RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.9

The Plane Ex 29.9 Q1

We know that distance of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \text{ unit}$$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and

plane $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$

$\vec{r}.\vec{n} - d = 0$

So, required distance

$$D = \left|\frac{\left(2\hat{i} - \hat{j} - 4\hat{k}\right)\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}\right|$$

$$= \left|\frac{(2)(3) + (-1)(-4) + (-4)(12) - 9}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{6 + 4 - 48 - 9}{\sqrt{169}}\right|$$

$$= \left|-\frac{47}{13}\right|$$

$$= \frac{47}{13} \text{ units}$$

Required distance is $\frac{47}{13}$ units

The Plane Ex 29.9 Q2

We know that, distance of a point $\vec{a}$ to a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

Let D_1 be the distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$

from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then

$$D_1 = \left|\frac{\left(\hat{i} - \hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right| \qquad \left[\text{Using equation } (i)\right]$$

$$= \left|\frac{(1)(5) + (-1)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{5 - 2 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|-\frac{9}{\sqrt{78}}\right|$$

$$D_1 = \frac{9}{\sqrt{78}} \text{ units} \qquad ---(ii)$$

Again, let D_2 be the distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from the plane

$\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then, using equation (i), we get

$$D_2 = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

$$= \frac{9}{\sqrt{78}} \text{ units} \qquad ---(iii)$$

From equation (ii) and (iii)

$D_1 = D_2$

Distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

$=$ Distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

The Plane Ex 29.9 Q3

We know that, distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \quad ---(i)$$

So, distance of point $(2, 3, -5)$ from the plane $x + 2y - 2z - 9 = 0$ is given by

$$D = \left|\frac{2 + (2)(3) - 2(-5) - 9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right| \quad \text{[Using equation (i)]}$$

$$= \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

$$= \left|\frac{9}{\sqrt{9}}\right|$$

$$= \left|\frac{9}{3}\right|$$

$D = 3$ units

The Plane Ex 29.9 Q4

Given equation of plane is

$x + 2y - 2z + 8 = 0$ --- (i)

We know that, equation of the plane parallel to plane (i) is given by

$x + 2y - 2z + \lambda = 0$ --- (ii)

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \quad \text{--- (iii)}$$

Given, $D = 2$ unit is the distance of the plane (ii) from the point $(2, 1, 1)$, so
Using (i),

$$2 = \left|\frac{2 + (2)(1) - 2(1) + \lambda}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right|$$

$$2 = \left|\frac{2 + 2 - 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$2 = \left|\frac{2 + \lambda}{\sqrt{9}}\right|$$

Squaring both the sides, we get

$$4 = \frac{(2 + \lambda)^2}{9}$$

$$36 = (2 + \lambda)^2$$

$$2 + \lambda = \pm 6$$

$\Rightarrow \quad 2 + \lambda = 6 \quad or \quad 2 + \lambda = -6$

$\Rightarrow \quad \lambda = 4 \quad or \quad \lambda = -8$

Put $\lambda = 4$ in equation (ii),
$x + 2y - 2z + 4 = 0$

Put $\lambda = -8$ in equation (ii),
$x + 2y - 2z - 8 = 0$

Hence, equation of the required plane are
$x + 2y - 2z + 4 = 0$
$x + 2y - 2z - 8 = 0$

The Plane Ex 29.9 Q5

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \quad ---(i)$$

Let D_1 be the distance of the point $(1,1,1)$ from plane $3x + 4y - 12z + 13 = 0$, so using (i), we get

$$D_1 = \left|\frac{(3)(1) + (4)(1) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$

$$= \left|\frac{3 + 4 - 12 + 13}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{8}{\sqrt{169}}\right|$$

$$D = \frac{8}{13} \text{ units} \quad ---(ii)$$

Let D_2 be the distane of a point $(-3,0,1)$ from the plane $3x + 4y - 12z + 13 = 0$, so using equation (i),

$$D_2 = \left|\frac{(3)(-3) + (4)(0) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$

$$= \left|\frac{-9 + 0 - 12 + 13}{\sqrt{9 + 4 + 144}}\right|$$

$$= \left|-\frac{8}{\sqrt{169}}\right|$$

$$D_2 = \frac{8}{13} \text{ units} \quad ---(iii)$$

Hence, from equation (ii) and (iii)

$$D_1 = D_2$$

The Plane Ex 29.9 Q6

Given equation of plane is

$x - 2y + 2z - 3 = 0 \qquad ---(i)$

We know that, equation of a plane parallel to plane (i) is given by,

$x - 2y + 2z + \lambda = 0 \qquad ---(ii)$

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given that, distance of plane (ii) from a point $(1,1,1)$ is one unit, so using (iii),

$$1 = \left|\frac{(1) - 2(1) + 2(1) + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}}\right|$$

$$= \left|\frac{1 - 2 + 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$1 = \left|\frac{1 + \lambda}{\sqrt{9}}\right|$$

$$1 = \left|\frac{1 + \lambda}{3}\right|$$

Squaring both the sides,

$$1 = \frac{(1 + \lambda)^2}{9}$$

$$9 = (1 + \lambda)^2$$

$$1 + \lambda = \pm 3$$

$\Rightarrow \quad 1 + \lambda = 3 \quad$ or $\quad 1 + \lambda = -3$

$\Rightarrow \quad \lambda = 2 \quad$ or $\quad \lambda = -4$

Put the value of λ in equation (ii) to get the equations of required planes,

$x - 2y + 2z + 2 = 0$

$x - 2y + 2z - 4 = 0$

The Plane Ex 29.9 Q7

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \quad ---(i)$$

So, distance of point $(2,3,5)$ from xy-plane (we know that equation of xy-plane is $z = 0$) is

$$= \left|\frac{(2)(0) + (3)(0) + (5)(1) + 0}{\sqrt{(0)^2 + (0)^2 + (1)^2}}\right| \quad [\text{Using } (i)]$$

$$= \left|\frac{0 + 0 + 5}{\sqrt{0 + 0 + 1}}\right|$$

= 5 unit

Distance of the point $(2,3,5)$ from xy-plane = 5 unit

The Plane Ex 29.9 Q8

We know that, distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \quad ---(i)$$

So, distance of point $(3\hat{i} + 3\hat{j} + 3\hat{k})$ from plane $\vec{r}.(5\hat{i} + 2\hat{j} + 3\hat{k}) + 9 = 0$ is

$$D = \left|\frac{(3\hat{i} + 3\hat{j} + 3\hat{k})(5\hat{i} + 2\hat{j} + 3\hat{k}) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

Therefore, required distance is

$$= \frac{9}{\sqrt{78}} \text{ units}$$

The Plane Ex 29.9 Q9

Distance of point (1,1,1) from origin is $\sqrt{3}$

Distance of point (1,1,1) from plane is $\left|\frac{1+\lambda}{\sqrt{3}}\right|$

Product = $\left|\frac{1+\lambda}{\sqrt{3}}\right| \times \sqrt{3} = 5$

$|1+\lambda| = 5$

so λ=4 or -6

The Plane Ex 29.9 Q10

Consider

$$3x - 4y + 12z - 6 = 0 \qquad \ldots\ldots (1)$$
$$4x + 3z - 7 = 0 \qquad \ldots\ldots (2)$$

The distance of a point (x_1, y_1, z_1) from the plane $3x - 4y + 12z - 6 = 0$ is

$$D_1 = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$
$$= \left|\frac{3x_1 - 4y_1 + 12z_1 - 6}{\sqrt{3^2 + (-4)^2 + 12^2}}\right|$$
$$= \left|\frac{3x_1 - 4y_1 + 12z_1 - 6}{\sqrt{169}}\right|$$
$$= \left|\frac{3x_1 - 4y_1 + 12z_1 - 6}{13}\right|$$

The distance of the point (x_1, y_1, z_1) from the plane $4x + 3z - 7 = 0$ is

$$D_2 = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$
$$= \left|\frac{4x_1 + 3z_1 - 7}{\sqrt{4^2 + 3^2}}\right|$$
$$= \left|\frac{4x_1 + 3z_1 - 7}{\sqrt{25}}\right|$$
$$= \left|\frac{4x_1 + 3z_1 - 7}{5}\right|$$

Since the point (x_1, y_1, z_1) are equidistant from the planes $3x - 4y + 12z - 6 = 0$ and $4x + 3z - 7 = 0$

So

$$D_1 = D_2$$
$$\left|\frac{3x_1 - 4y_1 + 12z_1 - 6}{13}\right| = \left|\frac{4x_1 + 3z_1 - 7}{5}\right|$$
$$\frac{3x_1 - 4y_1 + 12z_1 - 6}{13} = \pm\frac{4x_1 + 3z_1 - 7}{5}$$

Taking positive sign

$$\frac{3x_1 - 4y_1 + 12z_1 - 6}{13} = \frac{4x_1 + 3z_1 - 7}{5}$$
$$15x_1 - 20y_1 + 60z_1 - 30 = 52x_1 + 39z_1 - 91$$
$$37x_1 + 20y_1 - 21z_1 - 61 = 0$$

Taking negative sign

$$\frac{3x_1 - 4y_1 + 12z_1 - 6}{13} = -\frac{4x_1 + 3z_1 - 7}{5}$$
$$15x_1 - 20y_1 + 60z_1 - 30 = -52x_1 - 39z_1 + 91$$
$$67x_1 - 20y_1 + 99z_1 - 121 = 0$$

The Plane Ex 29.9 Q11

The equation of any plane passing through $A(2,5,-3)$

is $a(x-2)+b(y-5)+c(z+3)=0$....(1)

The above plane passes through the point $B(-2,-3,5)$

and hence, we have,

$a(-2-2)+b(-3-5)+c(5+3)=0$

$\Rightarrow -4a-8b+8c=0$...(2)

Again the required plane passes through the point $C(5,3,-3)$

and hence, we have,

$a(5-2)+b(3-5)+c(-3+3)=0$

$\Rightarrow 3a-2b+0c=0$...(3)

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{(-8)\times 0-(-2)\times 8}=\frac{b}{3\times 8-(-4)\times 0}=\frac{c}{(-4)\times(-2)-3\times(-8)}=\lambda\text{(say)}$$

$$\Rightarrow \frac{a}{0+16}=\frac{b}{24+0}=\frac{c}{8+24}=\lambda$$

$$\Rightarrow \frac{a}{16}=\frac{b}{24}=\frac{c}{32}=\lambda$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\lambda$$

$\Rightarrow a=2\lambda, b=3\lambda$ and $c=4\lambda$

Substituting the above values in equation (1), we have,

$2\lambda(x-2)+3\lambda(y-5)+4\lambda(z+3)=0$

Since $\lambda\neq 0$, we have,

$2(x-2)+3(y-5)+4(z+3)=0$

$\Rightarrow 2x-4+3y-15+4z+12=0$

$\Rightarrow 2x+3y+4z-7=0$

Thus the equation of the plane is

$2x+3y+4z-7=0$

The distance from the point $P(7,2,4)$ to the plane is

$$d=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\therefore \text{Distance, } d=\left|\frac{2x+3y+4z-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{2\times 7+3\times 2+4\times 4-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{29}{\sqrt{29}}\right|$$

$\Rightarrow d_{(7,2,4)}=\sqrt{29}$ units

The Plane Ex 29.9 Q12

Given that a plane is making intercepts $-6,3$ and 4 respectively on the coordinate axes.

Thus the equation of the plane is

$$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1 \quad ...(1)$$

We need to find the length of the perpendicular from the origin on the plane.

If the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is at a distance 'p', then

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \quad ...(2)$$

Comparing equation (1) with the general equation, we get,

$a=-6, b=3$ and $c=4$

Thus, equation (2) becomes,

$$\frac{1}{p^2}=\frac{1}{(-6)^2}+\frac{1}{3^2}+\frac{1}{4^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{4+16+9}{144}$$

$$\Rightarrow \frac{1}{p^2}=\frac{29}{144}$$

$$\Rightarrow p^2=\frac{144}{29}$$

$$\Rightarrow p=\frac{12}{\sqrt{29}} \text{ units}$$