RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.3

The Plane 29.3 Q1

We know that, vector equation of a plane passing through a point $\bar{a}$ and normal to $\bar{n}$ is given by,

$$\left(\vec{r} - \bar{a}\right).\bar{n} = 0 \qquad ---(i)$$

Here,

$\bar{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\bar{n} = 4\hat{i} + 2\hat{j} - 3\hat{k}$

Put, $\bar{a}$ and $\bar{n}$ in equation (i)

$$\left[\vec{r} - \left(2\hat{i} - \hat{j} + \hat{k}\right)\right].\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$

$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + \hat{k}\right).\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$

$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[(2)(4) + (-1)(2) + (1)(-3)\right] = 0$$

$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[8 - 2 - 3\right] = 0$$

$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - 3 = 0$$

So, equation of required plane is given by,

$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 3$$

The Plane 29.3 Q2(i)

Given the vector equation of a plane,

$$\vec{r}.\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$

let, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$
$$(x)(12) + (y)(-3) + (z)(4) + 5 = 0$$

$$12x - 3y + 4z + 5 = 0$$

Cartesian form of the equation of the plane is given by

$$12x - 3y + 4z + 5 = 0$$

The Plane 29.3 Q2(ii)

Here, equation of the plane is,

$$\vec{r}.\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$

let, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$
$$(x)(-1) + (y)(1) + (z)(2) = 9$$

$$-x + y + 2z = 9$$

Cartesian form of the equation of plane is,

$$-x + y + 2z = 9$$

The Plane 29.3 Q3

We have to find vector equation of coordinate planes.
For xy-plane.

It passes through origin and is perpendicular to z-axis, so
Put $\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{k}$ in the vector equation of plane passing through point $\vec{a}$ and perpendicular to vector $\vec{n}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{k} = 0$$

$$\vec{r}.\hat{k} = 0 \qquad \text{---(i)}$$

For xz-plane,
It passes through origin and perpendicular to y-axis, so

$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{j}$

Equation of xz-plane is given by
$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{j} = 0$$

$$\vec{r}.\hat{j} = 0$$

For yz-plane.

It passes through origin and is perpendicular to x-axis, so
$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$, $\vec{n} = \hat{i}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{i} = 0$$

$$\vec{r}.\hat{i} = 0$$

Hence, equation of xy, yz, zx-plane are given by

$$\vec{r}.\hat{k} = 0$$
$$\vec{r}.\hat{i} = 0$$
$$\vec{r}.\hat{j} = 0$$

The Plane 29.3 Q4(i)

Given, equation of plane is,

$2x - y + 2z = 8$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

So,

Vector equation of the plane is $\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

The Plane 29.3 Q4(ii)

Given, cartesian equation of the plane is,

$x + y - z = 5$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

$\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

So,

Vector equation of the plane is $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

The Plane 29.3 Q4(iii)

Given, cartesian equation of plane is,

$x + y = 3$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j}\right) = 3$

$\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

So,

Vector equation of the plane is $\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

The Plane 29.3 Q5

We know that, vector equation of a plane passing through point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---(\text{i})$$

The given plane is passing through the point $(1, -1, 1)$ and normal to the line joining $A(1, 2, 5)$ and $B(-1, 3, 1)$. So,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = \overrightarrow{AB}$

= Position vector of $\vec{B}$ − Position vector of A

$= \left(-\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$

$= -\hat{i} + 3\hat{j} + \hat{k} - \hat{i} - 2\hat{j} - 5\hat{k}$

$= -2\hat{i} + \hat{j} - 4\hat{k}$

Put, $\vec{n}$ and $\vec{a}$ in equation (i),

$$\left[\vec{r} - \left(\hat{i} - \hat{j} + \hat{k}\right)\right].\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$$

$$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left(\hat{i} - \hat{j} + \hat{k}\right)\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$$

$$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[(1)(-2) + (-1)(1) + (1)(-4)\right] = 0$$

$$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-2 - 1 - 4\right] = 0$$

$$\vec{r}\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-7\right] = 0$$

$$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) + 7 = 0$$

$$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = -7$$

Multiplying by (-1) on both the sides

$$\vec{r}\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$$

$$(x)(2) + (y)(-1) + (2)(4) = 7$$

$$2x - y + 4z = 7$$

So, vector and cartesian equation the plane is

$$\vec{r}.\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7,\ 2x - y + 4z = 7$$

The Plane 29.3 Q6

Here, it is given that $\bar{n} = \sqrt{3}$ and $\bar{n}$ makes equal angle with coordinate axes.

Let, $\bar{n}$ has direction cosine as l, m and n and it makes angle of α, β and γ with the coordinate axes. So

Here, $\alpha = \beta = \gamma$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n = p\text{(Say)}$

We know that,

$$l^2 + m^2 + n^2 = 1$$

$$p^2 + p^2 + p^2 = 1$$

$$3p^2 = 1$$

$$p^2 = \frac{1}{3}$$

$$p = \pm\frac{1}{\sqrt{3}}$$

So,

$$l = \pm\frac{1}{\sqrt{3}}$$

$$\cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Now, $\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

It gives, α is an obtuse angle so, neglect it.

Again, $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

It gives, α is an acute angle, so

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

$$\therefore \quad l = m = n = \frac{1}{\sqrt{3}}$$

So,

$$\vec{n} = |\vec{n}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$$

$$= \sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$\vec{n} = \hat{i} + \hat{j} + \hat{k}$$

And, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

We know that, vector equation of a plane passing through the point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$

$$\left[\vec{r} - \left(2\hat{i} + \hat{j} - \hat{k}\right)\right].\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$$

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[(2)(1) + (1)(1) + (-1)(1)\right] = 0$$

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[2 + 1 - 1\right] = 0$$

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - 2 = 0$$

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$$

Put, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$$

$$(x)(1) + (y)(1) + (z)(1) = 2$$

$$x + y + z = 2$$

So, vector and cartesian equation of the plane is,

$$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2,\ x + y + z = 2$$

The Plane 29.3 Q7

Here, it is given that foot of the perpendicular drawn from origin O to the plane is $P(12,-4,3)$

It means, the required plane is passing through $P(12,-4,3)$ and perpendicular to OP.

We know that, equation of a plane passing through $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=12\hat{i}-4\hat{j}+3\hat{k}$

And, $\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P – Position vector of O

$$=\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$

$$\vec{n}=12\hat{i}-4\hat{j}+3\hat{k}$$

Put, value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\right].\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[(12)(12)+(-4)(-4)+(3)(3)\right]=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[144+16+9\right]=0$$

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$

$$(x)(12)+(y)(-4)+(z)(3)=169$$

$$12x-4y+3z=169$$

So, the vector and cartesian equation of the required plane is,

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=169,\ 12x-4y+3z=169$$

The Plane 29.3 Q8

Given that, the plane is passing through $P(2,3,1)$ having 5,3,2 as the direction ratios of the normal to the plane.

We know that,
Equation of a plane passing through a point $\vec{a}$ and $\vec{n}$ is a vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

So, $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$
$\vec{n}=5\hat{i}+3\hat{j}+2\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\right]\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[(2)(5)+(3)(3)+(1)(2)\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[10+9+2\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$
$$(x)(5)+(y)(3)+(z)(2)=21$$

$$5x+3y+2z=21$$

The Plane 29.3 Q9

Here, given that P is the point $(2,3,-1)$ and required plane is passing through P at right angles to OP

It means, the plane is passing through P and OP is the vector normal to the plane.

We know that, equation of a plane, passing through a point $\vec{a}$ and $\vec{n}$ is vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$

$\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P $-$ Position vector of O

$=\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\vec{n}=2\hat{i}+3\hat{j}-\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right].\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0$$

$$\vec{r}\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}-\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[(2)(2)+(3)(3)+(-1)(-1)\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[4+9+1\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-14=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

$$(x)(2)+(y)(3)+(z)(-1)=14$$

$$2x+3y-z=14$$

Equation of required plane is,

$$2x+3y-z=14$$

The Plane 29.3 Q10

Here, given equation of plane is,

$2x + y - 2z = 3$

Dividing by 3 on both the sides,

$$\frac{2x}{3} + \frac{y}{3} - \frac{2z}{3} = \frac{3}{3}$$

$$\frac{x}{\frac{3}{2}} + \frac{y}{3} + \frac{z}{-\frac{3}{2}} = 1 \qquad ---(i)$$

We know that, if a, b, c are the intercepts by a plane on the coordinate axes, new equation of the plane is,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$a = \frac{3}{2}, b = 3, c = -\frac{3}{2}$

Again, given equation of plane is,

$$2x + y - 2z = 3$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

$$\vec{r}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

So, vector normal to the plane is given by

$$\bar{n} = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\left|\bar{n}\right| = \sqrt{(2)^2 + (1)^2 + (-2)^2}$$

$$= \sqrt{4 + 1 + 4}$$

$$= \sqrt{9}$$

$$\left|\bar{n}\right| = 3$$

Direction vector of $\bar{n} = 2, 1, -2$

Direction vector of $\bar{n} = \frac{2}{\left|\bar{n}\right|}, \frac{1}{\left|\bar{n}\right|}, \frac{-2}{\left|\bar{n}\right|}$

$$= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$$

So,

Intercepts by the plane on coordinate axes are $= \frac{3}{2}, 3, -\frac{3}{2}$

Direction cosine of normal to the plane are $= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

The Plane 29.3 Q11

Here, given that, the required plane passes through the point $(1,-2,5)$ and is perpendicular to the line joining origin O to the point $P\left(3\hat{i}+\hat{j}-\hat{k}\right)$.

We know that, equation of a plane passing through a point $\bar{a}$ and perpendicular to a vector $\bar{n}$ is given by,

$$\left(\vec{r}-\bar{a}\right).\bar{n}=0 \qquad ---(i)$$

Here, $\bar{a}=\hat{i}-2\hat{j}+5\hat{k}$

$\bar{n}=\overrightarrow{OP}$

$=$ Position vector of P – Position vector of O

$=\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\bar{n}=3\hat{i}+\hat{j}-\hat{k}$

Put, the value of $\bar{a}$ and $\bar{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\right]\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[(1)(3)+(-2)(1)+(5)(-1)\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[3-2-5\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[-4\right]=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)+4=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$

The Plane 29.3 Q12

We have to find the equation of plane that bisects $A(1,2,3)$ and $B(3,4,5)$ perpendicularly

We know that, equation of a plane passing through the point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}$ = mid-point of AB

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$= \frac{\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}+4\hat{j}+5\hat{k}}{2}$$

$$\vec{a} = \frac{4\hat{i}+6\hat{j}+8\hat{k}}{2}$$

$$\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$$

And, $\vec{n} = \overrightarrow{AB}$

$= \text{Position vector of } B - \text{Position vector of } A$

$= \left(3\hat{i}+4\hat{j}+5\hat{k}\right) - \left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$= 3\hat{i}+4\hat{j}+5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$

$$\vec{n} = 2\hat{i}+2\hat{j}+2\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}-\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[(2)(2)+(3)(2)+(4)(2)\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[4+6+8\right]=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-18=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=18$$

The Plane 29.3 Q13(i)

Given, two equation of plane are,

$x - y + z - 2 = 0$ and
$3x + 2y - z + 4 = 0$

$x - y + z = 2$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right) = 2$
$\vec{r}.\vec{n_1} = 2$ --- (i)

$3x + 2y - z = -4$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}.\vec{n_2} = -4$ --- (i)

From equation (i) and (ii), we get that
$\vec{n_1}$ is normal to equation (i) and
$\vec{n_2}$ is normal to equation (ii).

Now,
$\vec{n_1}.\vec{n_2} = \left(\hat{i} - \hat{j} + \hat{k}\right).\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$
$= (1)(3) + (-1)(2) + (1)(-1)$
$= 3 - 2 - 1$
$= 3 - 3$

$= 0$

$\vec{n_1}.\vec{n_2} = 0$

So, $\vec{n_1}$ is perpendicular to $\vec{n_2}$

The Plane 29.3 Q13(ii)

Given, two vector equation of plane are,

$\vec{r}.\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 5$

$\vec{r}.\overrightarrow{n_1} = 5$

So, $\overrightarrow{n_1} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

And, $\vec{r}.\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$

$\vec{r}.\overrightarrow{n_2} = 5$

So, $\overrightarrow{n_2} = 2\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $\overrightarrow{n_1}.\overrightarrow{n_2}$

$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right)$

$= (2)(2) + (-1)(-2) + (3)(-2)$

$= 4 + 2 - 6$

$= 6 - 6$

$= 0$

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$

Hence, normals to planes $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ are perpendicular.

The Plane 29.3 Q14

Given, equation of plane is,

$$2x + 2y + 2z = 3$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$$
$$\vec{r}.\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$$

$$\vec{r}.\vec{n} = d$$

Normal to the plane $\vec{n} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

Direction ratio of $\vec{n} = 2,2,2$

Direction cosine of $\vec{n} = \frac{2}{|\vec{n}|}, \frac{2}{|\vec{n}|}, \frac{2}{|\vec{n}|}$

$$|\vec{n}| = \sqrt{(2)^2 + (2)^2 + (2)^2}$$
$$= \sqrt{4+4+4}$$
$$= \sqrt{12}$$

$$|\vec{n}| = 2\sqrt{3}$$

Direction cosine of $|\vec{n}| = \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

So, $l = \frac{1}{\sqrt{3}}$, $m = \frac{1}{\sqrt{3}}$, $n = \frac{1}{\sqrt{3}}$

Let, α, β, γ be the angle that normal $\vec{n}$ makes with the coordinate axes respectively.

$$l = \cos\alpha = \frac{1}{\sqrt{3}}$$
$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad ---\text{(i)}$$

$$m = \cos\beta = \frac{1}{\sqrt{3}}$$
$$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad ---\text{(ii)}$$

$$n = \cos\gamma = \frac{1}{\sqrt{3}}$$
$$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad ---\text{(iii)}$$

From equation (i), (ii) and (iii),

$\alpha = \beta = \gamma$

So, normal to the plane, $\vec{n}$ is equally inclined with the coordinate axes.

The Plane 29.3 Q15

Given, equation of plane is,

$12x - 3y + 4z = 1$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$

$\vec{r}.\vec{n} = 1$

So, normal to the plane is

$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$

$= \sqrt{144 + 9 + 16}$

$= \sqrt{144 + 25}$

$= 169 = 13$

Unit vector $\hat{n} = \dfrac{12\hat{i} - 3\hat{j} + 4\hat{k}}{13}$

$= \dfrac{12\hat{i}}{13} - \dfrac{3}{13}\hat{j} + 4\hat{k}$

A vector normal to the plane with magnitude

$26 = 26\hat{n}$

$= 26\left(\dfrac{12\hat{i}}{13} - 3\hat{j} + 4\hat{k}\right)$

Required vector $= 24\hat{i} - 6\hat{j} + 8\hat{k}$

The Plane 29.3 Q16

Given that, line drawn from $A(4,-1,2)$ meets a plane at right angle, at the point $B(-10,5,4)$.

We know that,
Equation of a plane passing through the point $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}$ = Position vector B

$$\vec{a}=-10\hat{i}+5\hat{j}+4\hat{k}$$

$$\begin{aligned}\vec{n} &= \overrightarrow{AB}\\ &= \text{Position vector of } B - \text{Position vector of } A\\ &= \left(-10\hat{i}+5\hat{j}+4\hat{k}\right)-\left(4\hat{i}-\hat{j}+2\hat{k}\right)\\ &= -10\hat{i}+5\hat{j}+4\hat{k}-4\hat{i}+\hat{j}-2\hat{k}\end{aligned}$$

$$\vec{n}=-14\hat{i}+6\hat{j}+2\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(-10\hat{i}+5\hat{j}+4\hat{k}\right)\right].\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}.\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left(-10\hat{i}+5\hat{j}+4\hat{k}\right)\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left[(-10)(-14)+(5)(6)+(4)(2)\right]=0$$
$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left[140+30+8\right]=0$$
$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-178=0$$

$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=178$$

The Plane 29.3 Q17

We have to find the equation of plane which bisects the line joining the points $A(-1,2,3)$ and $B(3,-5,6)$ at right angles.

Let, C be the mid-point of AB

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here, $\vec{a}$ = Position vector of C

= Mid-point of A and B

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$\vec{a} = \frac{-\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}-5\hat{j}+6\hat{k}}{2}$$

$$= \frac{2\hat{i}}{2} - \frac{3\hat{j}}{2} + \frac{9\hat{k}}{2}$$

$$\vec{a} = \hat{i} - \frac{3}{2}\hat{j} + \frac{9}{2}\hat{k}$$

$\vec{n} = \overrightarrow{AB}$

= Position vector of B – Position vector of A

$$= \frac{\left(3\hat{i}-5\hat{j}+6\hat{k}\right)-\left(-\hat{i}+2\hat{j}+3\hat{k}\right)}{2}$$

$$= \frac{3\hat{i}-5\hat{j}+6\hat{k}+\hat{i}-2\hat{j}-3\hat{k}}{2}$$

$$= \frac{4\hat{i}-7\hat{j}+3\hat{k}}{2}$$

$$= \frac{4}{2}\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

$$\vec{n} = 2\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\right]\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[(1)(2)+\left(-\frac{3}{2}\right)\left(-\frac{7}{2}\right)+\left(\frac{9}{2}\right)\left(+\frac{3}{2}\right)\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[2+\frac{21}{4}+\frac{27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[\frac{29+27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\frac{56}{4}=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

$$(x)(2)+(y)\left(-\frac{7}{2}\right)+(z)\left(+\frac{3}{2}\right)-14=0$$

$$2x-\frac{7y}{2}+\frac{3z}{2}-14=0$$

$$\frac{4x-7y+3z-28}{2}=0$$

$$4x-7y+3z=28$$

Equation of required plane is,

$$4x-7y+3z=28$$

The Plane Ex 29.3 Q18

Vector equation of the plane:

Given that the required plane passes through the point (5, 2, − 4) having the position vector
$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

Also given that the required plane is perpendicular to the line with direction ratios 2, 3 and − 1.

Thus the vector equation of the normal vector to the plane is $\vec{n} = 2\hat{i} + 3\hat{j} - \hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = (5\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 10 + 6 + 4$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

The Cartesian equation of the plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 2$

$\Rightarrow 2x + 3y - z = 20$

The Plane Ex 29.3 Q19

Consider the point $P(1, 2, -3)$.

Thus the position vector of the point P is

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

Direction ratios of the line OP, where O is the origin, are 1, 2 and -3

Thus the vector equation of the normal vector, OP, to th

plane is $\vec{n} = \hat{i} + 2\hat{j} - 3\hat{k}$.

We know that the vector equation of the plane passing

through a point having position vector $\vec{a}$ and normal to

vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$$\vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 1 + 4 + 9$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$$

$$\Rightarrow x + 2y - 3z = 14$$

The Plane Ex 29.3 Q20

O is the origin and the coordinates of A are (a, b, c).

$\overrightarrow{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

∴ The direction the direction ratios of OA are proportional to, a, b, c.

∴ Direction cosines are,

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

The equation of the line passing through A(a, b, c) and perpendicular to $\overline{OA}$

$$\left\{x\hat{i} + y\hat{j} + z\hat{k} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \bullet a\hat{i} + b\hat{j} + c\hat{k} = 0$$

$$ax + by + cz = a^2 + b^2 + c^2$$