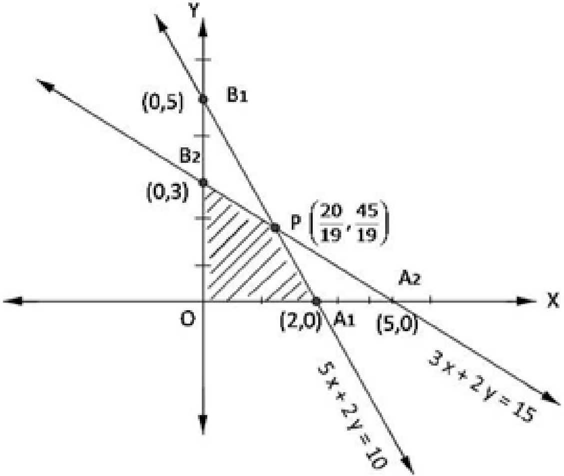
RD Sharma Solutions Class 12 Maths Chapter 30 Ex 30.2

Linear Programming Ex 30.2 Q1

Converting the given inequations into equations, we get

$$3x + 5y = 15,\ 5x + 2y = 10,\ x = 0,\ y = 0$$



Region represented by $5x + 2y \le 10$: The line meets coordinate axes at $A_1(2,0)$ and $B_1(0,5)$ respectively. Join these points to obtain the line $5x + 2y = 10$, clearly, $(0,0)$ satisfies the in eqation $5x + 2y \le 10$, so, the region in xy-plane that contains the origin represents the solution set if the given in equation.

Region represented by $3x + 5y \le 10$: The line meets coordinate axes at $A_2(5,0)$ and $B_2(0,3)$ respectively. Join these points to obtain the line $3x + 5y = 15$, clearly, $(0,0)$ satisfies the in eqation $3x + 5y \le 15$, so, the region in xy-plane contains the origin represents the solution set if the given in equation.

Region represented by $x \ge 0$, $y \ge 0$: It clearly represents first quadrant of xy-plane.
Common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $O(0,0)$, $A(2,0)$, $P\left(\frac{20}{19}, \frac{45}{19}\right)$, $B_2(0,3)$.

The value of $Z = 5x + 3y$ at

$O(0,0) \quad = 5 \times + 3 \times 0$

$A(2,0) \quad = 5 \times 2 + 3 \times 0 = 10$

$P\left(\frac{20}{19}, \frac{45}{19}\right) \quad = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

$B_2(0,3) \quad = 5 \times 0 + 3 \times 3 = 9$

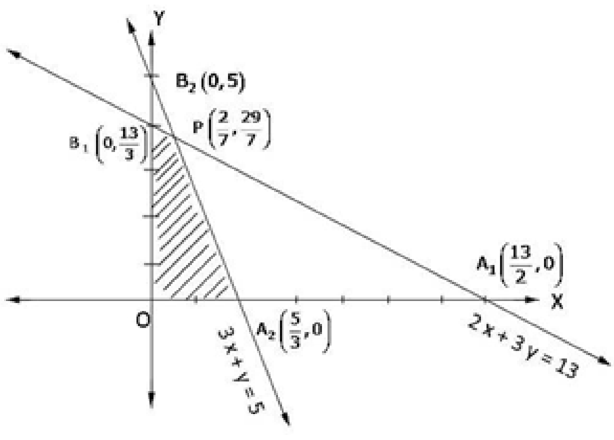
Clearly, Z is maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$

So, $x = \frac{20}{19}$, $y = \frac{45}{19}$, maximum $Z = \frac{235}{19}$

Linear Programming Ex 30.2 Q3

Converting the given inequations into equations, we get

$2x + 3y = 13$, $3x + y = 5$, and $x = 0$, $y = 0$



Region represented by $2x+3y \le 13$: The line meets coordinate axes at $A_1\left(\frac{13}{2},0\right)$ and $B_1\left(0,\frac{13}{3}\right)$ respectively. Join these points to obtain the line $2x+3y=13$, clearly, $(0,0)$ satisfies the in eqation $2x+3y \le 13$, so, the region in xy-plane that contains origin represents the solution set of $2x+3y \le 13$.

Region represented by $3x+y \le 5$: The line meets coordinate axes at $A_2\left(\frac{5}{3},0\right)$ and $B_2(0,5)$ respectively. Join these points to obtain the line $3x+y=5$, clearly, $(0,0)$ satisfies the in eqation $3x+y \le 5$, so, the region in xy-plane that contains origin represents the solution set of $3x+y \le 5$.

Region represented by $x, y \ge 0$: It clearly represent first quadrant of xy-plane. The common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $0(0,0)$, $A\left(\frac{5}{3},0\right)$, $P\left(\frac{2}{7},\frac{29}{7}\right)$, $B_2\left(0,\frac{13}{3}\right)$.

The value of $Z=9x+3y$ at

$0(0,0) \quad = 9(0)+3(0)=0$

$A_1\left(\frac{5}{3},0\right) \quad = 9\left(\frac{5}{3}\right)+3(0)=15$

$P\left(\frac{2}{7},\frac{29}{7}\right) \quad = 9\left(\frac{2}{7}\right)+3\left(\frac{29}{7}\right)=15$

$B_2\left(0,\frac{13}{3}\right) \quad = 9(0)+3\left(\frac{13}{3}\right)=13$

Clearly, Z is maximum at at every point on the line joining A_1 and P, so

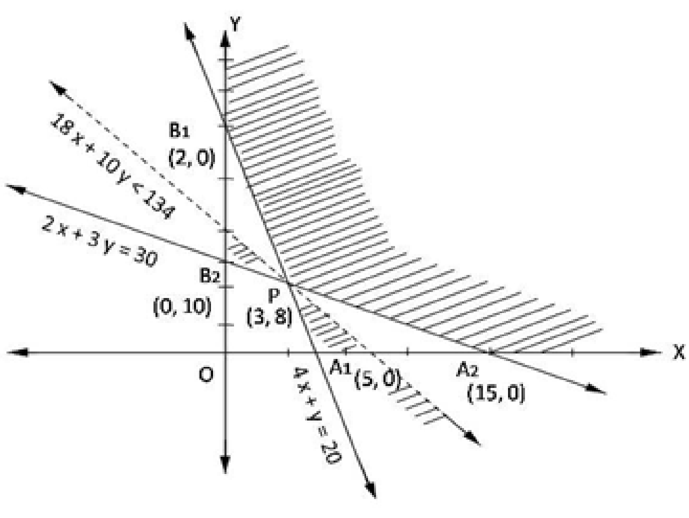
$x=\frac{5}{3}$ or $\frac{2}{7}$, $y=0$ or $\frac{29}{7}$

and maximum $Z=15$.

Linear Programming Ex 30.2 Q3

Converting given inequations into equations as

$$4x+y=20,\ 2x+3y=30,\ x=0,\ y=0$$



Region represented by in equation $4x + y \geq 20$: The line $4x + y = 20$ meets the coordinate axes at $A_1(5,0)$ and $B_1(0,20)$. Joining A_1B_1 we get $4x + y = 20$. Clearly, $(0,0)$, also does not satisfies the in eqation, so the region does not containing the origin represents the in equality $4x + y \geq 20$ in the xy-plane.

Region represented by in equation $2x + 3y \geq 30$: The line $2x + 3y = 30$ meets the coordinate axes at $A_2(15,0)$ and $B_2(0,20)$. Obtain line $2x + 3y = 30$ by joining A_2 and B_2. Clearly, $(0,0)$, does not satisfies the in eqation $2x + 3y \geq 30$, so the region does not containing the origin represents the in equality $2x + 3y \geq 30$ in the xy-plane.

Region represented by $x, y \geq 0$: $x, y \geq 0$ represents the first quadrant of xy-plane.

The shaded region is the feasible region with corner points $A_2(15,0)$, $P(3,8)$, $B_1(0,20)$ where P is obtained by solving $2x + 3y = 30$ and $4x + y = 20$ simultaneously.

The value of $Z = 18x + 10y$ at

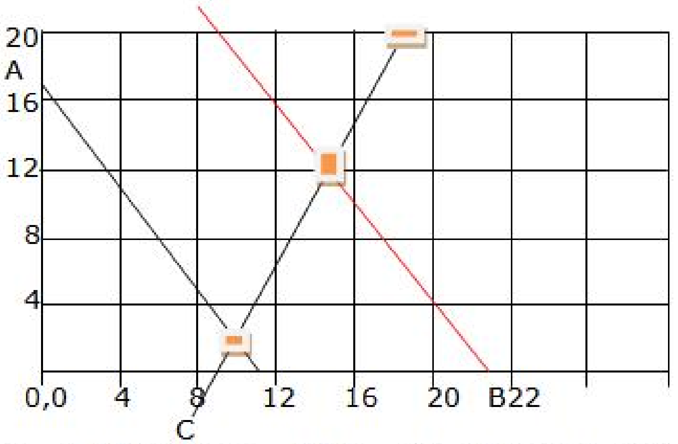
$A_2(15,0)$	$= 18(15) + 10(8) = 270$
$P(3,8)$	$= 18(3) + 10(8) = 134$
$B_1(0,20)$	$= 18(0) + 10(20) = 200$

Clearly, Z is manimum at $x = 3$ and $y = 8$. The minimum value of Z is 134.

We observe that open half plane represented by $18x + 10y < 134$ does not have points in common with the solution region. So Z has

Minimum value $= 134$ at $x = 3$, $y = 8$

Linear Programming Ex 30.2 Q4



2x-y ≥ 18 ; when x = 12, y = 6 & when y=0, x=9
3x+2y ≤ 34 ; when x = 0, y = 17 & when y=0, x=34/3

Plotting these points gives line AB and CD
The feasible area is the unbounded area D-E-12

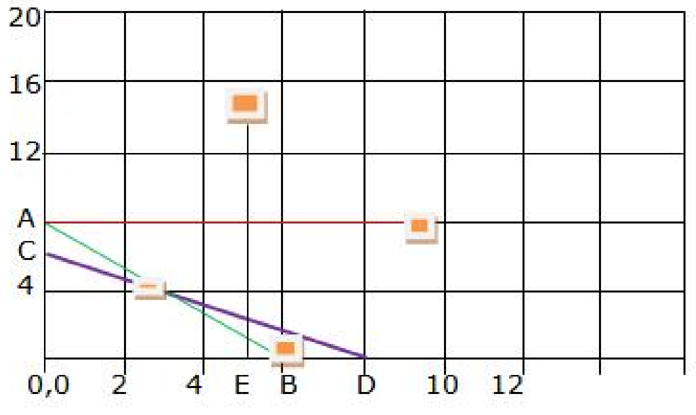
Corner point	Value of Z = 50x + 30y
10, 2	560
11.3, 17	1076.66

The maximize value of Z = 50x+30y, occurs at x = 34/3, y = 17

Since we have an unbounded region as the feasible area plot 50x + 30y > 1076.66

Since the region D-F-B has common points with region D-E-12 the problem has no optimal maximum value.

Linear Programming Ex 30.2 Q5



3x+4y ≤ 24 ; when x = 0, y = 6 & when y=0, x=8, line AB

8x+6y ≤ 48 ; when x = 0, y = 8 & when y=0, x=6, line CD

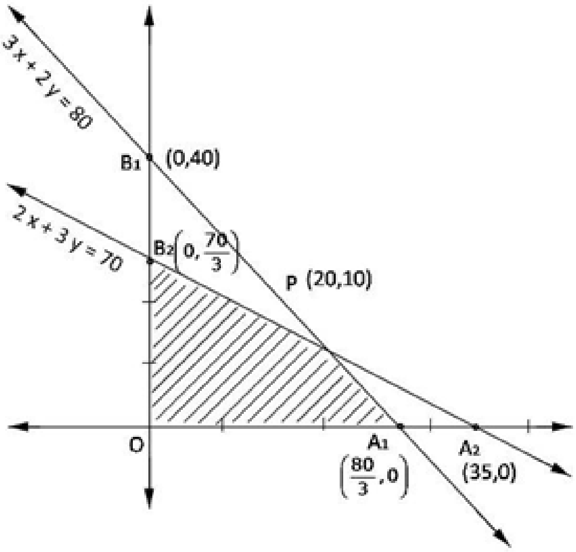
Plotting x≤5 gives line EF; Plotting y≤6 gives line AG
The feasible area is 0,0-C-H-G-E

Corner point	Value of Z = 4x + 3y
0, 0	0
0, 6	18
3.4, 3.4	24
5, 1	23
5,0	20

Linear Programming Ex 30.2 Q6

Converting the inequations into equations as

$3x + 2y = 80,\ 2x + 3y = 70,\ x = y = 0$



Region represented by $3x + 2y \le 80$: Line $3x + 2y = 80$ meets coordinate axes at $A_1\left(\frac{80}{3}, 0\right)$ and $B_1(0, 40)$, clearly, $(0,0)$ satisfies the $3x + 2y \le 80$, so, region containing the origin represents by $3x + 2y \le 80$ in xy-plane

Region represented by $2x + 3y \le 70$: Line $2x + 3y = 70$ meets the coordinate axes at $A_2(35,0)$ and $B_2\left(0, \frac{70}{3}\right)$, clearly, $(0,0)$ satisfies the $2x + 3y \le 70$ so, the region containing the origin represents by $2x + 3y \le 70$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

So, shaded area OA_1PB_2 represents the feasible region.

Coordinate of $P(20,10)$ can be obtained by solving $3x + 2y = 80$ and $2x + 3y = 70$

Now, the value of $Z = 15x + 10y$ at

$O(0,0) \qquad = 15(0) + 10(0) = 0$

$A_1\left(\frac{80}{3}, 0\right) \qquad = 15\left(\frac{80}{3}\right) + 10(0) = 400$

$P(20,10) \qquad = 15(20) + 10(10) = 400$

$B_2\left(0, \frac{70}{3}\right) \qquad = 15(0) + 10\left(\frac{70}{3}\right) = \frac{700}{3}$

So, maximum $Z = 400$ is on each and every point on the line joining A_1P, so we can have,

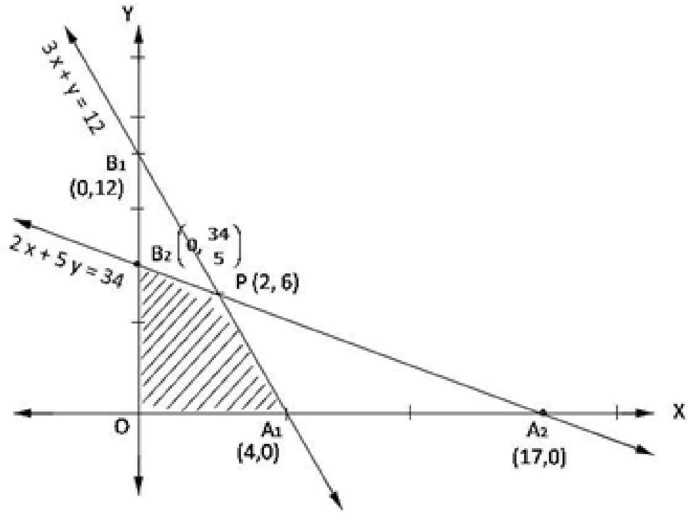
maximum $Z = 400$ at $x = \frac{80}{3}$ and $y = 0$

maximum $Z = 400$ at $x = 20$ and $y = 10$

Linear Programming Ex 30.2 Q7

Converting the given inequations into equations

$$3x + y = 12,\ 2x + 5y = 34,\ x = y = 0$$



Region represented by $3x + y \le 12$: Line $3x + y = 12$ meets the coordinate axes at $A_1(4,0)$ and $B_1(0,12)$, clearly, $(0,0)$ satisfies $3x + y \le 12$, so, region containing origin is represented by $3x + y \le 12$ in xy-plane

Region represented by $2x + 5y \le 34$: Line $2x + y = 34$ meets coordinate axes at $A_2(17,0)$ and $B_2\left(0, \frac{34}{5}\right)$, clearly, $(0,0)$ satisfies the $2x + 5y \le 34$ so, region containing origin represents $2x + 5y \le 34$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

Therefore, shaded area OA_1PB_2 is the feasible region.

The coordinate of $P(2,6)$ is obtained by solving $2x + 5y = 34$ and $3x + y = 12$

The value of $Z = 10x + 6y$ at

$O(0,0) \quad = 10(0) + 6(0) = 0$

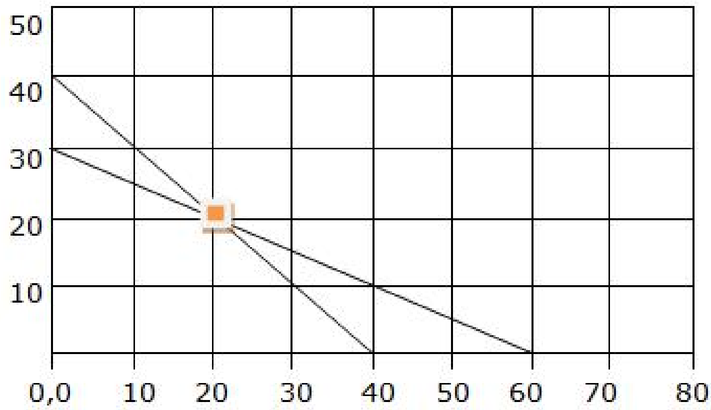
$A_1(4,0) \quad = 10(4) + 6(0) = 40$

$P(2,6) \quad = 10(2) + 6(6) = 56$

$B_2\left(0, \frac{34}{5}\right) \quad = 10(0) + 6\left(\frac{34}{5}\right) = \frac{204}{5} = 40\frac{4}{5}$

Hence, maximum $Z = 56$ at $x = 2,\ y = 6$

Linear Programming Ex 30.2 Q8



$2x+2y \le 80$; when $x=0$, $y=40$ and when $y=0$, $x=40$
$2x+4y \le 120$; when $x=0$, $y=30$ and when $y=0$, $x=60$

The intersection of the two plotted lines gives (20, 20)
Feasible area is 30-C-40

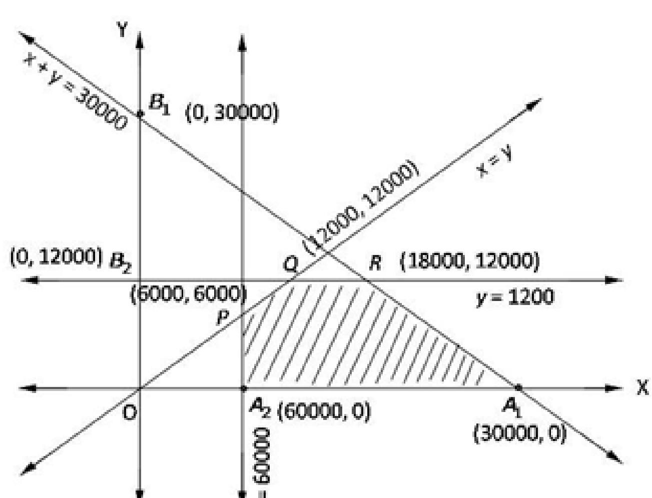
Corner point	Value of Z = 3x + 4y
0, 0	0
0, 30	120
20, 20	140
40, 0	120

The maxima is obtained at $x=20$, $y=20$ and is 140

Linear Programming Ex 30.2 Q9

Converting the given inequations into equations,

$$x + y = 30000,\ y = 12000,\ x = 6000,\ x = y,\ x = y = 0$$



Region represented by $x + y \le 30000$: Line $x + y = 30000$ meets the coordinate axes at $A_1(30000, 0)$ and $B_1(0, 30000)$, clearly $(0,0)$ satisfies $x + y \le 30000$, so, region containing the origin represents $x + y \le 30000$ in xy-plane

Region represented by $y \le 12000$: Line $y = 12000$ is parallel to x-axis and meets y-axis at $B_2(0, 12000)$. Clearly $(0,0)$ satisfies $y \le 12000$, so, region containing origin represents $y \le 12000$ in xy-plane.

Region represented by $x \le 6000$: Line $x = 6000$ is parallel to y-axis and meets x-axis at $A_2(6000, 0)$. Clearly $(0,0)$ satisfies $x \le 6000$, so, region containing origin represents $x \le 6000$ in xy-plane.

Region represented by $x \ge y$: Line $x = y$ passes through origin and point $Q(12000, 12000)$. Clearly, $A_2(6000, 0)$ satisfies $x \ge y$, so, region containing $A_2(6000, 0)$ represents $x \ge y$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

Shaded region A_2A_1QP represents the feasible region.

Coordinates of $R(18000, 12000)$ is obtained by solving $x + y = 30000$ and $y = 12000$, $Q(12000, 12000)$ is obtained by solving $x = y$ and $y = 12000$, $P(6000, 6000)$ is obtained by solving $x = y$ and $x = 6000$.

The value of $Z = 7x + 10y$ at

$A_2(6000, 0) \quad = 7(6000) + 10(0) = 42000$

$A_1(30000, 0) \quad = 7(30000) + 10(0) = 210000$

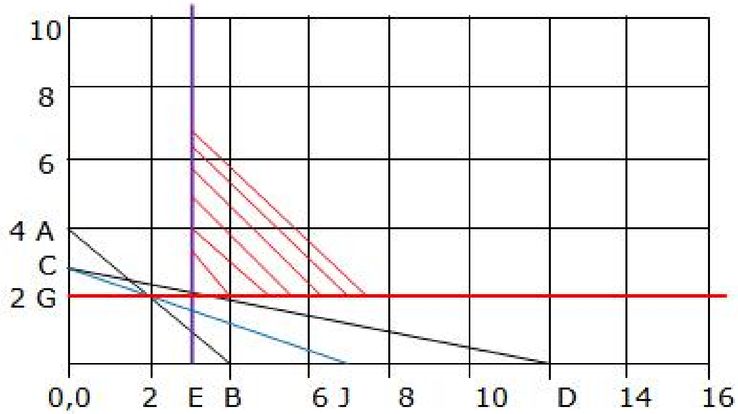
$R(18000, 12000) \quad = 7(18000) + 10(12000) = 246000$

$Q(12000, 12000) \quad = 7(12000) + 10(12000) = 204000$

$P(6000, 6000) \quad = 7(6000) + 10(6000) = 102000$

So, maximum $Z = 246000$ at $x = 18000$, $y = 12000$

Linear Programming Ex 30.2 Q10



2x+2y≥8 ;When x=0, y=4 & when y=0, x=4 line AB
x+4y≥12; When x=0, y=3 & when y=0, x=12 line CD
x≥3, y≥2 are the lines parallel to Y-axis and X-axis resp.

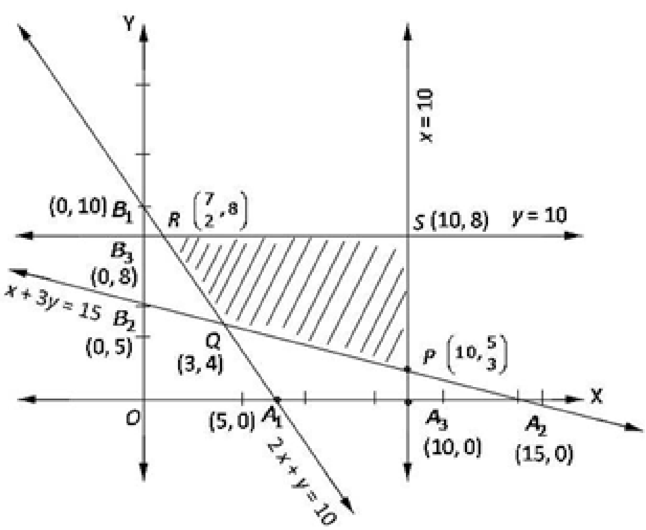
The diverging shaded area in red lines is the area of feasible solution. This area is unbounded.
Z = 2x+4y @ (3,2) = 14.

Plot 2x+4y > 14 line CJ to see if there is any common region. There is no common region so there is no optimal solution.

Linear Programming Ex 30.2 Q11

Converting the given inequations into equations,

$$2x + y = 10,\ x + 3y = 15,\ x = 10,\ y = 8,\ x = y = 0$$



Region represented by $2x + y \geq 10$: Line $2x + y = 10$ meets coordinate axes at $A_1(5,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $2x + y \geq 10$, so, region not containing origin represents $2x + y \geq 10$ in xy - plane.

Region represented by $x + 3y \geq 15$: Line $x + 3y = 15$ meets coordinate axes at $A_2(15,0)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $x + 3y \geq 15$, so, region not containing origin represents $x + 3y \geq 15$ in xy - plane.

Region represented by $x \leq 10$: Line $x = 10$ is parallel to y-axis and meet x-axis at $A_3(10,0)$. Clearly $(0,0)$ satisfies $x \leq 10$, so region containing origin represent $x \leq 10$ in xy-plane.

Region represented by $y \leq 8$: Line $y = 8$ is parallel to x-axis and meet y-axis at $B_3(0,8)$, clearly $(0,0)$ satisfies $y \leq 8$, so region containing origin represent $y \leq 8$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

Shaded region $QPSR$ is the feasible region. $Q(3,4)$ is obtained by solving $2x + y = 10$ and $x + 3y = 15$, $P\left(10, \frac{5}{3}\right)$ is obtained by solving $x + 3y = 15$ and $x = 10$, $R\left(\frac{7}{2}, 8\right)$ is obtained by $2x + y = 10$ and $y = 8$.

The value of $Z = 5x + 3y$ at

$$P\left(10, \frac{5}{3}\right) = 5(10) + 3\left(\frac{5}{3}\right) = 55$$

$$Q(3,4) = 5(3) + 3(4) = 27$$

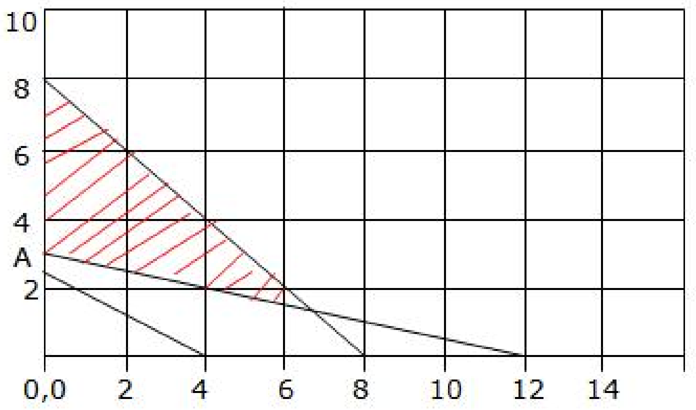
$$R\left(\frac{7}{2}, 8\right) = 5\left(\frac{7}{2}\right) + 3(8) = \frac{83}{2} = 41\frac{1}{2}$$

$$S(10,8) = 5(10) + 3(8) = 74$$

So,

Minimum $Z = 27$ at $x = 3$, $y = 4$

Linear Programming Ex 30.2 Q12



x + y≤8 ; when x=0, y=8 & when y=0, x=8, line 8-8
x + 4y≥12; when x=0, y=3 & when y=0, x=12 line A-12
5x+8y=20; when x=0, y=5/2 & when y=0, x=4

The shaded area in red is the area of feasible solution.

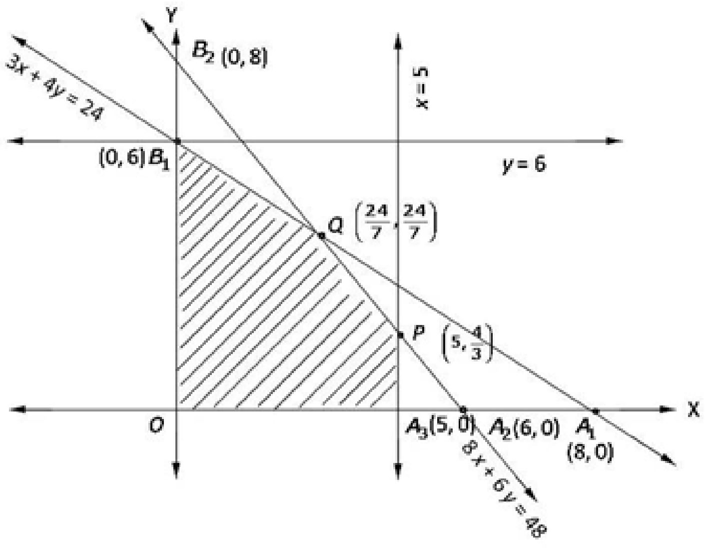
Corner point	Value of Z = 30x + 20y
0, 3	60
0, 8	160
6.66, 1.33	226.66

The maxima is obtained at x=6.66, y=1.33 and is 226.66

Linear Programming Ex 30.2 Q13

Converting the given inequations into equations,

$$3x + 4y = 24,\ 8x + 6y = 48,\ x = 5,\ y = 6,\ x = y = 0$$



Region represented by $3x + 4y \le 24$: Line $3x + 4y = 24$ meets coordinate axes at $A_1(8, 0)$ and $B_1(0, 6)$, clearly $(0,0)$ satisfies $3x + 4y \le 24$, so region containing origin represents $3x + 4y \le 24$ in xy -plane.

Region represented by $8x + 6y \le 48$: Line $8x + 6y = 48$ meets coordinate axes at $A_2(6, 0)$ and $B_2(0, 8)$. Clearly, $(0,0)$ satisfies $8x + 6y \le 48$, so region containing origin represents $8x + 6y \le 48$ in xy -plane.

Region represented $x \le 5$: Line $x = 5$ is parallel to y-axis and meets x-axis at $A_3(5, 0)$. Clearly $(0,0)$ satisfies $x \le 5$, so region containing origin represent $x \le 5$ in xy-plane.

Region represented by $y \le 6$: Line $y = 6$ is parallel to x-axis and meets y-axis at $B_1(0, 6)$. Clearly $(0,0)$ satisfies $y \le 6$, so, region containing origin represents $y \le 6$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

So, shaded region QA_3PQB represents feasible region.

Coordinate of $P\left(5, \frac{4}{3}\right)$ is obtained by solving $8x + 6y = 48$ and $x = 5$, coordinate of $Q\left(\frac{24}{7}, \frac{24}{7}\right)$ is obtained by solving $3x + 4y = 24$ and $8x + 6y = 48$.

The value of $Z = 4x + 3y$ at

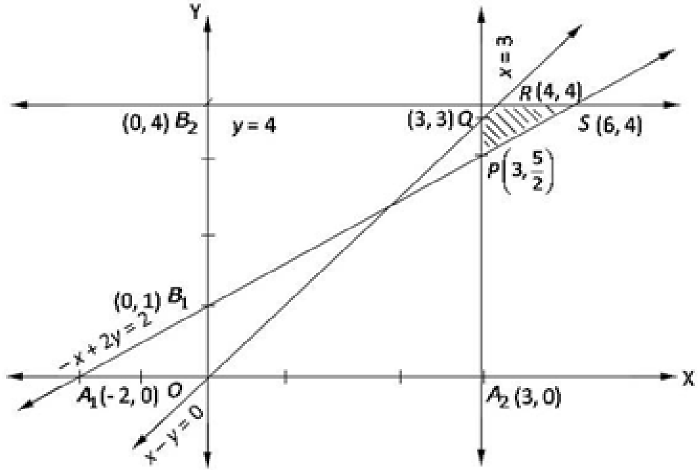
$O(0, 0)$	$= 4(0) + 3(0) = 0$
$A_3(5, 0)$	$= 4(5) + 3(0) = 20$
$P\left(5, \frac{4}{3}\right)$	$= 4(5) + 3\left(\frac{4}{3}\right) = 24$
$Q\left(\frac{24}{7}, \frac{24}{7}\right)$	$= 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24$
$B_1(0, 6)$	$= 4(0) + 3(6) = 18$

So, maximum $Z = 24$ at $x = 5$, $y = \frac{4}{3}$ or $x = \frac{24}{7}$, $y = \frac{24}{7}$ or at every point joining PQ.

Linear Programming Ex 30.2 Q14

Converting the given inequations into equations,

$$x - y = 0,\ -x + 2y = 2,\ x = 3,\ y = 4,\ x = y = 0$$



Region represented by $x - y \geq 0$: $x - y = 0$ is a line passing through origin and $R(4, 4)$. Clearly, $(3,0)$ satisfies $x - y \geq 0$, so, region containing $(3,0)$ represents $x - y \geq 0$ in xy-plane.

Region represented by $-x + 2y \geq 2$: Line $-x + 2y = 2$ meets coordinate axes at $A_1(-2, 0)$ and $B_1(0, 1)$. Clearly, $(0,0)$ does not satisfy $-x + 2y \geq 2$, so, region not containing origin represents $-x + 2y \geq 2$ in xy-plane.

Region represented $x \geq 3$: Line $x = 3$ is parallel to y-axis and meets x-axis at $A_2(3, 0)$. Clearly, $(0,0)$ does not satisfy $x \geq 3$, so region not containing origin represent $x \geq 3$ in xy-plane.

Region represented by $y \leq 4$: Line $y = 4$ is parallel to x-axis and meets y-axis at $B_2(0, 4)$. Clearly $(0,0)$ satisfies $y \leq 4$, so region containing origin represents $y \leq 4$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

So, shaded region $PQRS$ represents feasible region.

The coordinate of $P\left(3, \frac{5}{2}\right)$ is obtained by solving $x = 3$ and $-x + 2y = 2$, $Q(3, 3)$ by solving $x = 3$ and $x - y = 0$, $R(4, 4)$ by solving $x = 4$ and $x - y = 0$, $S(6, 4)$ by solving $y = 4$ and $-x + 2y = 2$

The value of $Z = x - 5y + 20$ at

$$P\left(3, \frac{5}{2}\right) = 3 - 5\left(\frac{5}{2}\right) + 20 = \frac{21}{2} = 11\frac{1}{2}$$

$$Q(3, 3) = 3 - 5(3) + 20 = 8$$

$$R(4, 4) = 4 - 5(4) + 20 = 4$$

$$S(6, 4) = 6 - 5(4) + 20 = 6$$

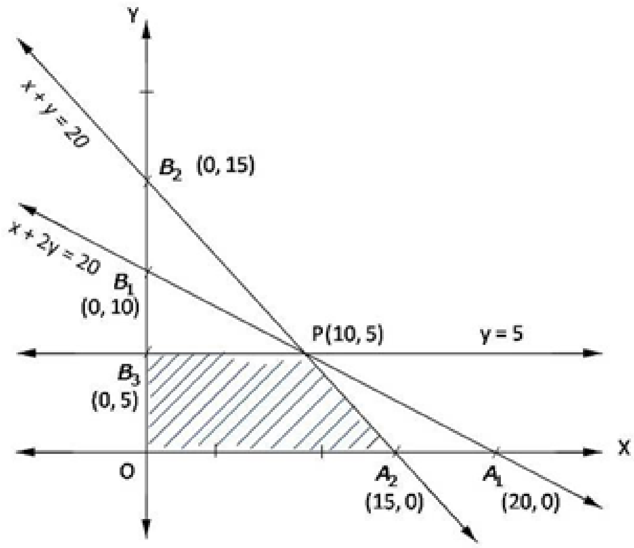
Hence,

Minimum $Z = 4$ at $x = 4$ and $y = 4$

Linear Programming Ex 30.2 Q15

Converting the given inequations into equations:-

$$x + 2y = 20,\ x + y = 15,\ y = 5,\ x = y = 0$$



Region represented by $x + 2y \le 20$: Line $x + 2y = 20$ meets coordinate axes at $A_1(20, 0)$ and $B_1(0, 10)$, clearly, $(0,0)$ satisfies $x + 2y \le 20$, so region containing origin represents $x + 2y \le 20$ in xy -plane.

Region represented by $x + y \le 15$: Line $x + y = 15$ meets coordinate axes at $A_2(15, 0)$ and $B_2(0, 15)$, clearly, $(0,0)$ satisfies $x + y \le 15$, so region containing origin represents $x + y \le 15$ in xy - plane.

Region represented by $y \le 5$: Line $y = 5$ is parallel to x-axis and meets at $B_3(0, 5)$ on y-axis. Clearly $(0,0)$ satisfies $y \le 5$, so region containing origin represents $y \le 5$ in xy-plane.

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane.

So, shaded region OA_2PB_3 represents the feasible region.

Coordinate of $P(10, 5)$ is obtained by solving $x + 2y = 20$ and $y = 5$.

The value of $Z = 3x + 5y$ at

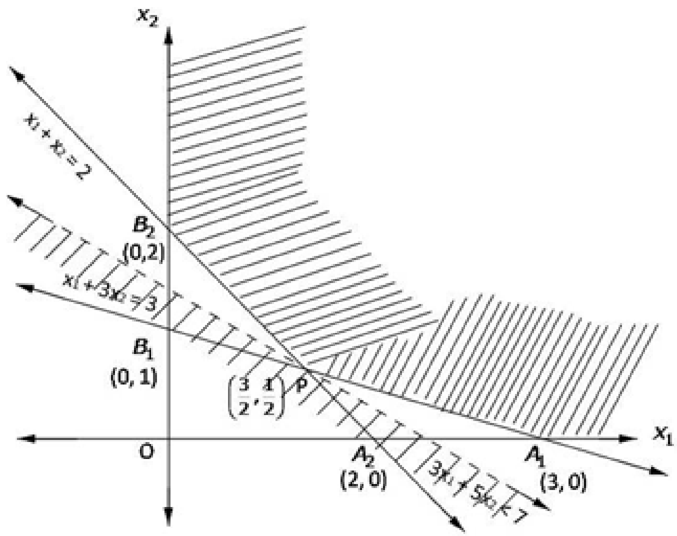
$$O(0,0) \quad = 3(0) + 5(0) = 0$$
$$A_2(15,0) \quad = 3(15) + 5(0) = 45$$
$$P(10,5) \quad = 3(10) + 5(5) = 55$$
$$B_3(0,5) \quad = 3(0) + 5(5) = 25$$

Hence, maximum $Z = 55$ at $x = 10$ and $y = 5$

Linear Programming Ex 30.2 Q16

Converting the given inequations into equations,

$$x_1 + 3x_2 = 3,\ x_1 + x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $x_1 + 3x_2 \geq 3$: Line $x_1 + 3x_2 = 3$ meets the coordinate axes at $A_1(3,0)$ and $B_1(0,1)$, clearly, $(0,0)$ does not satisfy $x_1 + 3x_2 \geq 3$, so, region not containing $(3,0)$ represents $x_1 + 3x_2 \geq 3$ in x_1x_2-plane.

Region represented by $x_1 + x_2 \geq 2$: Line $x_1 + x_2 = 2$ meets the coordinate axes at $A_2(2,0)$ and $B_2(0,2)$, clearly, $(0,0)$ does not satisfy $x_1 + x_2 \geq 2$, so, region not containing origin represents $x_1 + x_2 \geq 2$ in x_1x_2-plane.

Region represented $x_1, x_2 \geq 0$: It represents the first quadrant in x_1x_2-plane.

The unbounded shaded region with corner points $A_1(3,0)$, $B_2(0,2)$, and $P\left(\frac{3}{2},\frac{1}{2}\right)$.

$P\left(\frac{3}{2},\frac{1}{2}\right)$ is obtained by $x_1 + x_2 = 2$ and $x_1 + 3x_2 = 3$.

The value of $Z = 3x_1 + 5x_2$ at

$A_1(3,0) \quad = 3(3) + 5(0) = 9$

$P\left(\frac{3}{2},\frac{1}{2}\right) \quad = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 7$

$B_2(0,2) \quad = 3(0) + 5(2) = 10$

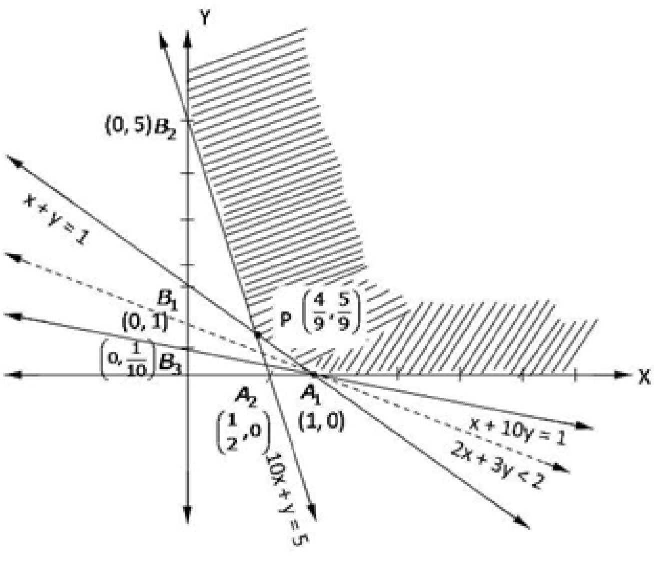
The smallest value of $Z = 7$,
region has no point in common, so smallest value is the minimum value.

Hence, minimum $Z = 7$ at $x = \frac{3}{2}$ and $y = \frac{1}{2}$

Linear Programming Ex 30.2 Q17

Converting the given inequations into equations

$x + y = 1,\ 10x + y = 5,\ x + 10y = 1,\ x = y = 0$



Region represented by $x + y \geq 1$: Line $x + y = 1$ meets coordinate axes at $A_1(1, 0)$ and $B_1(0,1)$, clearly, $(0,0)$ does not satisfy $x + y \geq 1$, so region not containing origin represents $x + y \geq 1$ in xy -plane.

Region represented by $10x + y \geq 5$: Line $10x + y = 5$ meets coordinate axes at $A_2\left(\frac{1}{2}, 0\right)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $10x + y \geq 5$, so region not containing origin represents $10x + y \geq 5$ in xy -plane.

Region represented by $x + 10y \geq 1$: Line $x + 10y = 1$ meets coordinate axes $A_1(1,0)$ and $B_3\left(0, \frac{1}{10}\right)$. Clearly, $(0,0)$ does not satisfy $x + 10y \geq 1$, so, region not containing origin represents $x + 10y \geq 1$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

So, unbounded shaded represents feasible region. Its corner points are $A_1(1,0)$, $P\left(\frac{4}{9}, \frac{5}{9}\right)$ and $B_2(0,5)$.

The coordinate of $P\left(\frac{4}{9}, \frac{5}{9}\right)$ is obtained by solving $10x + y = 5$ and $x + y = 1$.

The value of $Z = 2x + 3y$ at

$A_1(1,0) \quad = 2(1) + 3(0) = 2$

$P\left(\frac{4}{9}, \frac{5}{9}\right) \quad = 2\left(\frac{4}{8}\right) + 3\left(\frac{5}{9}\right) = \frac{23}{9} = 2\frac{5}{9}$

$B_2(0,5) \quad = 2(0) + 3(5) = 15$

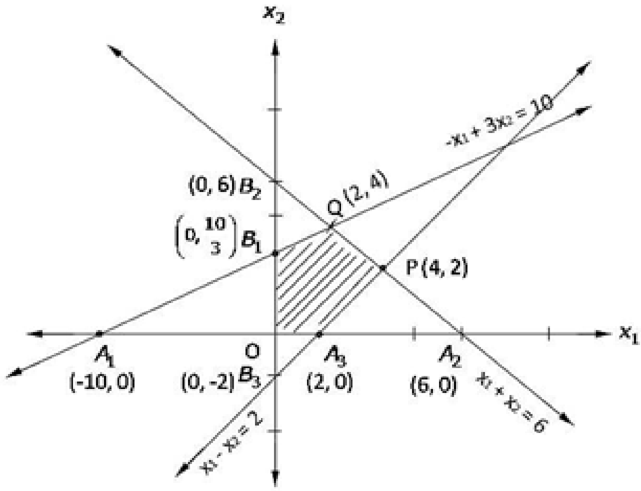
The smallest value of Z is 2. Now, open half plane $2x + 3y < 2$ has no point in common with feasible region so, smallest value of Z is the minimum value.

Hence, maximum $Z = 2$ at $x = 1$ and $y = 0$

Linear Programming Ex 30.2 Q18

Converting the given inequations into equations,

$$-x_1 + 3x_2 = 10,\ x_1 + x_2 = 6,\ x_1 = x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $-x_1 + 3x_2 \le 10$: Line $-x_1 + 3x_2 = 10$ meets coordinate axes at $A_1(-10, 0)$ and $B_1\left(0, \frac{10}{3}\right)$, clearly, $(0,0)$ satisfies $-x_1 + 3x_2 \le 10$, so region containing origin represents $-x_1 + 3x_2 \le 10$ in x_1x_2 -plane.

Region represented by $x_1 + x_2 \le 6$: Line $x_1 + x_2 = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ satisfies $x_1 + x_2 \le 6$, so region containing origin represents $x_1 + x_2 \le 6$ in x_1x_2 -plane.

Region represented by $x_1 - x_2 \le 2$: Line $x_1 - x_2 = 2$ meets coordinate axes at $A_3(2,0)$ and $B_3(0,-2)$. Clearly, $(0,0)$ satisfies $x_1 - x_2 \le 2$, so, region containing origin represents $x_1 - x_2 \le 2$ in x_1x_2 -plane.

Region represented $x_1, x_2 \ge 0$: It represents first quadrant in x_1x_2-plane.

So, shaded region OA_3PQB_1 represents feasible region.

Coordinate of $P(4,2)$ is obtained by solving $x_1 + x_2 = 6$ and $x_1 - x_2 = 2$, $Q(2,4)$ by solving $x_1 + x_2 = 6$ and $-x_1 + 3x_2 = 10$

The value of $Z = -x_1 + 2x_2$ at

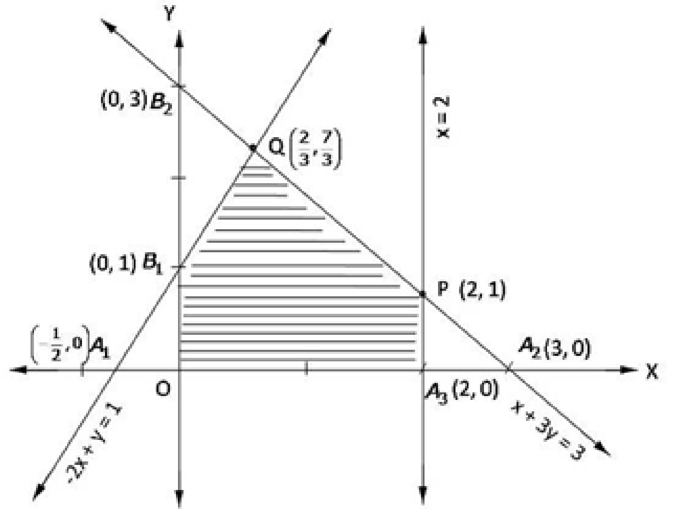
$$O(0,0) \quad = -(0) + 2(0) = 0$$
$$A_3(2,0) \quad = -(2) + 2(0) = -2$$
$$P(4,2) \quad = -(4) + 2(2) = 0$$
$$Q(2,4) \quad = -(2) + 2(4) = 6$$
$$B_1\left(0, \frac{10}{3}\right) \quad = -(0) + 2\left(\frac{10}{3}\right) = \frac{20}{3} = 6\frac{2}{3}$$

Hence, maximum $Z = \frac{20}{3}$ at $x = 0$ and $y = \frac{10}{3}$

Linear Programming Ex 30.2 Q19

Converting the given inequations into equations,

$$-2x + y = 1,\ x = 2,\ x + y = 3,\ x = y = 0$$



Region represented by $-2x + y \leq 1$: Line $-2x + y = 1$ meets coordinate axes at $A_1\left(\frac{-1}{2}, 0\right)$ and $B_1(0, 1)$, clearly, $(0,0)$ satisfies $-2x + y \leq 1$, so region containing origin represents $-2x + y \leq 1$ in xy-plane.

Region represented by $x \leq 2$: Line $x = 2$ is parallel to y-axis and meets x-axis at $A_3(2, 0)$. Clearly, $(0,0)$ satisfies $x \leq 2$, so region containing origin represents $x \leq 2$ in xy-plane.

Region represented by $x + y \leq 3$: Line $x + y = 3$ meets coordinate axes at $A_2(3, 0)$ and $B_2(0, 3)$. Clearly, $(0,0)$ satisfies $x + y \leq 3$, so region containing origin represents $x + y \leq 3$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

So, shaded region OA_3PQB, represents the feasible region.

Coordinates of $P(2, 1)$ is obtained by solving $x + y = 3$ and $x = 2$, $Q\left(\frac{2}{3}, \frac{7}{3}\right)$ by solving $-2x + y = 1$ and $x + y = 3$.

The value of $Z = x + y$ at

$O(0, 0) \quad = 0 + 0 = 0$

$A_3(2, 0) \quad = 2 + 0 = 2$

$P(2, 1) \quad = 2 + 1 = 2$

$Q\left(\frac{2}{3}, \frac{7}{3}\right) \quad = \frac{2}{3} + \frac{7}{3} = 3$

$B_1(0, 1) \quad = 0 + 1 = 1$

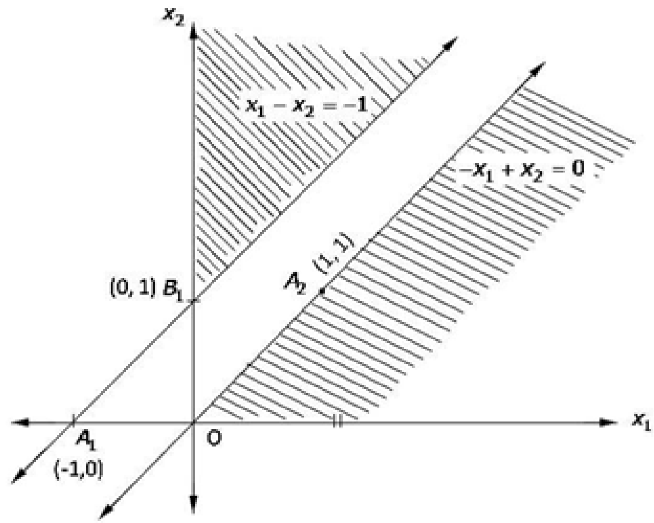
So, maximum $Z = 3$ is at every point on the line joining PQ.

Hence, maximum $Z = 3$ at $x = 2$ and $y = 1$ Or $x = \frac{2}{3}$ and $y = \frac{7}{3}$

Linear Programming Ex 30.2 Q20

Converting the given inequations into equations,

$$x_1 - x_2 = -1,\ -x_1 + x_2 = 0,\ x_1 = x_2 = 0$$



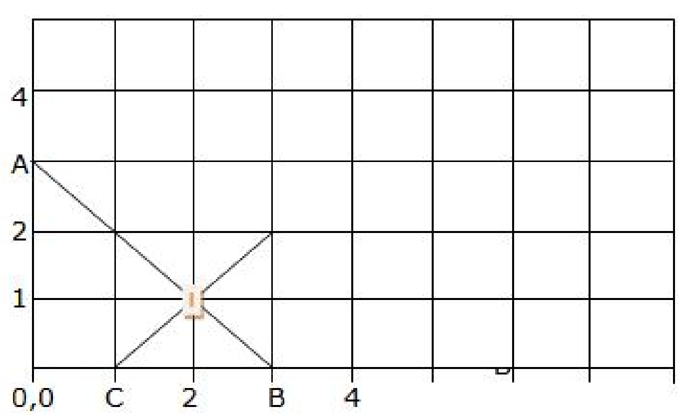
Region represented by $x_1 - x_2 \le -1$: Line $x_1 - x_2 = -1$ meets coordinate axes at $A_1(-1, 0)$ and $B_1(0, 1)$, clearly, $(0,0)$ does not satisfy $x_1 - x_2 \le -1$, so region not containing origin represents $x_1 - x_2 \le -1$ in x_1x_2-plane.

Region represented by $-x_1 + x_2 \le 0$: Line $-x_1 + x_2 = 0$ passes through origin and $A_2(1,1)$. Clearly, $(0,0)$ does not satisfy $-x_1 + x_2 \le 0$, so, region not containing $(0,1)$ represents $-x_1 + x_2 \le 0$ in x_1x_2-plane.

Since, there is not common shaded region represented by $x_1 - x_2 \le -1$ and $-x_1 + x_2 \le 0$ which can form feasible region.

Hence, maximum $Z = 3x_1 + 4x_2$ does not exists.

Linear Programming Ex 30.2 Q21



x-y≤1; when x = 0, y=1 & when y=0, x=2
x + y≥3; when x = 0, y=3 & when y=0, x=3, line AB
a unbounded region A-C-D is obtained using the constraints.

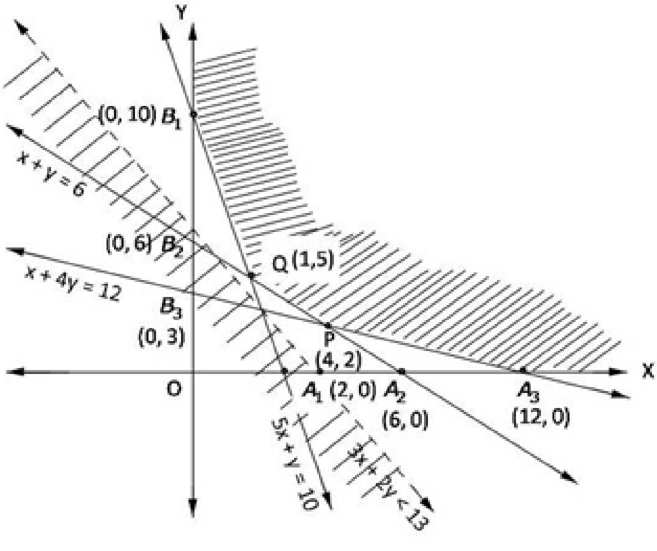
Corner point	Value of Z = 3x + 3y
0, 3	9
2, 1	9

So an optimal solution does not exist.

Linear Programming Ex 30.2 Q22

Converting the given inequations into equations

$5x + y = 10,\ x + y = 6,\ x + 4y = 12,\ x = y = 0$



Region represented by $5x + y \geq 10$: Line $5x + y = 10$ meets coordinate axes at $A_1(2,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $5x + y \geq 10$, so region not containing origin represents $5x + y \geq 10$ in xy -plane.

Region represented by $x + y \geq 6$: Line $x + y = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ does not satisfy $x + y \geq 6$, so region not containing origin represents $x + y \geq 6$ in xy -plane.

Region represented by $x + 4y \geq 12$: Line $x + 4y = 12$ meets coordinate axes at $A_3(12,0)$ and $B_3(0,3)$. Clearly, $(0,0)$ does not satisfy $x + 4y \geq 12$, so, region not containing origin $x + 4y \geq 12$ in xy -plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

The unbounded shaded region with corner points $A_3(12,0)$, $P(4,2)$, $Q(1,5)$, $B_1(0,10)$ represents feasible region. Point P is obtained by solving $x + 4y = 12$ and $x + y = 6$, Q by solving $x + y = 6$ and $5x + y = 10$.

The value of $Z = 3x + 2y$ at

$A_3(12,0) = 3(12) + 2(0) = 36$

$P(4,2) = 3(4) + 2(2) = 16$

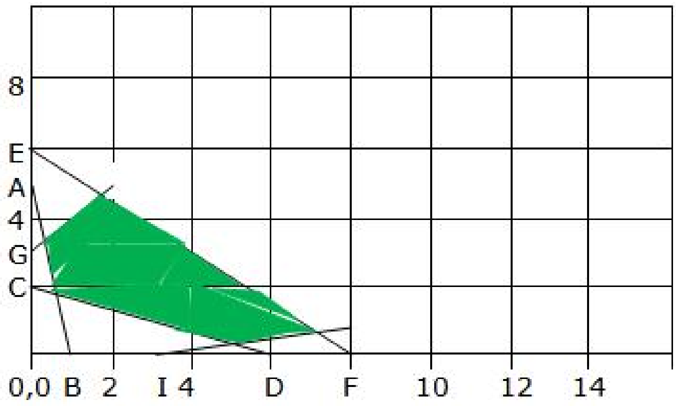
$Q(1,5) = 3(1) + 2(5) = 13$

$B(0,10) = 3(0) + 2(10) = 20$

Smallest value of $Z = 13$, Now open half plane $3x + 2y < 13$ has no point in common with feasible region, so, smallest value is the minimum value of Z, Hence

Minimum $Z = 13$ at $x = 1$, $y = 5$

Linear Programming Ex 30.2 Q23



x+3y≥6; or y=-0.333x+2; when x=0, y=2 & when y=0, x=6; line CD
x-3y≤3; or y=0.333x-1; when x=0, y=-1 & when y=0, x=3; line IJ
3x+4y≤24; or y=-0.75x+6; when x=0, y=6 & when y=0, x=8; line EF

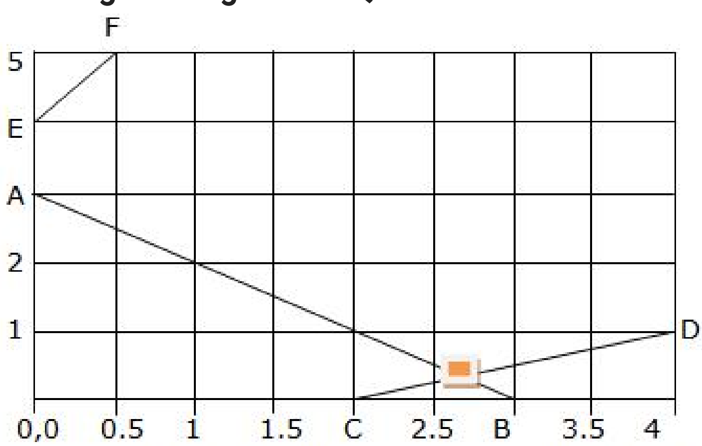
-3x+2y≤6; or y= 1.5x+3; when x=0, y=3 & when y=0, x=-2;line GH
5x+y≥5; or y=-5x+5; when x=0, y=5 & when y=0, x=1; line AB

The feasible area is shaded in green

Corner point	Value of Z = 2x + y	
4.5, 0.5		9.5
0.64, 1.78		3.07
6.46, 1.15	Maximum	14.07
1.33, 5		7.6667
0.30, 3.46		4.0769

Maximum value is 14.07 at the point (6.46, 1.15)
Minimum value is 3.07 at the point (0.64, 1.78)

Linear Programming Ex 30.2 Q24



-2x+y≤4; or y=2x+4; when x=0, y=4 & when y=0, x=-2 line EF x+y≥3; or y=-x+3; when x=0, y=3 & when y=0, x=3; line AB

x-2y≤2; or y=0.5x-1; when x=0, y=-1 & when y=0,

x=2 line CD

The feasible solution is the unbounded area with F-E-A-G-D

Corner point	Value of Z = 3x + 5y	
(2.67, 0.33)	Minimum	9.66
(0, 3)		15
(0, 4)		20

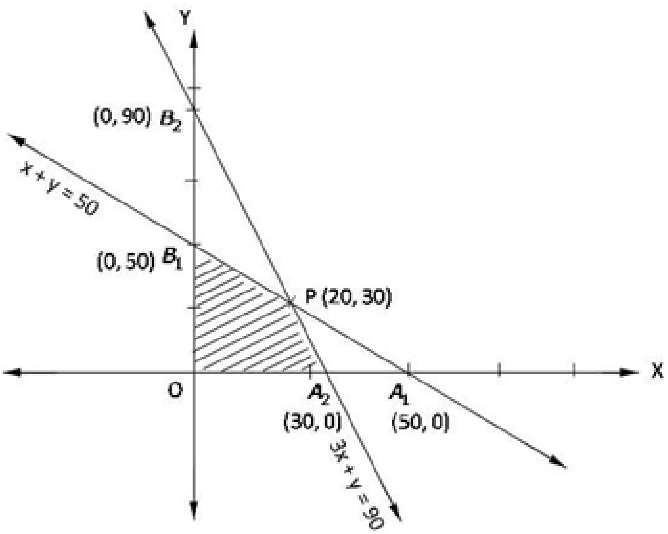
To check whether it is the minimal value plot the objective function with a value less than 9.66 or y=-0.6x-1.932

it can be seen that the values of x and y are always negative. So there is no optimal solution.

Linear Programming Ex 30.2 Q25

Converting the given inequations into equations,

$$x + y = 50,\ 3x + y = 90,\ x = y = 0$$



Region represented by $x + y \leq 50$: Line $x + y = 50$ meets coordinate axes at $A_1(50, 0)$ and $B_1(0, 50)$. Clearly, $(0,0)$ satisfies $x + y \leq 50$, so, region containing origin represents $x + y \leq 50$ in xy-plane.

Region represented by $3x + y \leq 90$: Line $3x + y = 90$ meets coordinate axes at $A_2(30,0)$ and $B_2(0,90)$. Clearly, $(0,0)$ satisfies $3x + y \leq 90$, so, region containing origin represents $3x + y \leq 90$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

Shaded region OA_2PB_1 represents the feasible region. $P(20,30)$ can be obtained by solving $x + y = 50$ and $3x + y = 90$.

The value of $Z = 60x + 15y$ at

$O(0,0) \quad = 60(0) + 15(0) = 0$

$A_2(30,0) \quad = 60(30) + 15(0) = 1800$

$P(20,30) \quad = 60(20) + 15(30) = 1650$

$B_1(0,50) \quad = 60(0) + 15(50) = 750$

Hence,

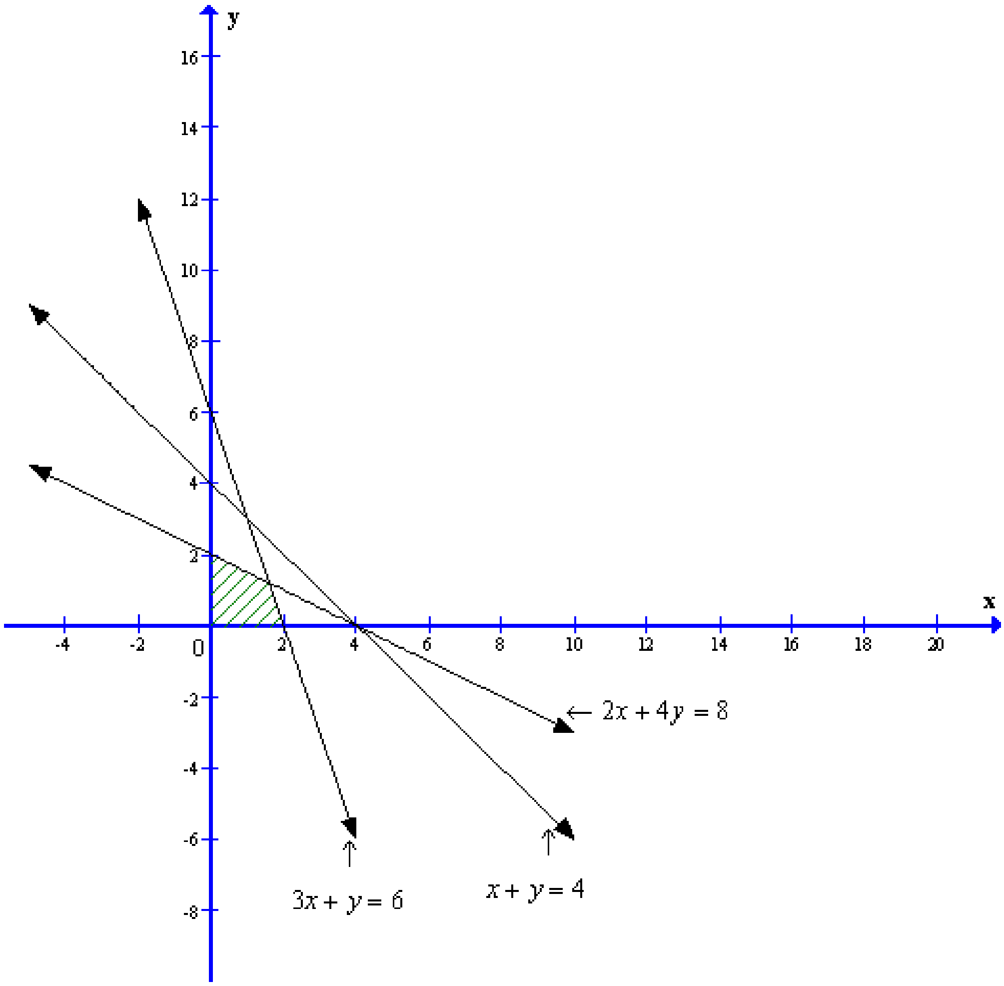
maximum Z is 1800 at $x = 30$ and $y = 0$.

Linear Programming Ex 30.2 Q26

Converting the inequations into equations, we obtain the lines

$2x + 4y = 8, 3x + y = 6, x + y = 4, x = 0, y = 0.$

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in the graph.



From the graph we can see the corner points as (0, 2) and (2, 0).

Now solving the equations $3x + y = 6$ and $2x + 4y = 8$ we get the values of x and y as $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Substituting $x = \frac{8}{5}$ and $y = \frac{6}{5}$ in $Z = 2x + 5y$ we get,

$$Z = 2\left(\frac{8}{5}\right) + 5\left(\frac{6}{5}\right)$$

$$Z = \frac{46}{5}$$

Hence maximum value of Z is $\frac{46}{5}$ at $x = \frac{8}{5}$ and $y = \frac{6}{5}$.