

RD Sharma
Solutions
Class 12 Maths
Chapter 30
Ex 30.1

Linear Programming Ex 30.1 Q1

The given data may be put in the following tabular form: -

Gadget	Foundry	Machine-shop	Profit
<i>A</i>	10	5	Rs 30
<i>B</i>	6	4	Rs 20
Firm's capacity per week	1000	600	

Let required weekly production of gadgets A and B be x and y respectively.

Given that, profit on each gadget A is Rs 30

So, profit on x gadget of type $A = 30x$

Profit on each gadget of type $B = Rs 20$

So, profit on y gadget of type $B = 20y$

Let Z denote the total profit, so

$$Z = 30x + 20y$$

Given, production of one gadget A requires 10 hours per week for foundry and gadget B requires 6 hours per week for foundry.

So, x units of gadget A requires $10x$ hours per week and y units of gadget B requires $6y$ hours per week, But the maximum capacity of foundry per week is 1000 hours, so

$$10x + 6y \leq 1000$$

This is first constraint.

Given, production of one unit gadget A requires 5 hours per week of machine shop and production of one unit of gadget B requires 4 hours per week of machine shop.

So, x units of gadget A requires $5x$ hours per week and y units of gadget B requires $4y$ hours per week, but the maximum capacity of machine shop is 600 hours per week

So, $5x + 4y \leq 600$

This is second constraint.

Hence, mathematical formulation of LPP is:

Find x and y which

Maximize $Z = 30x + 20y$

Subject to constraints,

$$10x + 6y \leq 1000$$

$$5x + 4y \leq 600$$

And, $x, y \geq 0$

[Since production cannot be less than zero]

Linear Programming Ex 30.1 Q2

The given information can be written in tabular form as below:

Product	Machine hours	Labour hours	Profit
A	1	1	Rs 60
B	-	1	Rs 80
Total capacity	400 for A	500	
Minimum supply of product B is 200 units.			

Let production of product A be x units and production of product B be y units.

Given, profit on one unit of product $A = \text{Rs } 60$

So, profit on x unit of product $A = \text{Rs } 60x$

Given, profit on one unit of product $B = \text{Rs } 80$

So, profit on y units of product $B = \text{Rs } 80y$

Let Z denote the total profit, so

$$Z = 60x + 80y$$

Given, minimum supply of product B is 200

So, $y \geq 200$ (First constraint)

Given that, production of one unit of product A requires 1 hour of machine hours, so x units of product A requires x hours but given total machine hours available for product A is 400 hours, so

$$x \leq 400 \quad (\text{Second constraint})$$

Given, each unit of product A and B requires one hour of labour hour, so x units of product A require x hours and y units of product B require y hours of labour hours but total labour hours available are 500, so

$$x + y \leq 500 \quad (\text{Third constraint})$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{Minimize } Z = 60x + 80y$$

Subject to constraints,

$$y \geq 200$$

$$x \leq 400$$

$$x + y \leq 500$$

$$x, y \geq 0$$

[Since production of product cannot be less than zero]

Linear Programming Ex 30.1 Q3

Product	Machine (M_1)	Machine (M_2)	Profit
A	4	2	3
B	3	2	2
C	5	4	4
Capacity maximum	2000	2500	

Let required production of product A, B and C be x, y and z units respectively.

Given, profit on one unit of product A, B and C are Rs 3, Rs 2, Rs 4, so

Profit on x unit of A , y unit of B and z unit of C are given by Rs. $3x, 2y, 4z$.

Let U be the total profit, so

$$U = 3x + 2y + 4z$$

Given, one unit of product A, B and C requires 4,3 and 5 minutes on machine M_1 . So, x units of product A , y units of B and z units of product C need $4x, 3y$ and $5z$ minutes on machine M_1 is 2000 minutes, so

$$4x + 3y + 5z \leq 2000 \quad (\text{First constraint})$$

Given, one unit of product A, B and C requires 2,2 and 4 minutes on machine M_2 . So, x units of A , y units of B and z units of C require $2x, 2y$ and $4z$ minutes on machine M_2 is 2500 minutes, so

$$2x + 2y + 4z \leq 2500 \quad (\text{Second constraint})$$

Also, given that firm must manufacture 100 A 's, 200 B 's and 50 C 's but not more than 150 A 's.

$$100 \leq x \leq 150$$

$$y \geq 200 \quad (\text{Other constraints})$$

$$z \geq 50$$

Hence, mathematical formulation of LPP is :-

Find x, y and z which

$$\text{maximize } U = 3x + 2y + 4z$$

Subject to constraints,

$$4x + 3y + 5z \leq 2000$$

$$2x + 2y + 4z \leq 2500$$

$$100 \leq x \leq 150$$

$$y \geq 200$$

$$z \geq 50$$

And, $x, y, z \geq 0$ [Since, x, y, z are non-negative]

Linear Programming Ex 30.1 Q4

Given information can be written in tabular form as below:

Product	M_1	M_2	Profit
A	1	2	2
B	1	1	3
Capacity	6 hours 40 min = 400 min.	10 hours = 600 min.	

Let required production of product A be x units and product B be y units.

Given, profit on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B will be Rs $2x$ and Rs $3y$ respectively.

Let total profit be Z , so

$$Z = 2x + 3y$$

Given, production of one unit of product A and B require 1 and 1 minute on machine M_1 respectively, so production of x units of product A and y units of product B require x minutes and y minutes on machine M_1 but total time available on machine M_1 is 400 minutes, so

$$x + y \leq 400 \quad (\text{First constraint})$$

Given, production of one unit of product A and B require 2 minutes and 1 minutes on machine M_2 respectively. So production of x units of product A and y units of product B require $2x$ minutes and y minutes respectively on machine M_2 but machine M_2 is available for 600 minutes, so

$$2x + y \leq 600 \quad (\text{Second constraint})$$

Hence, mathematical formulation of LPP is:-

Find x and y which

$$\text{maximize } Z = 2x + 3y$$

Subject to constraints,

$$x + y \leq 400$$

$$2x + y \leq 600$$

and, $x, y \geq 0$ [Since production of product can not be less than zero]

Linear Programming Ex 30.1 Q5

Plant	A	B	C	Cost
I	50	100	100	2500
II	60	60	200	3500
Monthly demand	2500	3000	7000	

Let plant I requires x days and plant II requires y days per month to minimize cost.

Given, plant I and II costs Rs 2500 perday and Rs 3500 perday respectively, so cost to run plant I and II is Rs 2500 x and Rs 3500 y per month.

Let Z be the total cost per month, so

$$Z = 2500x + 3500y$$

Given, production of tyre A from plant I and II is 50 and 60 respectively, so production of tyre A from plant I and II will be 50 x and 60 y respectively per month but the maximum demand of tyre A is 2500 per month so,

$$50x + 60y \geq 2500 \quad [\text{First constraint}]$$

Given, production of tyre B from plant I and II is 100 and 60 respectively, so production of tyre B from plant I and II will be 100 x and 60 y per month respectively but the maximum demand of tyre B is 3000 per month, so

$$100x + 60y \geq 3000 \quad [\text{Second constraint}]$$

Given, production of tyre C from plant I and II is 100 and 200 respectively. So production of tyre B from plant I and II will be 100 x and 200 y per month respectively but the maximum demand of tyre C is 7000 per day, so

$$100x + 200y \geq 7000 \quad [\text{Third constraint}]$$

Hence, mathematical formulation of LPP is..

Find x and y which

$$\text{Minimize } Z = 2500x + 3500y$$

Subject to constraint,

$$50x + 60y \geq 2500$$

$$100x + 60y \geq 3000$$

$$100x + 200y \geq 7000$$

And, $x, y \geq 0$ [Since number of days can not be less than zero]

Linear Programming Ex 30.1 Q6

Product	Man hours	Maximum demand	Profit
A	5	7000	60
B	3	10000	40
Total capacity	45000		

Let required production of product A be x units and production of product B be y units.

Given, profits on one unit of product A and B are Rs 60 and Rs 40 respectively, so profits on x units of product A and y units of product B are Rs $60x$ and Rs $40y$.

Let Z be the total profit, so

$$Z = 60x + 40y$$

Given, production of one unit of product A and B require 5 hours and 3 hours respectively man hours, so x unit of product A and y units of product B require $5x$ hours and $3y$ hours of man hours respectively but total man hours available are 45000 hours, so

$$5x + 3y \leq 45000 \quad (\text{First constraint})$$

Given, demand for product A is maximum 7000, so

$$x \leq 7000 \quad (\text{Second constraint})$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{maximize } Z = 60x + 40y$$

Subject to constraints,

$$5x + 3y \leq 45000$$

$$x \leq 7000$$

$$y \leq 10000$$

$$x, y \geq 0$$

[Since production can not be less than zero]

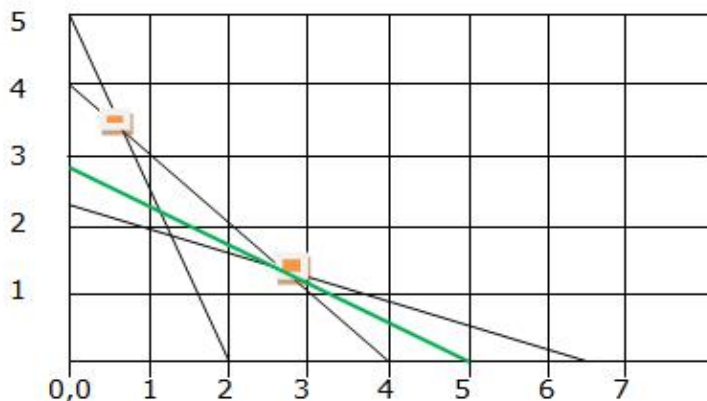
Let x and y be the packets of 25 gm of Food I and Food II purchased. Let Z be the price paid. Obviously price has to be minimized.

Take a mass balance on the nutrients from Food I and II,

$$\begin{array}{ll} \text{Calcium} & 10x + 4y \geq 20 \\ & 5x + 2y \geq 10 \quad \dots\dots(i) \\ \text{Protein} & 5x + 5y \geq 20 \\ & x + y \geq 4 \quad \dots\dots(ii) \\ \text{Calories} & 2x + 6y \geq 13 \quad \dots\dots(iii) \end{array}$$

These become the constraints for the cost function, Z to be minimized i.e., $0.6x + y = Z$, given cost of Food I is Rs 0.6/- and Rs 1/- per lb

From (i), (ii) & (iii) we get points on the X & Y-axis as $[0, 5]$ & $[2, 0]$; $[0, 4]$ & $[4, 0]$; $[0, 13/6]$ & $[6.5, 0]$
Plotting these



The smallest value of Z is 2.9 at the point $(2.75, 1.25)$. We cannot say that the minimum value of Z is 2.9 as the feasible region is unbounded.

Therefore, we have to draw the graph of the inequality $0.6x + y < 2.9$

Plotting this to see if the resulting line (in green) has any point common with the feasible region. Since there are no common points this is the minimum value of the function Z and the mix is

Food I = 2.75 lb; Food II = 1.25 lb; Price = Rs 2.9

When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Here the feasible region is the unbounded region

A-B-C-D

Computing the value of Z at the corner points of the feasible region ABHG

Point	Corner point	Value of $Z = 0.6x + y$
A	2, 5	6.2
B	0.67, 3.33	3.73
C	2.75, 1.25	2.9
D	6.5, 2.16	6.06

Linear Programming Ex 30.1 Q8

Given information can be tabulated as:-

Product	Grinding	Turning	Assembling	Testing	Profit
A	1	3	6	5	2
B	2	1	3	4	3
Maximum capacity	30 hours	60 hours	200 hours	200 hours	

Let required production of product A and B be x and y respectively

Given, profits on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B are given by $2x$ and $3y$ respectively. Let Z be total profit, so

$$Z = 2x + 3y$$

Given, production of 1 unit of product A and B require 1 hour and 2 hours of grinding respectively, so, production of x units of product A and y units of product B require x hours and $2y$ hours of grinding respectively but maximum time available for grinding is 30 hours, so

$$x + 2y \leq 30 \quad (\text{First constraint})$$

Given, production of 1 unit of product A and B require 3 hours and 1 hours of turning respectively, so x units of product A and y units of product B require $3x$ hours and y hours of turning respectively but total time available for turning is 60 hours, so

$$3x + y \leq 60 \quad (\text{Second constraint})$$

Given, production of 1 unit of product A and B require 6 hour and 3 hours of assembling respectively, so production of x units of product A and y units of product B require $6x$ hours and $3y$ hours of assembling respectively but total time available for assembling is 200 hours, so

$$6x + 3y \leq 200 \quad (\text{Third constraint})$$

Given, production of 1 unit of product A and B require 5 hours and 4 hours of testing respectively, so production of x units of product A and y units of product B require $5x$ hours and $4y$ hours of testing respectively but total time available for testing is 200 hours, so

$$5x + 4y \leq 200 \quad (\text{Fourth constraint})$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{maximize } Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \leq 30$$

$$3x + y \leq 60$$

$$6x + 3y \leq 200$$

$$5x + 4y \leq 200$$

and, $x, y \geq 0$

[Since production can not be negative]

Linear Programming Ex 30.1 Q9

Given information can be tabulated as below:

Foods	Vitamin A	Vitamin B	Cost
F_1	2	3	5
F_2	4	2	2.5
Minimum daily requirement	40	50	

Let required quantity of food F_1 be x units and quantity of food F_2 be y units.

Given, costs of one unit of food F_1 and F_2 are Rs 5 and Rs 2.5 respectively, so costs of x units of food F_1 and y units of food F_2 are Rs $5x$ and Rs $2.5y$ respectively.

Let Z be the total cost, so

$$Z = 5x + 2.5y$$

Given, one unit of food F_1 and food F_2 contain 2 and 4 units of vitamin A respectively, so x unit of Food F_1 and y units of food F_2 contain $2x$ and $4y$ units of vitamin A respectively, but minimum requirement of vitamin A is 40 unit, so

$$2x + 4y \geq 40 \quad (\text{First constraint})$$

Given, one unit of food F_1 and food F_2 contain 3 and 2 units of vitamin B respectively, so x unit of Food F_1 and y units of food F_2 contain $3x$ and $2y$ units of vitamin B respectively, but minimum daily requirement of vitamin B is 40 unit, so

$$3x + 2y \geq 50 \quad (\text{Second constraint})$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{Minimize } Z = 5x + 2.5y$$

Subject to constraint,

$$2x + 4y \geq 40$$

$$3x + 2y \geq 50$$

$$x, y \geq 0 \quad [\text{Since requirement of food } F_1 \text{ and } F_2 \text{ can not be less than zero.}]$$

Let the number of automobiles produced be x and let the number of trucks produced be y .

Let Z be the profit function to be maximized.
 $Z = 2,000x + 30,000y$

The constraints are on the man hours worked

$$\begin{array}{llll} \text{Shop A} & 2x + 5y \leq 180 & \text{(i)} & \text{assembly} \\ \text{Shop B} & 3x + 3y \leq 135 & \text{(ii)} & \text{finishing} \\ & x \geq 0 ; y \geq 0 & & \end{array}$$

Corner points can be obtained from

$$2x + 5y = 180 \Rightarrow x=0; y=36 \text{ and } x=90; y=0$$

$$3x + 3y \leq 135 \Rightarrow x=0; y=45 \text{ and } x=45; y=0$$

Solving (i) & (ii) gives $x = 15$ & $y = 30$

Corner point	Value of $Z = 2,000x + 30,000y$
0,0	0
0, 36	10,80,000
15, 30	9,30,000
45, 0	90,000

0 automobiles and 36 trucks will give max profit of 10,80,000/-

Linear Programming Ex 30.1 Q11

	Taylor A		Taylor B	Limit
Variable	x		y	
Shirts	$6x$	+	$10y$	≥ 60
Pants	$4x$	+	$4y$	≥ 32
Earn Rs.	150	+	200	Z

The above LPP can be presented in a table above.

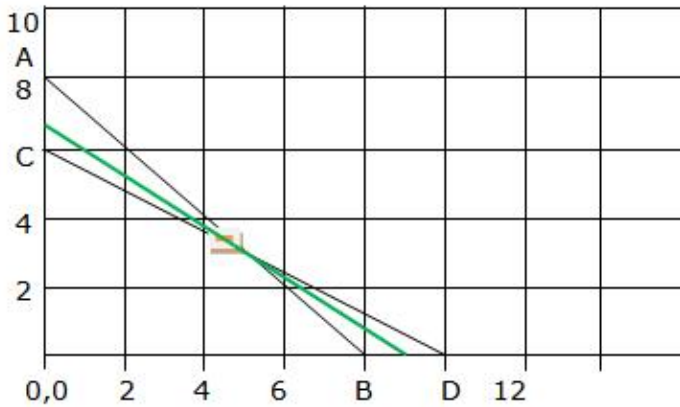
To minimize labour cost means to assume minimize the earnings i.e., $\text{Min } Z = 150x + 200y$

s.t. the constraints

$$\begin{array}{ll} x \geq 0; y \geq 0 & \text{at least 1 shirt \& pant is required} \\ 6x + 10y \geq 60 & \text{require at least 60 shirts} \\ 4x + 4y \geq 32 & \text{require at least 32 pants} \end{array}$$

Solving the above inequalities as equations we get,
 $x = 5$ and $y = 3$

other corner points obtained are $[0, 6]$ & $[10, 0]$
 $[0, 8]$ & $[8, 0]$



The feasible region is the open unbounded region A-E-D

Point E(5, 3) may not be the minimal value. So, plot $150x + 200y < 1350$ to see if there is a common region with A-E-D

The green line has no common point, therefore

Corner point	Value of $Z = 150x + 200y$
0,8	0
10, 0	1500
5, 3	1350

Stitching 5 shirts and 3 pants minimizes labour cost to Rs.1350/-

Linear Programming Ex 30.1 Q12

	Model 314		Model 535	Limit
Variable	x		y	
F class	20x	+	20y	≥ 160
T class	30x	+	60y	≥ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in a table above.

The flight cost is to be minimized i.e. $\text{Min } Z = x + 1.5y$
s.t. the constraints

$x \geq 2$ at least 2 planes of model 314 must be used

$y \geq 0$ at least 1 plane of model 535 must be used

$20x + 20y \geq 160$ require at least 160 F class seats

$30x + 60y \geq 300$ require at least 300 T class seats

Solving the above inequalities as equations we get,

When $x=0$, $y=8$ and when $y=0$, $x=8$

When $x=0$, $y=5$ and when $y=0$, $x=10$

We get an unbounded region 8-E-10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	Value of $Z = x + 1.5y$
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of $Z (x+1.5y < 9)$ is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.



Linear Programming Ex 30.1 Q13

Given information can be tabulated as below

Sets	Time requirement	Points
I	3	5
II	2	
III	4	6
Time for all three sets = $3\frac{1}{2}$ hours		
Time for Set I and Set II = $2\frac{1}{2}$ hours		
Number of questions maximum 100		

Let he should x, y, z questions from set I, II and III respectively.

Given, each question from set I, II, III earn 5, 4, 6 points respectively, so x questions of set I, y questions of set II and z questions of set III earn $5x, 4y$ and $6z$ points, let total point credit be U

$$\text{So, } U = 5x + 4y + 6z$$

Given, each question of set I, II and III require 3, 2 and 4 minutes respectively, so x questions of set I, y questions of set II and z questions of set III require $3x, 2y$ and $4z$ minutes respectively but given that total time to devote in all three sets is

$$3\frac{1}{2} \text{ hours} = 210 \text{ minutes and first two sets is } 2\frac{1}{2} \text{ hours} = 150 \text{ minutes}$$

So,

$$3x + 2y + 4z \leq 210 \quad (\text{First constraint})$$

$$3x + 2y \leq 150 \quad (\text{Second constraint})$$

Given, total number of questions cannot exceed 100

$$\text{So, } x + y + z \leq 100 \quad (\text{Third constraint})$$

Hence, mathematical formulation of LPP is

Find x and y which

$$\text{maximize } U = 5x + 4y + 6z$$

Subject to constraint,

$$3x + 2y + 4z \leq 210$$

$$3x + 2y \leq 150$$

$$x + y + z \leq 100$$

$$x, y, z \geq 0$$

[Since number of questions to solve from each set]
cannot be less than zero

Linear Programming Ex 30.1 Q14

Given information can be tabulated as below

Product	Yield	Cultivation	Price	Fertilizers
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg

Average 2000 kg/per acre
Total land = 100 Acre
Cost of fertilizers = Rs 0.50 per kg.
A total of 400 days of cultivation labour with Rs 20 per day

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z Acre respectively.

Given, costs of cultivation and harvesting of tomatoes, lettuce and radishes are $5 \times 20 = \text{Rs } 100$, $6 \times 20 = \text{Rs } 120$, $5 \times 20 = \text{Rs } 100$ respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes $100 \times 0.50 = \text{Rs } 50$, $100 \times 0.50 = \text{Rs } 50$ and $50 \times 0.50 = \text{Rs } 25$ respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are $\text{Rs } 100 + 50 = \text{Rs } 150x$, $\text{Rs } 120 + 50 = \text{Rs } 170y$ and radishes are $\text{Rs } 100 + 25 = \text{Rs } 125z$ respectively total selling price of tomatoes, lettuce and radishes, according to yield are $2000 \times 1 = \text{Rs } 2000x$, $3000 \times 0.75 = \text{Rs } 2250y$ and $100 \times 2 = \text{Rs } 2000z$ respectively.

Let U be the total profit,

So,

$$U = (2000x - 150x) + (2250y - 170y) + (2000z - 125z)$$

$$U = 1850x + 2080y + 1875z$$

Given, farmer has 100 acre form

So, $x + y + z \leq 100$ (First constraint)

Number of cultivation and harvesting days are 400

So, $5x + 6y + 5z \leq 400$

Hence, mathematical formulation of LPP is

Find x, y, z which

maximize $U = 1850x + 2080y + 1875z$

Subject to constraint,

$$x + y + z \leq 100$$

$$5x + 6y + 5z \leq 400$$

$$x, y, z \geq 0$$

[Since from used for cultivation cannot be less than zero.]

Given information can be tabulated as below:

Product	Department 1	Department 2	Selling price	Labour cost	Raw material cost
<i>A</i>	3	4	25	16	4
<i>B</i>	2	6	30	20	4
Capacity	130	260			

Let the required product of product *A* and *B* be x and y units respectively.

Given, labour cost and raw material cost of one unit of product *A* is Rs 16 and

Rs 4, so total cost of product *A* is Rs 16 + Rs 4 = Rs 20

And given selling price of 1 unit of product *A* is Rs 25,

So, profit on one unit of product

$$A = 25 - 20 = \text{Rs } 5$$

Again, given labour cost and raw material cost of one unit of product *B* is Rs 20 and Rs 4

So, that cost of product *B* is Rs 20 + Rs 4 = Rs 24

And given selling price of 1 unit of product *B* is Rs 30

So, profit on one unit of product *B* = 30 - 24 = Rs 6

Hence, profits on x unit of product *A* and y units of product *B* are Rs $5x$ and Rs $6y$ respectively.

Let Z be the total profit, so $Z = 5x + 6y$

Given, production of one unit of product *A* and *B* need to process for 3 and 4 hours

respectively in department 1, so production of x units of product *A* and y units of

product *B* need to process for $3x$ and $4y$ hours respectively in Department 1. But

total capacity of Department 1 is 130 hour,

So, $3x + 2y \leq 130$ (First constraint)

Given, production of one unit of product *A* and *B* need to process for 4 and 6 hours

respectively in department 2, so production of x units of product *A* and y units of

product *B* need to process for $4x$ and $6y$ hours respectively in Department 2 but total

capacity of Department 2 is 260 hours

So, $4x + 6y \leq 260$ (Second constraint)

Hence, mathematical formulation of LPP is,

Find x and y which

Maximize $Z = 5x + 6y$

Subject to constraint,

$$3x + 2y \leq 130$$

$$4x + 6y \leq 260$$

$$x, y \geq 0$$

[Since production cannot be less than zero]