RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.6

The Plane 29.6 Q1(i)

Given equation of two planes are

$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1 \qquad ---(i)$$

$$\vec{r}.\left(-\hat{i} + \hat{j}\right) = 4 \qquad ---(ii)$$

We know that, angle between two planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, $\vec{n_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{n_2} = -\hat{i} + \hat{j}$

$$\cos\theta = \frac{\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)\left(-\hat{i} + \hat{j}\right)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2}}$$

$$= \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{4 + 9 + 16}\sqrt{1 + 1}}$$

$$= \frac{-2 - 3 + 0}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane 29.6 Q1(ii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 6 \qquad ---(i)$$

$$\vec{r}.\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right) = 9 \qquad ---(ii)$$

We know that, angle between the planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, from equation (i) and (ii),

$\vec{n_1} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{n_2} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta = \frac{\left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)}{\sqrt{(2)^2 + (-1)^2 + (2)^2}\sqrt{(3)^2 + (6)^2 + (-2)^2}}$$

$$= \frac{(2)(3) + (-1)(6) + (2)(-2)}{\sqrt{4 + 1 + 4}\sqrt{9 + 36 + 4}}$$

$$\cos\theta = \frac{6 - 6 + 4}{\sqrt{9}\sqrt{49}}$$

$$= \frac{-4}{3.7}$$

$$= \frac{-4}{21}$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

The Plane 29.6 Q1(iii)

Given equation of planes are

$\vec{r}.\left(2\hat{i}+3\hat{j}-6\hat{k}\right)=5$ --- (i)

$\vec{r}.\left(\hat{i}-2\hat{j}+2\hat{k}\right)=9$ --- (ii)

We know that, angle between equation of planes

$\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \quad \text{--- (iii)}$$

From equation (i) and (ii),

$\vec{n_1}=2\hat{i}+3\hat{j}-6\hat{k}$

$\vec{n_2}=\hat{i}-2\hat{j}+2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta=\frac{\left(2\hat{i}+3\hat{j}-6\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{(2)^2+(3)^2+(-6)^2}\sqrt{(1)^2+(-2)^2+(2)^2}}$$

$$=\frac{(2)(1)+(3)(-2)+(-6)(2)}{\sqrt{4+9+36}\sqrt{1+4+4}}$$

$$=\frac{2-6-12}{\sqrt{49}\sqrt{9}}$$

$$=\frac{-16}{7.3}$$

$$=\frac{-16}{21}$$

$$\theta=\cos^{-1}\left(\frac{-16}{21}\right)$$

The Plane 29.6 Q2(i)

Given, equation of planes are,

$$2x - y + z = 4 \quad \text{---(i)}$$
$$x + y + 2z = 3 \quad \text{---(ii)}$$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{---(iii)}$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -1,\ c_1 = 1$
$a_2 = 1,\ b_2 = 1,\ c_2 = 2$

Put then values in equation (iii),

$$\cos\theta = \frac{(2)(1) + (-1)(1) + (1)(2)}{\sqrt{(2)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (1)^2 + (2)^2}}$$
$$= \frac{2 - 1 + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}$$
$$\cos\theta = \frac{4 - 1}{\sqrt{6}\sqrt{6}}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$
$$\theta = \frac{\pi}{3}$$

The Plane 29.6 Q2(ii)

Given, equation of two planes are,

$$x + y - 2z = 3 \qquad ---(i)$$
$$2x - 2y + z = 5 \qquad ---(ii)$$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 1,\ b_1 = 1,\ c_1 = -2$
$a_2 = 2,\ b_2 = -2,\ c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(2) + (1)(-2) + (-2)(1)}{\sqrt{(1)^2 + (1)^2 + (-2)^2}\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$\cos\theta = \frac{2 - 2 - 2}{\sqrt{1 + 1 + 4}\sqrt{4 + 4 + 1}}$$

$$= \frac{-2}{\sqrt{6}\sqrt{9}}$$

$$= \frac{-2}{3\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{-2}{3\sqrt{6}}\right)$$

The Plane 29.6 Q2(iii)

Given, equation of planes are,

$x - y + z = 5$ ---(i)

$x + 2y + z = 9$ ---(ii)

We know that, angle between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{---(iii)}$$

From equation (i) and (ii),

$a_1 = 1,\ b_1 = -1,\ c_1 = 1$

$a_2 = 1,\ b_2 = 2,\ c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(1) + (-1)(2) + (1)(1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (2)^2 + (1)^2}}$$

$$\cos\theta = \frac{1 - 2 + 1}{\sqrt{1 + 1 + 1}\sqrt{1 + 4 + 1}}$$

$$= \frac{0}{\sqrt{3}\sqrt{6}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

The Plane 29.6 Q2(iv)

Given, equation of planes are,

$2x - 3y + 4z = 1$ --- (i)

$-x + y = 4$ --- (ii)

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -3,\ c_1 = 4$
$a_2 = -1,\ b_2 = 1,\ c_2 = 0$

Put these values in equation (iii),

$$\cos\theta = \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2 + (0)^2}}$$

$$\cos\theta = \frac{-2 - 3 + 0}{\sqrt{4 + 9 + 16}\sqrt{1 + 1 + 0}}$$

$$= \frac{-5}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane Ex 29.6 Q2(v)

We know that the angle between the planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Therefore, the angle between $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

$$\cos\theta = \frac{2 \times 3 + 1 \times (-6) + (-2) \times (-2)}{\sqrt{2^2 + 1^2 + (-2)^2} \cdot \sqrt{3^2 + (-6)^2 + (-2)^2}}$$

$$\Rightarrow \cos\theta = \frac{6 - 6 + 4}{\sqrt{9} \cdot \sqrt{9 + 36 + 4}}$$

$$\Rightarrow \cos\theta = \frac{4}{3 \times 7}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

The Plane 29.6 Q3(i)

Given equation of planes are

$$\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 5 \qquad ---(i)$$
$$\vec{r}.\left(-\hat{i} - \hat{j} + \hat{k}\right) = 3 \qquad ---(ii)$$

We know that, planes

$\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ are perpendicular

if $\overrightarrow{n_1}.\overrightarrow{n_2} = 0 \qquad ---(iii)$

From equation (i) and (ii),

$$\overrightarrow{n_1} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\overrightarrow{n_2} = -\hat{i} - \hat{j} + \hat{k}$$

Put $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ in equation (iii),

$$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$$
$$\left(2\hat{i} - \hat{j} + \hat{k}\right)\left(-\hat{i} - \hat{j} + \hat{k}\right) = 0$$
$$(2)(-1) + (-1)(-1) + (1)(1) = 0$$
$$-2 + 1 + 1 = 0$$

$$0 = 0$$
$$LHS = RHS$$

Hence, planes are at right angle

The Plane 29.6 Q3(ii)

Given, equation of planes are,

$x - 2y + 4z = 10$
$18x + 17y + 4z = 49$

$\Rightarrow \quad x - 2y + 4z - 10 = 0 \quad$ ---(i)
$18x + 17y + 4z - 49 = 0 \quad$ ---(ii)

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are at right angles if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

From (i) and (ii),
$a_1 = 1, b_1 = -2, c_1 = 4$
$a_2 = 18, b_2 = 17, c_2 = 4$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(1)(18) + (-2)(17) + (4)(4) = 0$
$18 - 34 + 16 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angles

The Plane 29.6 Q4(i)

Here, given equation of planes are

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 7 \quad$ ---(i)
$\vec{r}.\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 26 \quad$ ---(ii)

We know that, planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ are perpendicular if

$\vec{n_1}.\vec{n_2} = 0 \quad$ ---(iii)

From equation (i) and (ii), we get
$\vec{n_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{n_2} = \lambda\hat{i} + 2\hat{j} - 7\hat{k}$

Since, (i) and (ii) are perpendicular, so from (iii),

$\vec{n_1}.\vec{n_2} = 0$
$\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$
$(1)(\lambda) + (2)(2) + (3)(-7) = 0$
$\lambda + 4 - 21 = 0$

$\lambda - 17 = 0$
$\lambda = 17$

The Plane 29.6 Q4(ii)

Given, that plane $2x - 4y + 3z - 5 = 0$ ---(i)
and $x + 2y + \lambda z - 5 = 0$ are ---(ii) perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

From equation (i) and (ii),
$a_1 = 2,\ b_1 = -4,\ c_1 = 3$
$a_2 = 1,\ b_2 = 2,\ c_2 = \lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(2)(1) + (-4)(2) + (3)(\lambda) = 0$
$2 - 8 + 3\lambda = 0$
$-6 + 3\lambda = 0$
$3\lambda = 6$
$\lambda = \frac{6}{3}$

$\lambda = 2$

The Plane 29.6 Q4(iii)

Given, that planes
$3x - 6y - 2z - 7 = 0$ ---(i)
and $2x + y - \lambda z - 5 = 0$ ---(ii)
are perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

From (i) and (ii),
$a_1 = 3,\ b_1 = -6,\ c_1 = -2$
$a_2 = 2,\ b_2 = 1,\ c_2 = -\lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(3)(2) + (-6)(1) + (-2)(-\lambda) = 0$
$6 - 6 + 2\lambda = 0$
$0 + 2\lambda = 0$
$2\lambda = 0$

$\lambda = 0$

The Plane 29.6 Q5

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ --- (i)

Given, plane is passing through $(-1, -1, 2)$,

$a(x + 1) + b(y + 1) + c(z - 2) = 0$ --- (ii)

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

Given, plane (ii) is perpendicular to plane
$3x + 2y - 3z = 1$ --- (iv)

So, using (ii),(iv) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(3) + (b)(2) + (c)(-3) = 0$

$3a + 2b - 3c = 0$ --- (v)

Also, plane (ii) is perpendicular to plane
$5x - 4y + z = 5$ --- (vi)

So, using (ii),(vi) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(5) + (b)(-4) + (c)(1) = 0$

$5a - 4b + c = 0$ --- (vii)

On solving (v) and (vii),
$$\frac{a}{(2)(1) - (-3)(-4)} = \frac{b}{(5)(-3) - (3)(1)} = \frac{c}{(3)(-4) - (2)(5)}$$
$$\frac{a}{2 - 12} = \frac{b}{-15 - 3} = \frac{c}{-12 - 10}$$
$$\frac{a}{-10} = \frac{b}{-18} = \frac{c}{-22} = \lambda \text{ (Say)}$$
$a = -10\lambda,\ b = -18\lambda,\ c = -22\lambda$

Put the value of a, b, c in equation (ii)

$a(x + 1) + b(y + 1) + c(z - 2) = 0$
$(-10\lambda)(x + 1) + (-18\lambda)(y + 1) + (-22\lambda)(z - 2) = 0$
$-10\lambda x - 10\lambda - 18\lambda y - 18\lambda - 22\lambda z + 44\lambda = 0$
$-10\lambda x - 18\lambda y - 22\lambda z + 16\lambda = 0$

Dividing by -2λ,
$5x + 9y + 11z - 8 = 0$

So, equation of required plane is,

$5x + 9y + 11z - 8 = 0$

The Plane 29.6 Q6

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$

Now, equation of plane passing through $(1, -3, -2)$,

$a(x - 1) + b(y + 3) + c(z + 2) = 0 \qquad ---(ii)$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Given, plane (ii) is perpendicular to plane
$x + 2y + 2z = 5 \qquad ---(iv)$

Using equation $(ii), (iv)$ in (iii),
$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0 \qquad ---(v)$

Also, plane (ii) is perpendicular to plane
$3x + 3y + 2z = 8 \qquad ---(vi)$

Using equation $(ii), (vi)$ in (iii),
$(a)(3) + (b)(3) + (c)(2) = 0$

$3a + 3b + 2c = 0 \qquad ---(vii)$

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(2) - (3)(2)} = \frac{b}{(3)(2) - (1)(2)} = \frac{c}{(1)(3) - (2)(3)}$$

$$\frac{a}{4 - 6} = \frac{b}{6 - 2} = \frac{c}{3 - 6}$$

$$\frac{a}{-2} = \frac{b}{4} = \frac{c}{-3} = \lambda \text{ (Say)}$$

$a = -2\lambda,\ b = 4\lambda,\ c = -3\lambda$

Put a, b, c in equation (ii)

$a(x - 1) + b(y + 3) + c(z + 2) = 0$
$(-2\lambda)(x - 1) + (4\lambda)(y + 3) + (-3\lambda)(z + 2) = 0$
$-2\lambda x + 2\lambda + 4\lambda y + 12\lambda - 3\lambda z - 6\lambda = 0$
$-2\lambda x + 4\lambda y - 3\lambda z + 8\lambda = 0$

Dividing by $(-\lambda)$,
$2x - 4y + 3z - 8 = 0$

Equation of required plane is,

$2x - 4y + 3z - 8 = 0$

The Plane 29.6 Q7

We know that equation of a plane passing through a point (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ ---(i)

Given that, plane is passing through origin, so

$a(x - 0) + b(y - 0) + c(z - 0) = 0$

$ax + by + cz = 0$ ---(ii)

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given that, plane (ii) is perpendicular to plane

$x + 2y - z = 1$ ---(iv)

Using (ii), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(-1) = 0$

$a + 2b - c = 0$ ---(v)

Given, plane (ii) is perpendicular to plane

$3x - 4y + z = 5$ ---(vi)

Using equation (ii), (vi) in (iii),

$(a)(3) + (b)(-4) + (c)(1) = 0$

$3a - 4b + c = 0$ ---(vii)

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(1) - (-4)(-1)} = \frac{b}{(3)(-1) - (1)(1)} = \frac{c}{(1)(-4) - (2)(3)}$$

$$\frac{a}{2 - 4} = \frac{b}{-3 - 1} = \frac{c}{-4 - 6}$$

$$\frac{a}{-2} = \frac{b}{-4} = \frac{c}{-10} = \lambda \text{ (Say)}$$

$a = -2\lambda,\ b = -4\lambda,\ c = -10\lambda$

Put a, b, c in equation (ii)

$ax + by + cz = 0$

$-2\lambda x - 4\lambda y - 10\lambda z = 0$

Dividing by -2λ,

$x + 2y + 5z = 0$

Equation of required plane is,

$x + 2y + 5z = 0$

The Plane 29.6 Q8

We know that equation of a plane passing through (x_1, y_1, z_1) is given by
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given that, plane is passing through $(1, -1, 2)$, so

$a(x - 1) + b(y + 1) + c(z - 2) = 0$ ---(i)

Plane (i) is also passing through $(2, -2, 2)$, so $(2, -2, 2)$ must satisfy the equation (i),
$a(2 - 1) + b(-2 + 1) + c(2 - 2) = 0$
$a - b = 0$ ---(ii)

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given that, plane (i) is perpendicular to plane
$6x - 2y + 2z - 9 = 0$ ---(iv)

Using plane (i), (iv) in equation (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(6) + (b)(-2) + (c)(2) = 0$

$6a - 2b + 2c = 0$ ---(v)
Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-1)(2) - (-2)(0)} = \frac{b}{(6)(0) - (1)(2)} = \frac{c}{(1)(-2) - (6)(-1)}$$
$$\frac{a}{-2 + 0} = \frac{b}{0 - 2} = \frac{c}{-2 + 6}$$
$$\frac{a}{-2} = \frac{b}{-2} = \frac{c}{4} = \lambda \text{ (Say)}$$
$\Rightarrow \quad a = -2\lambda,\ b = -2\lambda,\ c = 4\lambda$

Put a, b, c in equation (i)

$a(x - 1) + b(y + 1) + c(z - 2) = 0$
$(-2\lambda)(x - 1) + (-2\lambda)(y + 1) + (4\lambda)(z - 2) = 0$
$-2\lambda x + 2\lambda - 2\lambda y - 2\lambda + 4\lambda z - 8\lambda = 0$

$-2\lambda x - 2\lambda y + 4\lambda z - 8\lambda = 0$

Dividing by (-2λ),
$x + y - 2z + 4 = 0$

Equation of required plane is,

$x + y - 2z + 4 = 0$

The Plane 29.6 Q9

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here, the plane is passing through $(2, 2, 1)$

$a(x - 2) + b(y - 2) + c(z - 1) = 0 \quad ---(i)$

It is also passing through $(9, 3, 6)$, so it must satisfy the equation (i),

$a(9 - 2) + b(3 - 2) + c(6 - 1) = 0$

$7a + b + 5c = 0 \quad ---(ii)$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad ---(iii)$

Given that, plane (i) is perpendicular to plane

$2x + 6y + 6z = 1 \quad ---(iv)$

Using plane $(i), (iv)$ in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(2) + (b)(6) + (c)(6) = 0$

$2a + 6b + 6c = 0 \quad ---(v)$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(1)(6) - (5)(6)} = \frac{b}{(2)(5) - (7)(6)} = \frac{c}{(7)(6) - (2)(1)}$$

$$\frac{a}{6 - 30} = \frac{b}{10 - 42} = \frac{c}{42 - 2}$$

$$\frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -24\lambda,\ b = -32\lambda,\ c = 40\lambda$

Put a, b, c in equation (i),

$a(x - 2) + b(y - 2) + c(z - 1) = 0$

$(-24\lambda)(x - 2) + (-32\lambda)(y - 2) + (40\lambda)(z - 1) = 0$

$-24\lambda x + 48\lambda - 32\lambda y + 64\lambda + 40\lambda z - 40\lambda = 0$

$-24\lambda x - 32\lambda y + 40\lambda z + 72\lambda = 0$

Dividing by (-8λ),

$3x + 4y - 5z - 9 = 0$

Equation of required plane is,

$3x + 4y - 5z = 9$

The Plane 29.6 Q10

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given, the required plane is passing through $(-1, 1, 1)$,

$a(x + 1) + b(y - 1) + c(z - 1) = 0$ --- (i)

It is also passing through $(1, -1, 1)$, so it must satisfy the equation (i),
$a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$
$2a - 2b = 0$ --- (ii)

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

Given, plane (i) is perpendicular to plane
$x + 2y + 2z = 5$ --- (iv)

Using plane (i),(iv) in equation (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0$ --- (v)
Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-2)(2) - (2)(0)} = \frac{b}{(1)(0) - (2)(2)} = \frac{c}{(2)(2) - (1)(-2)}$$
$$\frac{a}{-4 - 0} = \frac{b}{0 - 4} = \frac{c}{4 + 2}$$
$$\frac{a}{-4} = \frac{b}{-4} = \frac{c}{6} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -4\lambda,\ b = -4\lambda,\ c = 6\lambda$
Put the value of a, b, c in equation (i),

$a(x + 1) + b(y - 1) + c(z - 1) = 0$
$(-4\lambda)(x + 1) + (-4\lambda)(y - 1) + (6\lambda)(z - 1) = 0$
$-4\lambda x + 4\lambda - 4\lambda y + 4\lambda + 6\lambda z - 6\lambda = 0$

$-4\lambda x - 4\lambda y + 6\lambda z - 6\lambda = 0$

Dividing by (-2λ), we get
$2x + 2y - 3z + 3 = 0$

The equation of required plane is,

$2x + 2y - 3z + 3 = 0$

The Plane Ex 29.6 Q11

The equation of the plane parallel to ZOX is y = constant.
Given that the $y-$intercept is 3.
Thus the equation of the plane is $y = 3$.

The Plane Ex 29.6 Q12

The equation of any plane passing through $(1, -1, 2)$
is $a(x-1) + b(y+1) + c(z-2) = 0$....(1)
Given that, plane (1) is perpendicular to the planes
$2x + 3y - 2z = 5$
and
$x + 2y - 3z = 8$
Therefore, we have,
$2a + 3b - 2c = 0$...(2)
and
$a + 2b - 3c = 0$...(3)
Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{3 \times (-3) - 2 \times (-2)} = \frac{b}{1 \times (-2) - 2 \times (-3)} = \frac{c}{2 \times 2 - 1 \times 3} = \lambda (\text{say})$$

$$\Rightarrow \frac{a}{-9+4} = \frac{b}{-2+6} = \frac{c}{4-3} = \lambda$$

$$\Rightarrow \frac{a}{-5} = \frac{b}{4} = \frac{c}{1} = \lambda$$

Thus, we have,
$a = -5\lambda, b = 4\lambda$ and $c = \lambda$
Substituting the above values in equation (1), we have,
$-5\lambda(x-1) + 4\lambda(y+1) + \lambda(z-2) = 0$
Since $\lambda \neq 0$, we have,
$-5(x-1) + 4(y+1) + (z-2) = 0$
$\Rightarrow -5x + 5 + 4y + 4 + z - 2 = 0$
$\Rightarrow -5x + 4y + z + 7 = 0$
$\Rightarrow 5x - 4y - z - 7 = 0$
$\Rightarrow 5x - 4y - z = 7$
Thus the required equation of the plane is $5x - 4y - z = 7$

The Plane Ex 29.6 Q13

Given that the equation of the required
plane is parallel to the plane
$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$...(1)
$\therefore$ Vector equation of any plane parallel to (1) is
$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = k$...(2)
Since the given plane passes through (a, b, c), then
$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = k$
$\Rightarrow a + b + c = k$...(3)

Substituting the above value of k in equation (2), we have,
$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$
Thus the required equation of the plane is $x + y + z = a + b + c$

The Plane Ex 29.6 Q14

The equation of any plane passing through $(-1,3,2)$
is $(x+1)+b(y-3)+c(z-2)=0....(1)$
Given that, Plane (1) is perpendicular to the planes
$x+2y+3z=5$
and
$3x+3y+z=0$
Therefore, we have,
$a+2b+3c=0...(2)$
and
$3a+3b+c=0...(3)$
Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times1-3\times3}=\frac{b}{3\times3-1\times1}=\frac{c}{1\times3-3\times2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{2-9}=\frac{b}{9-1}=\frac{c}{3-6}=\lambda$$

$$\Rightarrow \frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=\lambda$$

Thus, we have,
$a=-7\lambda, b=8\lambda$ and $c=-3\lambda$
Substituting the above values in equation (1), we have,
$-7\lambda(x+1)+8\lambda(y-3)-3\lambda(z-2)=0$
Since $\lambda\neq0$, we have,
$-7(x+1)+8(y-3)-3(z-2)=0$
$\Rightarrow -7x-7+8y-24-3z+6=0$
$\Rightarrow -7x+8y-3z-25=0$
$\Rightarrow 7x-8y+3z+25=0$

Thus the required equation of the plane is $7x-8y+3z+25=0$

The Plane Ex 29.6 Q15

The equation of any plane passing through $(2,1,-1)$
is $a(x-2)+b(y-1)+c(z+1)=0....(1)$
Also, the above plane passes through the point $(-1,3,4)$.
Thus, equation (1), becomes,
$a(-1-2)+b(3-1)+c(4+1)=0$
$\Rightarrow -3a+2b+5c=0...(2)$
Given that, Plane (1) is perpendicular to the plane
$x-2y+4z=10$
Therefore, we have,
$a-2b+4c=0...(3)$

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times4-5\times(-2)}=\frac{b}{1\times5-(-3)\times4}=\frac{c}{(-3)\times(-2)-1\times2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{8+10}=\frac{b}{5+12}=\frac{c}{6-2}=\lambda$$

$$\Rightarrow \frac{a}{18}=\frac{b}{17}=\frac{c}{4}=\lambda$$

Thus, we have,
$a=18\lambda, b=17\lambda$ and $c=4\lambda$
Substituting the above values in equation (1), we have,
$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$
Since $\lambda\neq0$, we have,
$18(x-2)+17(y-1)+4(z+1)=0$
$\Rightarrow 18x-36+17y-17+4z+4=0$
$\Rightarrow 18x+17y+4z-49=0$

Thus the required equation of the plane is $18x+17y+4z-49=0$