RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.11

The Plane Ex 29.11 Q1

$$\vec{r} = (2\hat{i} + 3\hat{j} + 9\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 5$$

Angle between line and plane is given by

$$\cos\theta = \left|\frac{2+3+4}{\sqrt{(1+1+1)(4+9+16)}}\right| = \frac{9}{\sqrt{87}}$$

The Plane Ex 29.11 Q2

We know that the angle (θ) between the line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given, equation of line is

$$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{1}$$

So, $a_1=1,\ b_1=-1,\ c_1=1$

Given equation of plane is $2x+y-z-4=0$
So, $a_2=2,\ b_2=1,\ c_2=-1$

Put these value in equation (i),

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{(1)(2)+(-1)(1)+(1)(-1)}{\sqrt{(1)^2+(-1)^2+(1)^2}\sqrt{(2)^2+(1)^2+(-1)^2}}$$

$$\sin\theta=\frac{2-1-1}{\sqrt{1+1+1}\sqrt{4+1+1}}$$

$$=\frac{0}{\sqrt{3}\sqrt{6}}$$

$\sin\theta=0$

$\theta=0^\circ$

angle between plane and line $=0^\circ$

The Plane Ex 29.11 Q3

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given that, line is passing through

$A(3,-4,-2)$ and $B(12,2,0)$, so direction ratios of line AB

$=(12-3,\ 2+4,\ 0+2)$

$=(9,\ 6,\ +2)$

So, $a_1=9,\ b_1=6,\ c_1=2$ $\qquad ---(ii)$

Given equation of plane is $3x-y+z=1$

$a_2=3,\ b_2=-1,\ c_2=1$ $\qquad ---(iii)$

Using (ii) and (iii) in equation (i),

Angle (θ) between plane and line is

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{(9)(3)+(6)(-1)+(2)(1)}{\sqrt{(9)^2+(6)^2+(2)^2}\sqrt{(3)^2+(-1)^2+(1)^2}}$$

$$=\frac{27-6+2}{\sqrt{81+36+4}\sqrt{9+1+1}}$$

$$=\frac{23}{\sqrt{121}\sqrt{11}}$$

$$=\frac{23}{11\sqrt{11}}$$

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

so, required angle between plane and line is given by

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

The Plane Ex 29.11 Q4

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to plane $\vec{r}.\vec{n} = d$ if

$\vec{b}.\vec{n} = 0$ --- (i)

Given, equation of line is $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$ and equation of plane $\vec{r}.\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 4$

So $\vec{b} = \left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$

$\vec{n} = \left(m\hat{i} + 3\hat{j} + \hat{k}\right)$

Put $\vec{b}$ and $\vec{n}$ in equation (i),

$$\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 0$$
$$(2)(m) + (-m)(3) + (-3)(1) = 0$$
$$2m - 3m - 3 = 0$$
$$-m - 3 = 0$$
$$-m = 3$$

$$m = -3$$

The Plane Ex 29.11 Q5

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is parallel if

$\vec{b}.\vec{n} = 0$ --- (i)

Given, equation of line $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and equation of plane $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$, so

$\vec{b} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + \hat{j} - \hat{k}$

Now,

$$\vec{b}.\vec{n} = \left(\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right)$$
$$= (1)(1) + (3)(1) + (4)(-1)$$
$$= 1 + 3 - 4$$
$$= 0$$

Since $\vec{b}.\vec{n} = 0$ so using (i), we get

Given line and plane are parallel

We know that, distance (D) of a plane $\vec{r}.\vec{n} - d = 0$ from a point $\vec{a}$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{\left|\vec{n}\right|}\right| \quad --- (ii)$$

We have to find distance between line and plane which is equal to the distance between point $\vec{a} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)$ from plane, so

$$D = \left| \frac{\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) - 7}{\sqrt{(1)^2 + (1)^2 + (-1)^2}} \right|$$

$$= \left| \frac{(2)(1) + (5)(1) + (7)(-1) - 7}{\sqrt{1+1+1}} \right|$$

$$= \left| \frac{2 + 5 - 7 - 7}{\sqrt{3}} \right|$$

$$= \left| \frac{-7}{\sqrt{3}} \right|$$

$$D = \frac{7}{\sqrt{3}}$$

So, required distance between plane and line is $D = \frac{7}{\sqrt{3}}$ unit

The Plane Ex 29.11 Q6

Required line is perpendicular to plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3$, so line is parallel to the normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$ of plane.

And it is passing through point $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

We know that equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = 0\hat{i} + 0\hat{j} + 0.\hat{k}$ and $\vec{b} = \vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

So, $\vec{r} = \left(0\hat{i} + 0\hat{j} + 0.\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Hence, equation required line is

$$\vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

The Plane Ex 29.11 Q7

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

So, equation of plane passing through $(2, 3, -4)$ is

$$a(x - 2) + b(y - 3) + c(z + 4) = 0 \qquad ---(ii) \qquad [\text{Using (i)}]$$

It is also passing through $(1, -1, 3)$, so,

$$a(1 - 2) + b(-1 - 3) + c(3 + 4) = 0$$
$$-a - 4b + 7c = 0$$
$$a + 4b - 7c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Here, equation (ii) is parallel to x-axis

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0} \qquad ---(v)$$

Using (ii) and (v) in equation (iv),

$$(a)(1) + (b)(0) + c(0) = 0$$
$$a = 0 \qquad ---(vi)$$

Put the value of a in equation (iii),

$$a - 4b + 7c = 0$$
$$0 - 4b + 7c = 0$$
$$-4b = -7c$$
$$4b = 7c$$

$$b = \frac{7}{4}c$$

Put the value of a and b in equation (ii),

$$a(x-2) + b(y-3) + c(z+4) = 0$$
$$0(x-2) + \left(\frac{7}{4}c\right)(y-3) + c(z+4) = 0$$
$$0 + \frac{7cy}{4} - \frac{21c}{4} + \frac{cz}{1} + \frac{4c}{1} = 0$$

$$7cy - 21c + 4cz + 16c = 0$$

Dividing by c,

$$7y + 4z - 5 = 0$$

Equation of required plane is

$$7y + 4z - 5 = 0$$

The Plane Ex 29.11 Q8

We know that equation a plane passing through the point (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given the required plane is passing through $(0,0,0)$, so using (i),

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$
$$ax + by + cz = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(3,-1,2)$,

$$3a - b + 2c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Given that, plane (ii) is parallel to line

$$\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}, \text{ so}$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(7) = 0$$
$$a - 4b + 7c = 0 \qquad ---(ii)$$

Solving equation (iii) and (v) by cross-multiplication, we get

$$\frac{a}{(-1)(7) - (-4)(2)} = \frac{b}{(1)(2) - (3)(7)} = \frac{c}{(3)(-4) - (1)(-1)}$$
$$\frac{a}{-7 + 8} = \frac{b}{2 - 21} = \frac{c}{-12 + 1}$$
$$\frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = \lambda \text{ (Say)}$$

$$\Rightarrow \qquad a = \lambda,\ b = -19\lambda,\ c = -11\lambda$$

Put the value of a, b, c in equation (ii),

$$ax + by + cz = 0$$
$$\lambda x - 19\lambda y - 11\lambda z = 0$$

Dividing by λ, we get

$$x - 19y - 11z = 0$$

Equation of required plane is

$$x - 19y - 11z = 0$$

The Plane Ex 29.11 Q9

We know that equation of a line passing through (x_1, y_1, z_1) is given by

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \qquad ---(i)$$

Here, required line is passing through $(1, 2, 3)$, is given by, [Using (i)]

$$\frac{x - 1}{a_1} = \frac{y - 2}{b_1} = \frac{z - 3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$$

Given, line (ii) is parallel to plane $x - y + 2z = 5$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$

$$a_1 - b_1 + 2c_1 = 0 \qquad ---(iv)$$

Also, given line (ii) is parallel to plane $3x + y + z = 6$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$$3a_1 + b_1 + c_1 = 0 \qquad ---(v)$$

Solving (iv) and (v) by cross-multiplication,

$$\frac{a_1}{(-1)(1)-(1)(2)} = \frac{b_1}{(3)(2)-(1)(1)} = \frac{c_1}{(1)(1)-(3)(-1)}$$

$$\frac{a_1}{-1-2} = \frac{b_1}{6-1} = \frac{c_1}{1+3}$$

$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow \qquad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$$

Put a_1, b_1, c_1 in equation (ii),

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$

Multiplying by λ,

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

Equation of required line is

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

The vector equation of the line is

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q10

Firstly we have to find the line of section of planes $5x+2y-4z+2=0$ and $2x+8y+2z-1=0$
Let a_1, b_1, c_1 be the direction ratios of the line $5x+2y-4z+2=0$ and $2x+8y+2z-1=0$

Since, line lies in both the planes, so it is perpendicular to both planes, so

$$5a_1+2b_1-4c_1=0 \qquad ---(i)$$
$$2a_1+8b_1+2c_1=0 \qquad ---(ii)$$

Solving equation (i) and (ii), by cross-multiplication

$$\frac{a_1}{(2)(2)-(-4)(8)}=\frac{b_1}{(2)(-4)-(5)(2)}=\frac{c_1}{(5)(8)-(2)(2)}$$
$$\frac{a_1}{4+32}=\frac{b_1}{-8-10}=\frac{c_1}{40-4}$$
$$\frac{a_1}{36}=\frac{b_1}{-18}=\frac{c_1}{36}$$
$$\frac{a_1}{2}=\frac{b_1}{-1}=\frac{c_1}{2}=\lambda\text{(Say)}$$

$$\Rightarrow \quad a_1=2\lambda,\ b_1=-\lambda,\ c_1=2\lambda$$

We know that, line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ is parallel to plane $a_2x+b_2y+c_2z+d_2=0$ if
$$a_1a_2+b_1b_2+c_1c_2=0 \qquad ---(iii)$$

Here line with direction ratio a_1, b_1, c_1 is parallel to plane $4x-2y-5z-2=0$,

$$a_1a_2+b_1b_2+c_1c_2$$
$$=(2)(4)+(-1)(-2)+(2)(-5)$$
$$=8+2-10$$

$$=0$$

Therefore, line of section is parallel to the plane.

The Plane Ex 29.11 Q11

Equation of line passing through $\vec{a}$ and parallel to $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Given that, required line is passing through $(1, -1, 2)$ is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\vec{b} \qquad ---(ii)$$

Since, line (i) is perpendicular to plane $2x - y + 3z - 5 = 0$, so normal to plane is parallel to the line.

In vector form,
$\vec{b}$ is parallel to $\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}$
So, $\vec{b} = \mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$ as μ is any scaler
Thus, equation of required line is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right)$$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \delta\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

The Plane Ex 29.11 Q12

We know that, equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given that, required plane is passing through $(2, 2, -1)$, so using (i),

$$a(x - 2) + b(y - 2) + c(z + 1) = 0 \qquad ---(ii)$$

Given, plane (ii) is passing through $(3, 4, 2)$,

$$a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$$
$$a + 2b + 3c = 0 \qquad ---(iii)$$

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$

Given that, plane (ii) is parallel to a line whose direction ratios are $7, 0, 6$ so using (iv), we get

$$(a)(7) + (b)(0) + (c)(6) = 0$$
$$7a + 0 + 6c = 0$$
$$7a + 6c = 0$$

$$a = -\frac{6c}{7}$$

Put the value of a in equation (iii),

$$a + 2b + 3c = 0$$
$$-\frac{6c}{7} + 2b + 3c = 0$$
$$-6c + 14b + 21c = 0$$
$$14b + 15c = 0$$

$$b = -\frac{15c}{14}$$

Put the value of a and b in equation (ii),
$$a(x-2) + b(y-2) + c(z+1) = 0$$
$$\left(-\frac{6c}{7}\right)(x-2) + \left(-\frac{15c}{14}\right)(y-2) + c(z+1) = 0$$
$$-\frac{6cx}{7} + \frac{12c}{7} - \frac{15cy}{14} + \frac{30c}{14} + cz + c = 0$$

Multiplying by $\left(\frac{14}{c}\right)$, we get

$$-12x + 24 - 15y + 30 + 14z + 14 = 0$$
$$-12x + 15y + 14z + 68 = 0$$

Multiplying by (-1),
Equation of required plane is,

$$12x + 15y - 14z - 68 = 0$$

The Plane Ex 29.11 Q13

We know that angle (θ) between line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad ---(i)$$

Given line is $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and equation of plane is $3x + 4y + z + 5 = 0$, so angle between plane and line is,

$$\sin\theta = \frac{(3)(3) + (-1)(4) + (2)(1)}{\sqrt{(3)^2 + (-1)^2 + (2)^2}\sqrt{(3)^2 + (4)^2 + (1)^2}}$$
$$= \frac{9 - 4 + 2}{\sqrt{9+1+4}\sqrt{9+16+1}}$$
$$= \frac{7 \times \sqrt{7}}{\sqrt{14}\sqrt{26} \times \sqrt{7}}$$

$$= \frac{7\sqrt{7}}{7\sqrt{52}}$$

$$\theta = \sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$$

The Plane Ex 29.11 Q14

We know that equation of plane passing through the intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the intersection of two planes $x - 2y + z - 1 = 0$ and $2x + y + z - 8 = 0$ is given by

$$\left(x - 2y + z - 1\right) + \lambda\left(2x + y + z - 8\right) = 0$$
$$x - 2y + z - 1 + 2\lambda x + \lambda y + \lambda z - 8\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-2 + \lambda\right) + z\left(1 + \lambda\right) - 1 - 8\lambda = 0 \qquad ---(i)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(ii)$

Given that plane (i) is parallel to line with direction ratio 1,2,1, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(1)(1 + 2\lambda) + (2)(-2 + \lambda) + (1)(1 + \lambda) = 0$$
$$1 + 2\lambda - 4 + 2\lambda + 1 + \lambda = 0$$
$$5\lambda - 2 = 0$$

$$\lambda = \frac{2}{5}$$

Put the value of λ in equation (i),

$$x\left(1 + \frac{4}{5}\right) + y\left(-2 + \frac{2}{5}\right) + z\left(1 + \frac{2}{5}\right) - 1 - \frac{16}{5} = 0$$

Multiplying by 5,

$$x\left(5 + 4\right) + y\left(-10 + 2\right) + z\left(5 + 2\right) - 5 - 16 = 0$$
$$9x - 8y + 7z - 21 = 0$$

So, equation of required plane is

$$9x - 8y + 7z - 21 = 0 \qquad ---(iii)$$

We know that distance (D) of a point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \frac{ax_1 + by_1 + cz_1 + d_1}{\sqrt{a^2 + b^2 + c^2}}$$

So, distance of point $(1,1,1)$ from plane (i) is given by

$$D = \left|\frac{(9)(1)+(-8)(1)+(7)(1)-21}{\sqrt{(9)^2+(-8)^2+(7)^2}}\right|$$

$$= \left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|$$

$$= \left|\frac{16-29}{\sqrt{194}}\right|$$

$$= \left|\frac{-13}{\sqrt{194}}\right|$$

$$D = \frac{13}{\sqrt{194}} \text{ units}$$

The Plane Ex 29.11 Q15

We know that line $\vec{r} = \vec{a} + \lambda \vec{b}$ is paralle to plane $\vec{r}.\vec{n} = d$ if

$\vec{b}.\vec{n} = 0$

Given, line is $\vec{r} = (\hat{i}+\hat{j}) + \lambda(3\hat{i}-\hat{j}+2\hat{k})$ and plane is $\vec{r}.(2\hat{j}+\hat{k}) = 3$, so

$\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$, $\vec{a} = (\hat{i}+\hat{j})$ and $\vec{n} = (2\hat{j}+\hat{k})$

Now, $\vec{b}\vec{n} = (3\hat{i}-\hat{j}+2\hat{k})(2\hat{j}+\hat{k})$

$= (3)(0)+(-1)(2)+(2)(1)$

$= 0-2+2$

$= 0$

Since, $\vec{b}.\vec{n} = 0$, so line is parallel to plane

Distance between point $\vec{a}$ and plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n}-d}{|\vec{n}|}\right| \qquad ---(i)$$

$\vec{a}$ is a point on the line. So distance between line and plane is equal to the distance between $\vec{a} = (\hat{i} + \hat{j})$ and plane $\vec{r}.(2\hat{j} + \hat{k}) = 3$, so using (i),

$$D = \left|\frac{(\hat{i} + \hat{j})(2\hat{j} + \hat{k}) - 3}{\sqrt{(2)^2 + (1)^2}}\right|$$

$$= \left|\frac{(1)(0) + (1)(2) + (0)(1) - 3}{\sqrt{4+1}}\right|$$

$$= \left|\frac{0 + 2 + 0 - 3}{\sqrt{5}}\right|$$

$$= \left|\frac{-1}{\sqrt{5}}\right|$$

$$= \frac{1}{\sqrt{5}} \text{ unit}$$

So, required distance $= \frac{1}{\sqrt{5}}$ unit

The Plane Ex 29.11 Q16

We know that line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} - d = 0$ are parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, line $\vec{r} = (-\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ and plane is $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) - 1 = 0$

So, $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$

Now, $\vec{b}.\vec{n} = (2\hat{i} + \hat{j} + 4\hat{k})(\hat{i} + 2\hat{j} - \hat{k})$

$= (2)(1) + (1)(2) + (4)(-1)$

$= 2 + 2 - 4$

$= 0$

Since, $\vec{b}.\vec{n} = 0$, so by equation (i), line is parallel to plane

Distance (D) between point $\vec{a}$ and plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(ii)$$

Distance between given line and plane

= Distance of point $\vec{a} = (-\hat{i} + \hat{j} + \hat{k})$ from $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) - 1 = 0$

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right|$$

$$= \left| \frac{\left(-\hat{i} + \hat{j} + \hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}} \right|$$

$$= \left| \frac{(-1)(1) + (1)(2) + (1)(-1) - 1}{\sqrt{1+4+1}} \right|$$

$$= \left| \frac{-1+2-1-1}{\sqrt{6}} \right|$$

$$= \left| \frac{-1}{\sqrt{6}} \right|$$

$$= \frac{1}{\sqrt{6}}$$

So, required distance = $\frac{1}{\sqrt{6}}$ units

The Plane Ex 29.11 Q17

We know that equation of plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0 \qquad ---(i)$$

So, equation of plane passing through the line of intersection of planes $3x - 4y + 5z - 10 = 0$ and $2x + 2y - 3z - 4 = 0$ is,

$$(3x - 4y + 5z - 10) + \lambda(2x + 2y - 3z - 4) = 0$$

$$(3 + 2\lambda)x + (-4 + 2\lambda)y + (5 - 3\lambda)z - 10 - 4\lambda = 0 \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ parallel to plane

$a_2x + b_2y + c_2z + d_2 = 0$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given that, plane (ii) is parallel to line $x = 2y = 3z$ or $\frac{x}{6} = \frac{y}{3} = \frac{z}{2}$

So,

$$(6)(3 + 2\lambda) + (3)(-4 + 2\lambda) + (2)(5 - 3\lambda) = 0$$

$$18 + 12\lambda - 12 + 6\lambda + 10 - 6\lambda = 0$$

$$12\lambda + 16 = 0$$

$$\lambda = -\frac{16}{12}$$

$$\lambda = -\frac{4}{3}$$

Put λ in equation (ii),

$$x(3+2\lambda)+y(-4+2\lambda)+z(5-3\lambda)-10-4\lambda=0$$

$$x\left(3-\frac{8}{3}\right)+y\left(-4-\frac{8}{3}\right)+z\left(5+\frac{12}{3}\right)-10+\frac{16}{3}=0$$

Multiplying by 3,

$$x(9-8)+y(-12-8)+z(15+12)-30+16=0$$

$$x-20y+27z-14=0$$

Equation of required plane is given by

$$x-20y+27z-14=0$$

The Plane Ex 29.11 Q18

The plane passes through the point $\vec{a}(1,2,-4)$

A vector in a direction perpendicular to

$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(\hat{i}-3\hat{j}+5\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$

is $\vec{n}=(2\hat{i}+3\hat{j}+6\hat{k})\times(\hat{i}+\hat{j}-\hat{k})$

$$\Rightarrow \vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 1 & 1 & -1\end{vmatrix}=-9\hat{i}+8\hat{j}-\hat{k}$$

Equation of the plane is $(\vec{r}-\vec{a}).\vec{n}=0$

$(\vec{r}-(\hat{i}+2\hat{j}-4\hat{k})).(-9\hat{i}+8\hat{j}-\hat{k})=0$

$\Rightarrow \vec{r}.(-9\hat{i}+8\hat{j}-\hat{k})=11$

Substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, we get the Cartesian form as

$-9x+8y-z=11$

The distance of the point $(9,-8,-10)$ from the plane

$$=\left|\frac{-9(9)+8(-8)-(-10)-11}{\sqrt{9^2+8^2+1^2}}\right|=\frac{146}{\sqrt{146}}=\sqrt{146}$$

The Plane Ex 29.11 Q19

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) \qquad ---(i)$$

Given that, required equation of plane is passing through $(3,4,1)$, so

$$a(x - 3) + b(y - 4) + c(z - 1) = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(0,1,0)$, so

$$a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$$
$$-3a - 3b - c = 0$$

$$3a + 3b + c = 0 \qquad ---(iii)$$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, line $\frac{x + 3}{2} = \frac{y - 3}{7} = \frac{z - 2}{5}$ is parallel to plane (ii), so

$$(2)(a) + (7)(b) + (c)(5) = 0$$
$$2a + 7b + 5c = 0 \qquad ---(iv)$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(3)(5) - (7)(1)} = \frac{b}{(2)(1) - (3)(5)} = \frac{c}{(3)(7) - (2)(3)}$$
$$\frac{a}{15 - 7} = \frac{b}{2 - 15} = \frac{c}{21 - 6}$$

$$\frac{a}{8} = \frac{b}{-13} = \frac{c}{15} = \lambda \text{(Say)}$$

$$\Rightarrow \quad a = 8\lambda,\ b = -13\lambda,\ c = 15\lambda$$

Put a, b, c in equation (ii),

$$a(x - 3) + b(y - 4) + c(z - 1) = 0$$
$$8\lambda(x - 3) + (-13\lambda)(y - 4) + (15\lambda)(z - 1) = 0$$
$$8\lambda x - 24\lambda - 13\lambda y + 52\lambda + 15\lambda z - 15\lambda = 0$$

$$8\lambda x - 13\lambda y + 15\lambda z + 13\lambda = 0$$

Dividing by λ, equation of required plane is,

$$8x - 13y + 15z + 13 = 0$$

The Plane Ex 29.11 Q20

Find the coordinates of the point where the line

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$

$\Rightarrow x = 3r+2, y = 4r-1, z = 2r+2$

Substituting in the equation of the plane $x - y + z - 5 = 0$,

we get $(3r+2) - (4r-1) + (2r+2) - 5 = 0$

$\Rightarrow r = 0$

$\Rightarrow x = 2, y = -1, z = 2$

Direction ratios of the line are 3,4,2

Direction ratios of a line perpendicular to the plane are 1, − 1, 1

$\sin\theta = \frac{3\times1+4\times-1+2\times1}{\sqrt{9+16+4}\sqrt{1+1+1}} = \frac{1}{\sqrt{87}}$

$\theta = \sin^{-1}\frac{1}{\sqrt{87}}$

The Plane Ex 29.11 Q21

We know that equation of line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$ ---(i)

Given that, line is passing through $(1,2,3)$.

So, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

It is given that line is perpendicular to plane $\vec{r}.\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$

So, normal to plane $\left(\vec{n}\right)$ is parallel to $\vec{b}$.

So, let $\vec{b} = \mu\vec{n} = \mu\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$

Put $\vec{a}$ and $\vec{b}$ in (i), equation of line is,

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left[\mu\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)\right]$

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \delta\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$ [As $\delta = \lambda\mu$]

Equation of required line is,

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \delta\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$

The Plane Ex 29.11 Q22

Direction ratios of the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

are (2,3,6)

Direction ratio of a line perpendicular to the plane

$10x + 2y - 11z = 3$ are 10, 2, − 11

If θ is the angle between the line and the plane, then

$\sin\theta = \frac{2\times10+3\times2+6\times-11}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+11^2}} = -\frac{40}{\sqrt{49}\sqrt{225}} = -\frac{40}{7\times15} = -\frac{8}{21}$

$\Rightarrow \theta = \sin^{-1}\left(-\frac{8}{21}\right)$

The Plane Ex 29.11 Q23

We know that, equation of line passing through (x_1, y_1, z_1) is given by

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \qquad ---(i)$$

Given that, required line is passing through $(1, 2, 3)$, so

$$\frac{x - 1}{a_1} = \frac{y - 2}{b_1} = \frac{z - 3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$
are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given line (ii) is parallel to

$\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$
$\Rightarrow \qquad x - y + 2z - 5 = 0$, so

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$

$$a_1 - b_1 + 2c_1 = 0 \qquad ---(iii)$$

Line (ii) is also parallel to plane
$\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$
$\Rightarrow \qquad 3x + y + z - 6 = 0$, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$$

$$3a_1 + b_1 + c_1 = 0 \qquad ---(iv)$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a_1}{(-1)(1)-(2)(1)} = \frac{b_1}{(3)(2)-(1)(1)} = \frac{c_1}{(1)(1)-(3)(-1)}$$
$$\frac{a_1}{-1-2} = \frac{b_1}{6-1} = \frac{c_1}{1+3}$$
$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow \qquad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$$

Put a_1, b_1, c_1 in equation (ii), so, equation line is given by

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$
$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

So, vector equation of required line is

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q24

Here, given mid line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$

So, normal vector of plane is parallel to line so,

Direction ratios of normal to plane are proparional to the direction ratios of line

Here,

$$\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$

cross multiplying the last two

$$-2\lambda = 4$$

$$\lambda = \frac{4}{-2}$$

$$\lambda = -2$$

The Plane Ex 29.11 Q25

The equation of a plane passing through $(-1, 2, 0)$ is

$a(x+1)+b(y-2)+c(z-0)=0.............(i)$

This passes through $(2, 2, -1)$

$\therefore\ a(2+1)+b(2-2)+c(-1-0)=0$

$3a-c=0............(ii)$

The plane in (i) is parallel to $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{1}$.

Theefore normal to the plane is perpendicular to the line.

$\therefore\ a(1)+b(2)+c(1)=0.........(iii)$

Solving (ii) and (iii) by cross multiplication we get,

$$\frac{a}{0-(-1)(2)}=\frac{b}{(1)(-1)-(3)(1)}=\frac{c}{(3)(2)-0}$$

$$\Rightarrow \frac{a}{2}=-\frac{b}{4}=\frac{c}{6}$$

$$\Rightarrow a=-\frac{b}{2}=\frac{c}{3}=\lambda(\text{say})$$

$$\Rightarrow a=\lambda, b=-2\lambda, c=3\lambda$$

Substituting $a=\lambda, b=-2\lambda, c=3\lambda$ in (i) we get,

$\lambda(x+1)-2\lambda(y-2)+3\lambda(z-0)=0$

$x-2y+3z+5=0$

$\therefore$ The required equation of the plane is $x-2y+3z+5=0$.