \Rightarrow \quad b = 3\beta \quad ---(ii)$$

$$\frac{c}{3} = \gamma \quad \Rightarrow \quad c = 3\gamma \quad ---(iii)$$

We know that, if a, b, c are intercepts by plane on coordinate axes, then equation of the plane is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Put a, b, c from equation (i), (ii) and (iii),

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$$

Multiplying by 3 on both the sides,

$$\frac{3x}{3\alpha} + \frac{3y}{3\beta} + \frac{3z}{3\gamma} = 3$$

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

The Plane 29.2 Q4

Intercepts on the coordinate axes are equal.

We know that, if a, b, c are intercepts on coordinate axes by a plane, then equation of the plane is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Here, it is givin that $a=b=c=p$ (Say)

$$\frac{x}{p}+\frac{y}{p}+\frac{z}{p}=1$$
$$\frac{x+y+z}{p}=1$$

$$x+y+z=p \qquad ---(i)$$

It is given that plane is passing through the point $(2,4,6)$, so, using equation (i)

$$x+y+z=p$$
$$2+4+6=p$$

$$12=p$$

Put, value of p in equation (i)

$$x+y+z=12$$

So, the required equation of the plane is given by,

$$x+y+z=12$$

The Plane 29.2 Q5

Here, it is given that plane meets the coordinate axes at A, B and C with centroid of $\triangle ABC$ is $(1,-2,3)$

The equation of plane with intercepts a, b and c on the coordinate axes is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(\text{i})$$

We know that, centroid of a triangle is given by

$$\text{Centroid} = \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}$$

$$(1,-2,3)=\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(1,-2,3)=\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

Comparing LHS and RHS,

$$\frac{a}{3}=1 \Rightarrow \qquad a=3 \qquad ---(\text{i})$$

$$\frac{b}{3}=-2 \Rightarrow \qquad b=-6 \qquad ---(\text{ii})$$

$$\frac{c}{3}=3 \Rightarrow \qquad c=9 \qquad ---(\text{iii})$$

Put, a, b, c is equation (i), we get the equation of required plane

$$\frac{x}{3}+\frac{y}{-6}+\frac{z}{9}=1$$

$$\frac{6x-3x+2z}{18}=1$$

$$6x-3x+2z=18$$