RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.5

Dividing by 4,

$$3x - 4z + 1 = 0$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(3\hat{i} + 0\hat{j} - 4\hat{k}\right) + 1 = 0$$

$$\vec{r}.\left(3\hat{i} - 4\hat{k}\right) + 1 = 0$$

Equation of the required plane,

$$\vec{r}.\left(3\hat{i} - 4\hat{k}\right) + 1 = 0$$

The Plane Ex 29.5 Q2

Let P(2,5, −3), Q(−2, −3,5) and R(5,3, −3) be the three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane. Therefore, $\overrightarrow{PQ} \times \overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$

$$\overrightarrow{PQ} = (-2-2)\hat{i} + (-3-5)\hat{j} + (5-(-3))\hat{k}$$
$$\Rightarrow \overrightarrow{PQ} = -4\hat{i} - 8\hat{j} + 8\hat{k}$$

Similarly,

$$\overrightarrow{PR} = (5-2)\hat{i} + (3-5)\hat{j} + (-3-(-3))\hat{k}$$
$$\Rightarrow \overrightarrow{PR} = 3\hat{i} - 2\hat{j} + 0\hat{k}$$

Thus

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}$$
$$= 16\hat{i} + 24\hat{j} + 32\hat{k}$$

The plane passes through the point P with position vector $\vec{p} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

Thus, its vector equation is

$$\{\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})\} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) - (32 + 120 - 96) = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) - 56 = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 56$$
$$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$$

The Plane Ex 29.5 Q3

Let A(a, 0, 0), B(0, b, 0) and C(0, 0, c) be three points on a plane having their position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same plane.
Therefore, $\overrightarrow{AB} \times \overrightarrow{AC}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$

$$\overrightarrow{AB} = (0-a)\hat{i} + (b-0)\hat{j} + (0-0)\hat{k}$$
$$\Rightarrow \overrightarrow{AB} = -a\hat{i} + b\hat{j} + 0\hat{k}$$

Similarly,

$$\overrightarrow{AC} = (0-a)\hat{i} + (0-0)\hat{j} + (c-0)\hat{k}$$
$$\Rightarrow \overrightarrow{AC} = -a\hat{i} + 0\hat{j} + c\hat{k}$$

Thus

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}$$
$$\vec{n} = bc\hat{i} + ac\hat{j} + ab\hat{k}$$
$$\Rightarrow \hat{n} = \frac{bc\hat{i} + ac\hat{j} + ab\hat{k}}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}$$

The plane passes through the point P with position vector $\vec{a} = a\hat{i} + 0\hat{j} + 0\hat{k}$

Thus, the vector equation in the normal form is

$$\{\vec{r} - \left((a\hat{i} + 0\hat{j} + 0\hat{k}\right)\} \cdot \left(\frac{bc\hat{i} + ac\hat{j} + ab\hat{k}}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}\right) = 0$$
$$\Rightarrow \vec{r} \cdot \frac{(bc\hat{i} + ac\hat{j} + ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{abc}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}$$
$$\Rightarrow \vec{r} \cdot \frac{\left(bc\hat{i} + ac\hat{j} + ab\hat{k}\right)}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{1}{\sqrt{\frac{b^2c^2 + a^2c^2 + a^2b^2}{a^2b^2c^2}}}$$
$$\Rightarrow \vec{r} \cdot \frac{(bc\hat{i} + ac\hat{j} + ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \ldots(1)$$

The vector equation of a plane normal to the unit vector $\hat{n}$ and at a distance 'd' from the origin is $\vec{r} \cdot \hat{n} = d \ldots(2)$
Given that the plane is at a distance 'p' from the origin.
Comparing equations (1) and (2), we have,

$$d = p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$
$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

The Plane Ex 29.5 Q4

Let P(1, 1, − 1), Q(6, 4, − 5) and R(− 4, − 2, 3) be three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ} \times \overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$

$\overrightarrow{PQ} = (6 - 1)\hat{i} + (4 - 1)\hat{j} + (-5 - (-1))\hat{k}$
$\Rightarrow \overrightarrow{PQ} = 5\hat{i} + 3\hat{j} - 4\hat{k}$
Similarly,
$\overrightarrow{PR} = (-4 - 1)\hat{i} + (-2 - 1)\hat{j} + (3 - (-1))\hat{k}$
$\Rightarrow \overrightarrow{PR} = -5\hat{i} - 3\hat{j} + 4\hat{k}$
Thus
Here, $\overrightarrow{PQ} = -\overrightarrow{PR}$
Therefore, the given points are collinear.
Thus, $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$ where, $5a + 3b - 4c = 0$
The plane passes through the point P with position vector $\vec{p} = \hat{i} + \hat{j} - \hat{k}$
Thus, its vector equation is
$\{\vec{r} - (\hat{i} + \hat{j} - \hat{k})\} \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = 0$, where, $5a + 3b - 4c = 0$

The Plane Ex 29.5 Q5

Let, A, B, C be the points with position vector $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$, $\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ and $\left(7\hat{i}+6\hat{k}\right)$ respectively. Then

$\overrightarrow{AB}$ = Position vector of B – Poosition vector of A

$$= \left(2\hat{i}-2\hat{j}-\hat{k}\right)-\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$$
$$= 2\hat{i}-2\hat{j}-\hat{k}-3\hat{i}-4\hat{j}-2\hat{k}$$

$$= -\hat{i}-6\hat{j}-3\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Poosition vector of B

$$= \left(7\hat{i}+6\hat{k}\right)-\left(2\hat{i}-2\hat{j}-\hat{k}\right)$$
$$= 7\hat{i}+6\hat{k}-2\hat{i}+2\hat{j}+\hat{k}$$

$$\overrightarrow{BC} = 5\hat{i}+2\hat{j}+7\hat{k}$$

A vector normal to A, B, C is a vector perpendicular to $\overrightarrow{AB}\times\overrightarrow{BC}$

$$\vec{n} = \overrightarrow{AB}\times\overrightarrow{BC}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -6 & -3 \\ 5 & 2 & 7 \end{vmatrix}$$

$$\vec{n} = \hat{i}\left(-42+6\right)-\hat{j}\left(-7+15\right)+\hat{k}\left(-2+30\right)$$

$$= -36\hat{i}-8\hat{j}+28\hat{k}$$

We know that, equation of a plane passing through vector $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-36\hat{i}-8\hat{j}+28\hat{k}\right) = \left(3\hat{i}+4\hat{j}+2\hat{k}\right)\left(-36\hat{i}-8\hat{j}+28\hat{k}\right)$$

$$= (3)(-36)+(4)(-8)+(2)(28)$$
$$= -108-32+56$$
$$= -140+56$$

$$\vec{r}.\left(-36\hat{i}-8\hat{j}+28\hat{k}\right) = -84$$

Dividing by (-4), we get

$$\vec{r}.\left(9\hat{i}+2\hat{j}-7\hat{k}\right) = 21$$

Equation of required plane is,

$$\vec{r}.\left(9\hat{i}+2\hat{j}-7\hat{k}\right) = 21$$