RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.15

The Plane Ex 29.15 Q1

$3x + 4y - 6z + 1 = 0$

Line passing through orgin and perpendicular to plane is given by

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{-6} = r(say)$$

So let the image of (0,0,0) is (3r, 4r, -6r)

Midpoint of (0,0,0) and (3r, 4r, -6r) lies on plane.

$$3\left(\frac{3r}{2}\right) + 2(4r) - 3(-6r) + 1 = 0$$

$$30.5r = -1$$

$$r = \frac{-2}{61}$$

So image is $\left(\frac{-6}{61}, \frac{-8}{61}, \frac{12}{61}\right)$

The Plane Ex 29.15 Q2

Here, we have to find reflection of the point $P(1,2,-1)$ in the plane $3x-5y+4z=5$

Let Q be the reflection of the point P and R be the mid-point of PQ.
Then, R lies on the plane $3x-5y+4z=5$.

Direction ratios of PQ are proportional to $3,-5,4$ and PQ is passing through $(1,2,-1)$.
So, equation of PQ is given by,

$$\frac{x-1}{3}=\frac{y-2}{-5}=\frac{z+1}{4}=\lambda\ (\text{Say})$$

Let Q be $(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

The coordinates of R are $\left(\frac{3\lambda+1+1}{2},\ \frac{-5\lambda+2+2}{2},\ \frac{4\lambda-1-1}{2}\right)=\left(\frac{3\lambda+2}{2},\ \frac{-5\lambda+4}{2},\ \frac{4\lambda-2}{2}\right)$

Since, R lies on the given plane $3x-5y+4z=5$

$$\therefore\quad 3\left(\frac{3\lambda+2}{2}\right)-5\left(-\frac{5\lambda+4}{2}\right)+4\left(\frac{4\lambda-2}{2}\right)=5$$

$$\Rightarrow\quad 9\lambda+6+25\lambda-20+16\lambda-8=10$$
$$\Rightarrow\quad 50\lambda-22=10$$
$$\Rightarrow\quad 50\lambda=10+22$$
$$\Rightarrow\quad 50\lambda=32$$
$$\Rightarrow\quad \lambda=\frac{16}{25}$$

So, $Q=(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

$$=\left(3\left(\frac{16}{25}\right)+1,\ -5\left(\frac{16}{25}\right)+2,\ 4\left(\frac{16}{25}\right)-1\right)$$

$$=\left(\frac{48}{25}+1,\ -\frac{16}{5}+2,\ \frac{64}{25}-1\right)$$

$$=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$$

So, reflection of $P(1,2,-1)=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$

The Plane Ex 29.15 Q3

We have to find foot of the perpendicular, say Q, drawn from $P(5,4,2)$ to the line

$$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda \quad \text{(say)}$$

Let Q be $(2\lambda-1, 3\lambda+3, -\lambda+1)$

Direction ratios of line PQ are $(2\lambda-1-5,\ 3\lambda+3-4, -\lambda+1-2)$ or $(2\lambda-6,\ 3\lambda-1, -\lambda-1)$

Here, line PQ is perpendicular to line given (AB).

So,

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(2\lambda-6)(2)+(3\lambda-1)(3)+(-\lambda-1)(-1)=0$$
$$4\lambda-12+9\lambda-3+\lambda+1=0$$
$$14\lambda-14=0$$
$$\lambda=\frac{14}{14}$$
$$\lambda=1$$

So, $Q=(2\lambda-1, 3\lambda+3, -\lambda+1)$

$$=(2(1)-1, 3(1)+3, -(1)+1)$$
$$=(2-1, 3+3, -1+1)$$
$$=(1,6,0)$$

Length of perpendicular PQ

$$=\sqrt{(5-1)^2+(4-6)^2+(2-0)^2} \qquad \text{[Using Distance formula]}$$
$$=\sqrt{16+4+4}$$
$$=\sqrt{24}$$
$$=2\sqrt{6}$$

So,

Foot of perpendicular is $(1,6,0)$

Length of the perpendicular is $2\sqrt{6}$ units

The Plane Ex 29.15 Q4

Here, we have to find image of the point $P(3,1,2)$ in the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=4$ or $2x-y+z=4$.

Let Q be the image of the point P.

So,
Direction ratios of normal to the point are $2,-1,1$

Direction ratios of line PQ perpendicular to $2,-1,1$ and PQ is passing through $(3,1,2)$.

So equation of PQ is

$$\frac{x-3}{2}=\frac{y-1}{-1}=\frac{z-2}{1}=\lambda \quad \text{(say)}$$

$$\left[\text{Using equation of line passing through } (x_1,y_1,z_1) \text{ is } \frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}\right]$$

General point on the line PQ is $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let Q be $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let R be the mid point of PQ. Then,

coordinates of R are $\left(\frac{2\lambda+3+3}{2},\frac{-\lambda+1+1}{2},\frac{\lambda+2+2}{2}\right)=\left(\frac{2\lambda+6}{2},\frac{-\lambda+2}{2},\frac{\lambda+4}{2}\right)$

Since, R lies on the plane $2x - y + z = 4$, we have,

$$2\left(\frac{2\lambda+6}{2}\right) - \left(\frac{-\lambda+2}{2}\right) + \left(\frac{\lambda+4}{2}\right) = 4$$

$\Rightarrow \quad 4\lambda + 12 + \lambda - 2 + \lambda + 4 = 8$

$\Rightarrow \quad 6\lambda = 8 - 14$

$\Rightarrow \quad \lambda = \frac{-6}{6}$

$\Rightarrow \quad \lambda = -1$

So,

Image of $P = Q\left(2(-1) + 3, -(-1) + 1, -1 + 2\right)$

Image of $P = (1, 2, 1)$

The equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$ is

$\bar{r} = 3\hat{i} + \hat{j} + 2\hat{k} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

The position vector of the image point is

$3\hat{i} + \hat{j} + 2\hat{k} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) = (3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+k)\hat{k}$

The position vector of the foot of the perpendicular is

$$\frac{\left[(3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+\lambda)\hat{k}\right] + \left[3\hat{i} + \hat{j} + 2\hat{k}\right]}{2}$$

$= (3+\lambda)\hat{i} + \left(1 - \frac{\lambda}{2}\right)\hat{j} + \left(2 + \frac{\lambda}{2}\right)\hat{k}$

Putting $\lambda = -1$ the position vector of the foot of the perpendicular is

$2\hat{i} + \frac{3}{2}\hat{j} + \frac{3}{2}\hat{k}$

The Plane Ex 29.15 Q5

$2x - 2y + 4z + 5 = 0$

$(1,1,2)$

$= \left|\frac{2-2+8+5}{\sqrt{1+1+4}}\right| = \frac{13}{\sqrt{6}}$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = k$

$x = 2k + 1$

$y = -2k + 1$

$z = 4k + 2$

Substitute $(x,y,z)=(2k+1, \ -2k+1, 4k+2)$ in plane equation

$2x - 2y + 4z + 5 = 0$

$4k + 2 + 4k - 2 + 16k + 8 + 5 = 0$

$24k = -13$

$k = \frac{-13}{24}$

$(x, y, z) = (\frac{-1}{12}, \frac{5}{3}, \frac{-1}{6})$

The Plane Ex 29.15 Q6

Here, we have to find distance of the point $P(1,-2,3)$ from the plane

$x - y + z = 5$ measured parallel to line AB, $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

Let Q be the mid point of the line joining P to plane.

Here, PQ is parallel to line AB

$\Rightarrow$ Direction ratios of line PQ are proportional to direction ratios of line AB

$\Rightarrow$ Direction ratios of line PQ are $2, 3, -6$ and PQ is passing through $P(1,-2,3)$.

So equation of PQ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 1}{2} = \frac{y + 2}{3} = \frac{z - 3}{-6} = \lambda \quad \text{(say)}$$

General point on line PQ is $(2\lambda + 1,\ 3\lambda - 2, -6\lambda + 3)$

Suppose coordinates of Q be $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

General point on line PQ is $(2\lambda + 1,\ 3\lambda - 2, -6\lambda + 3)$

Suppose coordinates of Q be $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Since Q lies on the plane $x - y + z = 5$

$$(2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) = 5$$

$$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$-7\lambda = 5 - 6$$

$$-7\lambda = -1$$

$$\lambda = \frac{1}{7}$$

Coordinate of Q = $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3) = \left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

Distance between $(1,-2,3)$ and plane = PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}}$$

$$= \sqrt{\frac{49}{49}}$$

$$= 1$$

Required distance = 1 unit

The Plane Ex 29.15 Q7

Let Q be the foot of the perpendicular.

Here, Direction ratios of normal to plane is $3, -1, -1$

$\Rightarrow$ Line PQ is parallel to normal to plane

$\Rightarrow$ Direction ratios of PQ are proportional to $3, -1, -1$ and PQ is passing through $P(2,3,7)$.

So,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{-1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (3\lambda + 2, \ -\lambda + 3, -\lambda + 7)$$

Coordinates of Q be $(3\lambda + 2, -\lambda + 3, -\lambda + 7)$

Point Q lies on the plane $3x - y - z = 7$.

So,

$$3(3\lambda + 2) - (-\lambda + 3) - (-\lambda + 7) = 7$$

$$9\lambda + 6 + \lambda - 3 + \lambda - 7 = 7$$

$$11\lambda = 7 + 4$$

$$11\lambda = 11$$

$$\lambda = \frac{11}{11}$$

$$\lambda = 1$$

$\therefore$ Coordinate of Q $= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$

$$= (3(1) + 2, -(1) + 3, -(1) + 7)$$

$$= (5, 2, 6)$$

Length of the perpendicular PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(2-5)^2 + (3-2)^2 + (7-6)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11}$$

The Plane Ex 29.15 Q8

Here, we have to find image of point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$

Let Q be the image of the point.

Here, Direction ratios of normal to plane are $2, -1, 1$

$\Rightarrow$ Direction ratios of PQ which is parallel to normal to the plane is proportional to $2, -1, 1$ and line PQ is passing through $P(1,3,4)$.

So, equation of line PQ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (2\lambda + 1, \ -\lambda + 3, \lambda + 4)$$

Let Q be $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Q is image of P, so R is the mid point of PQ

Coordinates of R $\left(\frac{2\lambda + 1 + 1}{2}, \frac{-\lambda + 3 + 3}{2}, \frac{\lambda + 4 + 4}{2}\right)$

$$= \left(\frac{2\lambda + 2}{2}, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

$$= \left(\lambda + 1, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

Point R is on th plane $2x - y + z + 3 = 0$

$$= 2(\lambda + 1) - \left(\frac{-\lambda + 6}{2}\right) + \left(\frac{\lambda + 8}{2}\right) = 0$$

$$4\lambda + 4 + \lambda - 6 + \lambda + 8 + 6 = 0$$

$$6\lambda = -12$$

$$\lambda = -2$$

So,

$$\text{Image } Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$$

$$= (-4 + 1, 2 + 3, -2 + 4)$$

$$= (-3, 5, 2)$$

Image of P $(1,3,4)$ is $(-3,5,2)$

The Plane Ex 29.15 Q9

Here, we have to find distance of a point A with position vector $\left(-\hat{i}-5\hat{j}-10\hat{k}\right)$ from the point of intersection of line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$ with plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Let the point of intersection of line and plane be $B\left(\vec{b}\right)$

The line and the plane will intersect when,

$$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left[\left(2+3\lambda\right)\hat{i}+\left(-1+4\lambda\right)\hat{j}+\left(2+12\lambda\right)\hat{k}\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left(2+3\lambda\right)\left(1\right)+\left(-1+4\lambda\right)\left(-1\right)+\left(2+12\lambda\right)\left(1\right)=5$$
$$2+3\lambda+1-4\lambda+2+12\lambda=5$$
$$11\lambda=5-5$$
$$\lambda=0$$

So, the point B is given by

$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\left(0\right)\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\overrightarrow{AB}=\vec{b}-\vec{a}$$
$$=\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(-\hat{i}-5\hat{j}-10\hat{k}\right) \qquad =\left(2\hat{i}-\hat{j}+2\hat{k}+\hat{i}+5\hat{j}+10\hat{k}\right)=\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\left|\overrightarrow{AB}\right| =\sqrt{\left(3\right)^2+\left(4\right)^2+\left(12\right)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Required distance = 13 units

The Plane Ex 29.15 Q10

$$x-2y+4z+5=0$$
$$(1,1,2)$$
$$=\left|\frac{1-2+4+5}{\sqrt{1+4+16}}\right|=\frac{8}{\sqrt{21}}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{4}=k$$
$$x=k+1$$
$$y=-2k+1$$
$$z=4k+2$$

Substitute (x,y,z)=(k+1, -2k+1, 4k+2) in plane equation

$$x-2y+4z+5=0$$
$$k+1+4k-2+16k+8+5=0$$
$$21k=-12$$
$$k=\frac{-12}{21}=\frac{-4}{7}$$
$$(x,y,z)=(\frac{3}{7},\frac{15}{7},\frac{-2}{7})$$

The Plane Ex 29.15 Q11

$2x-y+z+1=0$

$(3,2,1)$

$$=\left|\frac{6-2+1+1}{\sqrt{4+1+1}}\right|=\frac{6}{\sqrt{6}}=\sqrt{6}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-3}{2}=\frac{y-2}{-1}=\frac{z-1}{1}=k$$

$x=2k+3$

$y=-k+2$

$z=k-1$

Substitute $(x,y,z)=(2k+3,\ -k+2,\ k-1)$ in plane equation

$2x-y+z+1=0$

$4k+6+k-2+k-1+1=0$

$6k=-4$

$$k=\frac{-4}{6}=\frac{-2}{3}$$

$$(x,y,z)=\left(\frac{5}{3},\frac{8}{3},\frac{-5}{3}\right)$$

The Plane Ex 29.15 Q12

Given equation of the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}-2\hat{k}\right)+1=0$

Thus, the direction ratios normal to the plane are 6, −3 *and* −2

Hence the direction cosines to the normal to the plane are

$$\frac{6}{\sqrt{6^2+(-3)^2+(-2)^2}},\frac{-3}{\sqrt{6^2+(-3)^2+(-2)^2}},\frac{-2}{\sqrt{6^2+(-3)^2+(-2)^2}}$$

$$=\frac{6}{7},\frac{-3}{7},\frac{-2}{7}$$

$$=\frac{-6}{7},\frac{3}{7},\frac{2}{7}$$

The direction cosines of the unit vector perpendicular to the plane are same as the direction cosines of the normal to the plane.

Thus, the direction cosines of the unit vector perpendicular to the plane are: $\frac{-6}{7},\frac{3}{7},\frac{2}{7}$

The Plane Ex 29.15 Q13

Consider the given equation of the plane $2x-3y+4z-6=0$
The direction ratios of the normal to the plane are $2,\ -3$ *and* 4
Thus, the directio ratios of the line perpendicular to the plane are $2,\ -3$ *and* 4.
The equation of the line passing (x_1,y_1,z_1) *having direction ratios* a,b *and* c *is*

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Thus, the equation of the line passing through the origin with direction ratios $2,\ -3$ *and* 4 *is*

$$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{4}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{-3}=\frac{z}{4}=r, \text{ where } r \text{ is some constant}$$

Any point on the line is of the form $2r,\ -3r$ *and* $4r$
If the point $P(2r,\ -3r,\ 4r)$ *lies on the plane* $2x-3y+4z-6=0$,
it should satisfies the equation, $2x-3y+4z-6=0$
Thus, we have,

$$2(2r)-3(-3r)+4(4r)-6=0$$
$$\Rightarrow 4r+9r+16r-6=0$$
$$\Rightarrow 29r=6$$
$$\Rightarrow r=\frac{6}{29}$$

Thus, the coordinates of the point of intersection of the perpendicular from the origin and the plane are:

$$P\left(2\times\frac{6}{29},\ -3\times\frac{6}{29},\ 4\times\frac{6}{29}\right)=P\left(\frac{12}{29},\ \frac{18}{29},\ \frac{24}{29}\right)$$

The Plane Ex 29.15 Q14

The length of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

$$d = \left|\frac{2 - 3 + 8 + 5}{\sqrt{4 + 4 + 16}}\right| = \frac{12}{2\sqrt{6}} = \sqrt{6}$$

Let the foot of perpendicular be (x, y, z). So DR's are in proportional

$$\frac{x - 1}{2} = \frac{y - \frac{3}{2}}{-2} = \frac{z - 2}{4} = k$$

$$x = 2k + 1$$

$$y = -2k + \frac{3}{2}$$

$$z = 4k + 2$$

So using values of x, y, z in equation of the plane we have,

$$2(2k + 1) - 2\left(-2k + \frac{3}{2}\right) + 4(4k + 2) + 5 = 0$$

$$4k + 2 + 4k - 3 + 16k + 8 + 5 = 0$$

$$24k = -12$$

$$k = -\frac{1}{2}$$

$$(x, y, z) = \left(0, \frac{5}{2}, 0\right)$$