

**RD Sharma**  
**Solutions Class**  
**12 Maths**  
**Chapter 30**  
**Ex 30.4**

### Linear Programming Ex 30.4 Q1

Let he drives  $x$  km at a speed of 25 km/hr and  $y$  km at a speed of 40 km/hr.

Let  $Z$  be total distance travelled by him, so,

$$Z = x + y$$

Since he spend Rs 2 per km on petrol when speed is 25 km/hr and Rs 5 per km on petrol when speed is 40 km/hr, so, expence on  $x$  km and  $y$  km are Rs  $2x$  and Rs  $5y$  respectivley, but he has only Rs 100.,so

$$2x + 5y \leq 100 \quad (\text{first constraint})$$

$$\begin{aligned} \text{Time taken to travel } x \text{ km} &= \frac{\text{Distance}}{\text{speed}} \\ &= \frac{x}{25} \text{ hr} \end{aligned}$$

$$\text{Time taken to travel } y \text{ km} = \frac{y}{40} \text{ hr}$$

Given he has 1 hr to travel, so

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$\Rightarrow 40x + 25y \leq 1000$$

$$\Rightarrow 8x + 5y \leq 200 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = x + y$$

Subject to constriants,

$$2x + 5y \leq 100$$

$$8x + 5y \leq 200$$

$$x, y \geq 0$$

[Since distances can not be less than zero]

Region  $2x + 5y \leq 100$ : line  $2x + 5y = 100$  meets axes at  $A_1(50,0)$ ,  $B_1(0,20)$  respectively.

Region containing origin represents  $2x + 5y \leq 100$  as  $(0,0)$  satisfies  $2x + 5y \leq 100$ .

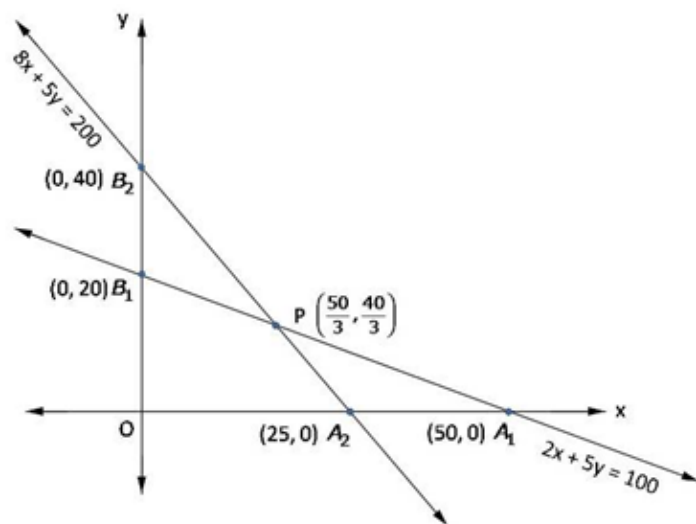
Region  $8x + 5y \leq 200$ : line  $8x + 5y = 200$  meets axes at  $A_2(25,0)$ ,  $B_2(0,40)$  respectively.

Region containing origin represents  $8x + 5y \leq 200$  as  $(0,0)$  satisfies  $8x + 5y \leq 200$ .

Region  $x, y \geq 0$ : it represent first quadrant

Shaded region  $OA_2PB_1$  represents feasible region.

Point  $P\left(\frac{50}{3}, \frac{40}{3}\right)$  is obtained by solving  $8x + 5y = 200$ ,  $2x + 5y = 100$



The value of  $Z = x + y$  at

$$O(0,0) = 0 + 0 = 0$$

$$A_2(25,0) = 25 + 0 = 25$$

$$P\left(\frac{50}{3}, \frac{40}{3}\right) = \frac{50}{3} + \frac{40}{3} = 30$$

$$B_1(0,20) = 0 + 20 = 20$$

$$\text{maximum } Z = 30 \text{ at } x = \frac{50}{3}, y = \frac{40}{3}$$

Distance travelled at speed of 25 km/hr =  $\frac{50}{3}$  km

and at speed of 40 km/hr =  $\frac{40}{3}$  km

maximum distance = 30 km.

Let required quantity of items A and B.

Given, profits on one item A and B are Rs 6 and Rs 4 respectively So, profits on  $X$  items of type A and  $Y$  items of type B are  $6x$  and  $Rs\ 4y$  respectively,

Let total profit be  $z$ , so,

$$Z = 6x + 4y$$

Given, machine I works 1 hour and 2 hours on item A and B respectively, so,

$x$  number of item A and  $y$  number of item B need  $x$  hour and  $2y$  hours on machine I respectively, but machine I works at most 12 hours, so

$$x + 2y \geq 12 \quad (\text{first constraint})$$

Given, machine II works 2 hours and 1 hours on item A and B respectively, so,

$x$  number of item A and  $y$  number of item B need  $2x$  hours and  $y$  hour on machine II, but machine II works maximum 12 hours, so

$$2x + y \geq 12 \quad (\text{second constraint})$$

Given, machine III works 1 hour and  $\frac{5}{4}$  hour on one item A and B respectively, so,

$x$  number of item A and  $y$  number of item B need  $x$  hour and  $\frac{5}{4}y$  hours respectively on machine III, but machine III works at least 5 hours, so

$$x + \frac{5}{4}y \geq 5$$

$$4x + 5y \geq 20 \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$z = 6x + 4y$$

subject to constraints,

$$x + 2y \geq 12$$

$$2x + y \geq 12$$

$$4x + 5y \geq 20$$

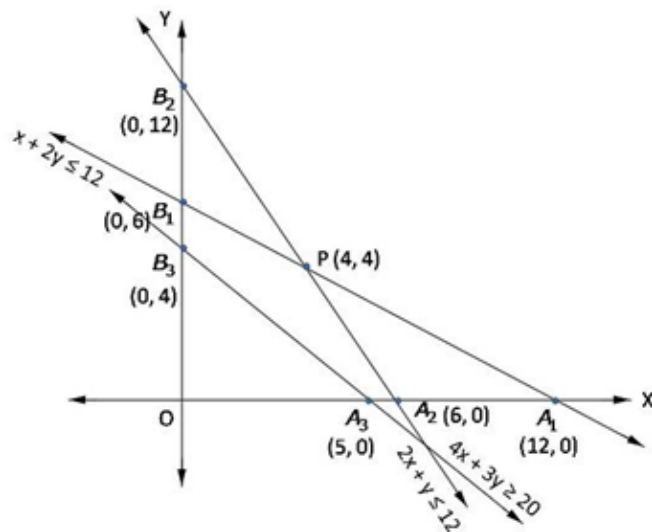
$$x, y \geq 0$$

[Since number of item A and B not be less than zero]

Region  $x + 2y \geq 12$ : line  $x + 2y = 12$  meets axes at  $A_1(12, 0), B_1(0, 6)$  respectively. Region containing origin represents  $x + 2y \geq 12$  as  $(0, 0)$  satisfies  $x + 2y \geq 12$ .

Region  $4x + 5y \geq 20$ : line  $4x + 5y = 20$  meets axes at  $A_3(5, 0), B_3(0, 4)$  respectively. Region not containing origin represents  $4x + 5y \geq 20$  as  $(0, 0)$  does not satisfy  $4x + 5y \geq 20$ .

Region  $x, y \geq 0$ : it represent first quadrant.



Shaded region  $A_2A_3PB_3B_1$  represents feasible region.

The value of  $Z = 6x + 4y$  at

$A_2(6,0)$	$= 6(6) + 4(0) = 36$
$A_3(5,0)$	$= 6(5) + 4(0) = 30$
$B_3(0,4)$	$= 6(0) + 4(4) = 16$
$B_2(0,6)$	$= 6(0) + 4(6) = 24$
$P(4,4)$	$= 6(4) + 4(4) = 40$

Hence,  $Z$  is maximum at  $x = 4, Y = 4$

Required number of product  $A = 4$ , product  $B = 4$

Maximum profit = Rs 40

Suppose tailor  $A$  and  $B$  work for  $x$  and  $y$  days respectively.

Since, tailor  $A$  and  $B$  earn Rs 15 and Rs 20 respectively So, tailor  $A$  and  $B$  earn is  $X$  and  $Y$  days Rs  $15x$  and  $20y$  respectively, let  $Z$  denote maximum profit that gives minimum labour cost, so,

$$Z = 15x + 20y$$

Since, Tailor  $A$  and  $B$  stitch 6 and 10 shirts respectively in a day, so, tailor  $A$  can stitch  $6x$  and  $B$  can stitch  $10y$  shirts in  $x$  and  $y$  days respectively, but it is desired to produce 60 shirts at least, so

$$6x + 10y \geq 60$$

$$3x + 5y \geq 30 \quad (\text{first constraint})$$

Since, Tailor  $A$  and  $B$  stitch 4 pants per day each, so, tailor  $A$  can stitch  $4x$  and  $B$  can stitch  $4y$  pants in  $x$  and  $y$  days respectively, but it is desired to produce at least 32 pants, so

$$4x + 4y \geq 32$$

$$x + y \geq 8 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which minimize

$$Z = 15x + 20y$$

subject to constraints,

$$3x + 5y \geq 30$$

$$x + y \geq 8$$

$$x, y \geq 0$$

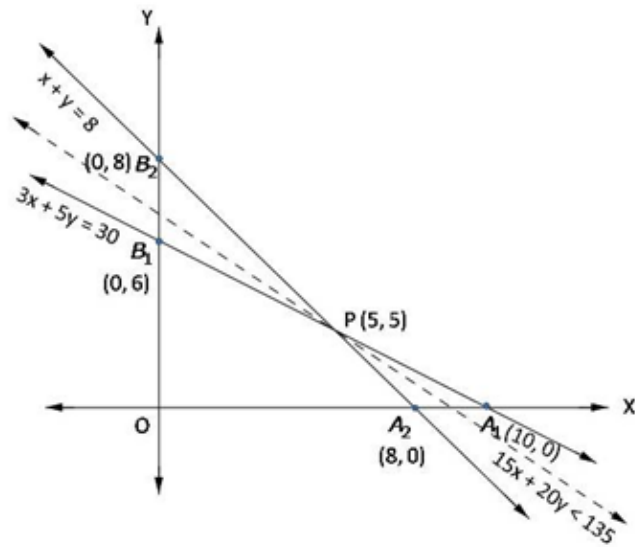
[Since  $x$  and  $y$  not be less than zero]

Region  $3x + 5y \geq 30$ : line  $3x + 5y = 30$  meets axes at  $A_1(10, 0)$ ,  $B_1(0, 6)$  respectively. Region not containing origin represents  $3x + 5y \geq 30$  as  $(0, 0)$  does not satisfy  $3x + 5y \geq 30$ .

Region  $x + y \geq 8$ : line  $x + y = 8$  meets axes at  $A_2(8, 0)$ ,  $B_2(0, 8)$  respectively. Region not containing origin represents  $x + y \geq 8$  as  $(0, 0)$  does not satisfy  $x + y \geq 8$ .

Region  $x, y \geq 8$ : it represent first quadrant.

Unbounded shaded region  $A_1P B_2$  represents feasible region with corner points  $A_1(10, 0)$ ,  $P(5, 3)$ ,  $B_2(0, 8)$ .



The value of  $Z = 15x + 20y$  at

$$A_1 (10, 0) = 15(10) + 20(0) = 150$$

$$P (5, 3) = 15(5) + 20(3) = 135$$

$$B_2 (0, 8) = 15(0) + 20(8) = 160$$

Smallest value of  $Z$  is 135, Now open half plane  $15x + 20y < 135$  has no point in common with feasible region, so smallest value is the minimum value. So,

$$Z = 135, \text{ at } x = 5, y = 3$$

Tailor A should work for 5 days and B should work for 3 days

### Linear Programming Ex 30.4 Q4

Let the factory manufacture  $x$  screws of type A and  $y$  screws of type B on each day. Therefore,

$$x \geq 0 \text{ and } y \geq 0$$

The given information can be compiled in a table as follows.

	Screw A	Screw B	Availability
<b>Automatic Machine (min)</b>	4	6	$4 \times 60 = 120$
<b>Hand Operated Machine (min)</b>	6	3	$4 \times 60 = 120$

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are

$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$\text{Total profit, } Z = 7x + 10y$$

The mathematical formulation of the given problem is

$$\text{Maximize } Z = 7x + 10y \dots (1)$$

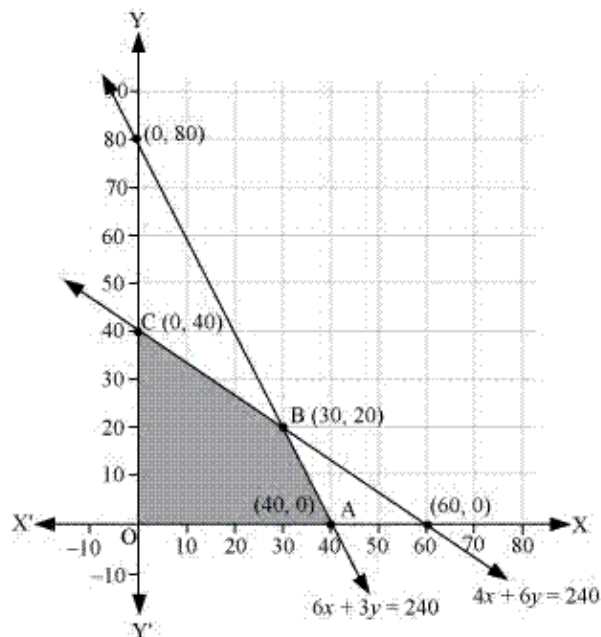
subject to the constraints,

$$4x + 6y \leq 240 \dots (2)$$

$$6x + 3y \leq 240 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	$Z = 7x + 10y$	
A(40, 0)	280	
B(30, 20)	410	→ Maximum
C(0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.



Let required number of belt  $A$  and  $B$  be  $x$  and  $y$ .

Given, profit on belt  $A$  and  $B$  be Rs 2 and Rs 1.50 per belt, So, profit on  $x$  belt of type  $A$  and  $Y$  belt fo type  $B$  be  $2x$  and  $1.5y$  respectively,

Let  $Z$  be total profit, so,

$$Z = 2x + 1.5y$$

Since, each belt of type  $A$  requires twice as much time as belt  $B$ . Let each belt  $B$  require 1 hour to make, so,  $A$  requires 2 hours. For  $x$  and  $y$  belts of type  $A$  and  $B$ . It required  $2x$  and  $y$  hours to make but total time available is equal to production 1000 belt  $B$  that is 1000 hours, so,

$$2x + y \leq 1000 \quad (\text{first constraint})$$

Given supply of leather only for 800 belts per day (both  $A$  and  $B$  combined), so

$$x + y \leq 800 \quad (\text{second constraint})$$

Buckels available for  $A$  is only 400 and for  $B$  only 700, so,

$$x \leq 400 \quad (\text{third constraint})$$

$$y \leq 700 \quad (\text{fourth constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which miximize

$$Z = 2x + 1.5y$$

subject to constraints,

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x \leq 400$$

$$y \leq 700$$

$$x, y \geq 0$$

[Since number of belt can not be less than zero]

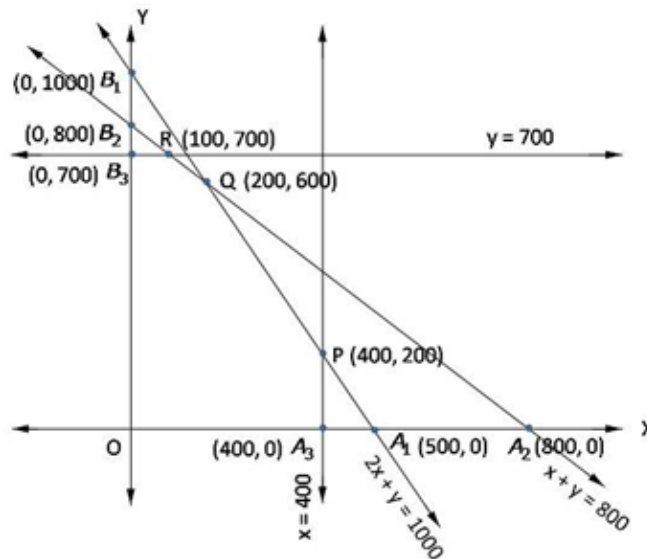
Region  $2x + y \leq 1000$ : line  $2x + y = 1000$  meets axes at  $A_1(500, 0)$ ,  $B_1(0, 1000)$  respectively. Region containing origin represents  $2x + y \leq 1000$  as  $(0, 0)$  satisfies  $2x + y \leq 1000$ .

Region  $x + y \leq 800$ : line  $x + y = 800$  meets axes at  $A_2(800, 0)$ ,  $B_2(0, 800)$  respectively. Region containing origin represents  $x + y \leq 800$  as  $(0, 0)$  satisfies  $x + y \leq 800$ .

Region  $x \leq 400$ : line  $x = 400$  meets axes is parallel to  $y$  axis and meet  $x$  - axis at  $A_3(400,0)$ . Region containing origin represents  $x \leq 400$  as  $(0,0)$  satisfies  $x \leq 400$ .

Region  $y \leq 700$ : line  $y = 700$  is parallel to  $x$ - axis and meet  $y$  - axis at  $B_3(0,700)$ . Region containing origin represents  $y \leq 700$  as  $(0,0)$  satisfies  $y \leq 700$ .

Region  $x, y \geq 0$ : it represent first quadrant.



Shaded region  $OA_3PQR B_3$  is feasible region,  $P$  is points of intersections of  $2x + y = 1000$  and  $x = 400$ ,  $Q$  is the point of intersection of  $x + y = 800$  and  $2x + y = 1000$ ,  $R$  is not point of intersection of  $y = 700$ ,  $x + y = 800$ .

The value of  $Z = 2x + 1.5y$  at

$$\begin{aligned}
 O(0,0) &= 2(0) + 1.5(0) = 0 \\
 A_3(400,0) &= 2(400) + 1.5(0) = 800 \\
 P(400,200) &= 2(400) + 1.5(200) = 1100 \\
 Q(200,600) &= 2(200) + 1.5(600) = 1300 \\
 R(100,700) &= 2(100) + 1.5(700) = 1250 \\
 B_3(0,700) &= 2(0) + 1.5(700) = 1050
 \end{aligned}$$

Therefore, maximum  $Z = 1300$ , at  $x = 200, y = 600$

Required number belt  $A = 200$ , belt  $B = 600$   
 maximum profit = Rs 1300

## Linear Programming Ex 30.4 Q6

Let required number of deluxe model and ordinary model be  $x$  and  $y$  respectively.

Since, profits on each model of deluxe and ordinary type model are Rs 15 and Rs 10 respectively. So, profits on  $x$  deluxe models and  $y$  ordinary models are  $15x$  and  $10y$

Let  $Z$  be total profit, then,

$$Z = 15x + 10y$$

Since, each deluxe and ordinary model require 2 and 1 hour of skilled men, so,  $x$  deluxe and  $y$  ordinary models required  $2x$  and  $y$  hours of skilled men but time available by skilled men is  $5 \times 8 = 40$  hours, So,

$$2x + y \leq 40 \quad \text{(first constraint)}$$

Since, each deluxe and ordinary model require 2 and 3 hours of semi-skilled men, so,  $x$  deluxe and  $y$  ordinary models require  $2x$  and  $3y$  hours of semi-skilled men respectively but total time available by semi-skilled men is  $10 \times 8 = 80$  hours, So,

$$2x + 3y \leq 80 \quad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 15x + 10y$$

subject to constraints,

$$2x + y \leq 40$$

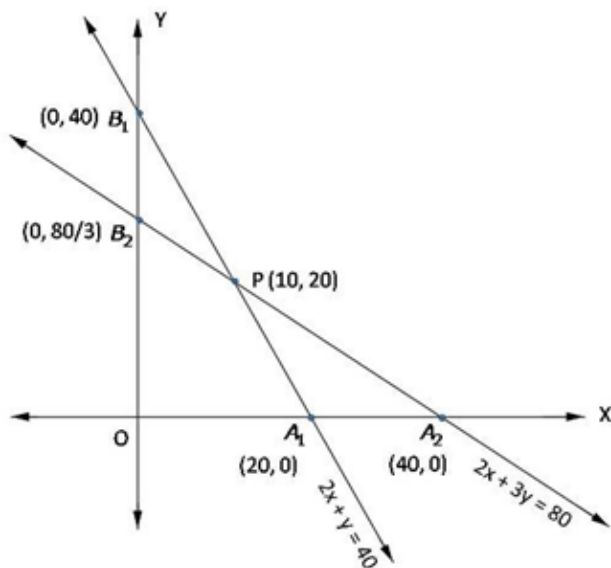
$$2x + 3y \leq 80$$

$$x, y \geq 0$$

[Since number of deluxe and ordinary models can not be less than zero]

Region  $2x + y \leq 40$ : line  $2x + y = 40$  meets axes at  $A_1(20, 0)$ ,  $B_1(0, 40)$  respectively. Region containing origin represents  $2x + y \leq 40$  as  $(0, 0)$  satisfies  $2x + y \leq 40$ .

Region  $2x + 3y \leq 80$ : line  $2x + 3y = 80$  meets axes at  $A_2(40, 0)$ ,  $B_2\left(0, \frac{80}{3}\right)$  respectively. Region containing origin represents  $2x + 3y \leq 80$ .



The value of  $Z = 15x + 10y$  at

$$O(0,0) = 15(0) + 10(0) = 0$$

$$A_1(20,0) = 15(20) + 10(0) = 300$$

$$P(10,20) = 15(10) + 10(20) = 350$$

$$B_2\left(0, \frac{80}{3}\right) = 15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$$

Therefore, maximum  $Z = 350$ , at  $x = 10, y = 20$

Required number deluxe model = 10

number of ordinary model = 600

maximum profit = Rs 350

Let required number of tea-cups of type  $A$  and  $B$  are  $x$  and  $y$  respectively.

Since, profits on each tea-cups of type  $A$  and  $B$  are 75 paise and 50 paise So, profits on  $x$  tea-cups of type  $A$  and  $y$  tea-cups of type  $B$  are  $75x$  and  $50y$  respectively, Let total profit on tea-cups be  $Z$ , so,

$$Z = 75x + 50y$$

Since, each tea-cup of type  $A$  and  $B$  require to work machine I for 12 and 6 minutes respectively so,  $x$  tea cups of type  $A$  and  $y$  tea cups of type  $B$  require to work on machine I for  $12x$  and  $6y$  minutes respectively .

Total time available on machine I is  $6 \times 60 = 360$  minutes. so,

$$12x + 6y \geq 360 \quad (\text{first constraint})$$

Since, each tea-cup of type  $A$  and  $B$  require to work machine II for 18 and 0 minutes respectively so,  $x$  tea cups of type  $A$  and  $y$  tea cups of  $B$  require to work on machine II for  $18x$  and  $0y$  minutes respectively but Total time available on machine II is  $6 \times 60 = 360$  minutes. so,

$$18x + 0y \geq 360 \quad (\text{second constraint})$$

$$x \leq 20$$

Since, each tea-cup of type  $A$  and  $B$  require to work machine III for 6 and 9 minutes respectively so,  $x$  tea cups of type  $A$  and  $y$  tea cups of  $B$  require to work on machine III for  $6x$  and  $9y$  minutes respectively Total time available on machine III is  $6 \times 60 = 360$  minutes. so,

$$6x + 9y \geq 360 \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 75x + 50y$$

subject to constraints,

$$12x + 6y \leq 360$$

$$x \leq 20$$

$$6x + 9y \leq 360$$

$$x, y \geq 0$$

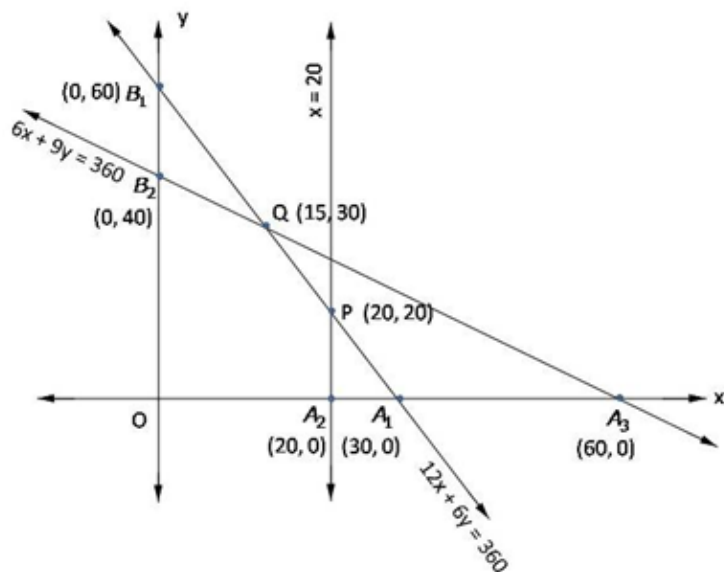
[Since production of tea cups can not be less than zero]

Region  $12x + 6y \leq 360$ : line  $12x + 6y = 360$  meets axes at  $A_1(30,0)$ ,  $B_1(0,60)$  respectively. Region containing origin represents  $12x + 6y \leq 360$  as  $(0,0)$  satisfies  $12x + 6y \geq 360$ .

Region  $x \leq 20$ : line  $x = 20$  is parallel to  $y$  - axes and meets  $x$  - axes at  $A_2(20,0)$ . Region containing origin represents  $x \leq 20$  as  $(0,0)$  satisfies  $x \leq 20$ .

Region  $6x + 9y \leq 360$ : line  $6x + 9y = 360$  meets axes at  $A_3(60,0)$ ,  $B_2(0,40)$  respectively. Region containing origin represents  $6x + 9y \leq 360$  as  $(0,0)$  satisfies  $6x + 9y \geq 360$ .

Region  $x, y \geq 0$  : it represents first quadrant.



Shaded region  $OA_2PQB_2$  is the feasible region. P is point obtained by solving  $x = 20$  and  $12x + 6y = 360$  and Q is point obtained by solving  $12x + 6y = 360$  and  $6x + 9y = 360$ .

The value of  $Z = 75x + 50y$  at

$$\begin{aligned} O(0, 0) &= 75(0) + 50(0) = 0 \\ A_2(20, 0) &= 75(20) + 50(0) = 1500 \\ P(20, 20) &= 75(20) + 50(20) = 2500 \\ Q(15, 30) &= 75(15) + 50(30) = 2624 \\ B_2(0, 40) &= 75(0) + 50(40) = 2000 \end{aligned}$$

Hence, Z is maximum at  $x = 15, Y = 30$

Therefore,

15 teacups of type A and 30 tea-cups of type B are needed to maximize profit

## Linear Programming Ex 30.4 Q8

Let required number of machine  $A$  and  $B$  are  $x$  and  $y$  respectively.

Since, production of each machine  $A$  and  $B$  are 60 and 40 units daily respectively, So, productions by  $x$  number of machine  $A$  and  $y$  number of machine  $B$  are  $60x$  and  $40y$  respectively, Let  $Z$  denote total output daily, so,

$$Z = 60x + 40y$$

Since, each machine of type  $A$  and  $B$  require 1000 sq.m and 1200 sq.m area so,  $x$  machine of type  $A$  and  $y$  machine of type  $B$  require  $100x$  and  $1200y$  sq.m area but, Total area available for machine is 7600 sq.m. so,

$$1000x + 1200y \leq 7600$$

$$5x + 6y \leq 38 \quad (\text{first constraint})$$

Since, each machine of type  $A$  and  $B$  require 12 men and 8 men to work respectively so,  $x$  machine of type  $A$  and  $y$  machine of type  $B$  require  $12x$  and  $8y$  men to work respectively but, Total 72 men available for work so,

$$12x + 8y \leq 72$$

$$3x + 2y \leq 18 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 60x + 40y$$

subject to constraints,

$$5x + 6y \leq 38$$

$$3x + 2y \leq 18$$

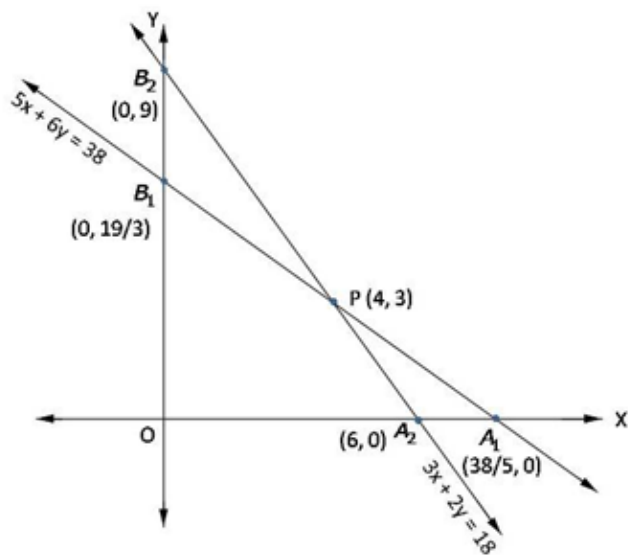
$$x, y \geq 0$$

[Number of machines can not be less than zero]

Region  $5x + 6y \leq 38$ : line  $5x + 6y = 38$  meets axes at  $A_1\left(\frac{38}{5}, 0\right)$ ,  $B_1\left(0, \frac{19}{3}\right)$  respectively. Region containing origin represents  $5x + 6y \leq 38$  as origin satisfies  $5x + 6y \geq 38$ .

Region  $3x + 2y \leq 18$ : line  $3x + 2y = 18$  meets axes at  $A_2(6, 0)$ ,  $B_2(0, 9)$  respectively. Region containing origin represents  $3x + 2y \leq 18$  as  $(0, 0)$  satisfies  $3x + 2y \leq 18$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $O A_2 P B_1$  is the feasible region.  $P(4, 3)$  is obtained by solving  $3x + 2y = 18$  and  $5x + 6y = 38$

The value of  $Z = 60x + 40y$  at

$$O(0, 0) = 60(0) + 40(0) = 0$$

$$A_2(6, 0) = 60(6) + 40(0) = 360$$

$$P(4, 3) = 60(4) + 40(3) = 360$$

$$B_1\left(0, \frac{19}{3}\right) = 60(0) + 40\left(\frac{19}{3}\right) = \frac{760}{3}$$

Therefore maximum  $Z = 360$  at  $x = 4, Y = 3$  or  $x = 6, y = 0$

Output is maximum when 4 machines of type A and 3 machine of type B or 6 machines of type A and no machine of type B.

## Linear Programming Ex 30.4 Q9



Let number of goods  $A$  and  $B$  are  $x$  and  $y$  respectively.

Since, profits on each  $A$  and  $B$  are Rs 40 and Rs 50 respectively. So, profits on  $x$  of type  $A$  and  $y$  of type  $B$  are  $40x$  and  $50y$  respectively, Let  $Z$  be total profit on  $A$  and  $B$ , so,

$$Z = 40x + 50y$$

Since, each  $A$  and  $B$  require 3 gm and 1 gm of silver respectively. so,  $x$  of type  $A$  and  $y$  type  $B$  require  $3x$  and  $y$  gm silver respectively but, Total silver available is 9 gm. so,

$$3x + y \leq 9 \quad (\text{first constraint})$$

Since, each  $A$  and  $B$  require 1 gm and 2 gm of gold respectively. so,  $x$  of type  $A$  and  $y$  type  $B$  require  $x$  and  $2y$  gm of gold respectively but, Total gold available is 8 gm, so,

$$x + 2y \leq 8 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 40x + 50y$$

Subject to constraints,

$$3x + y \leq 9$$

$$x + 2y \leq 8$$

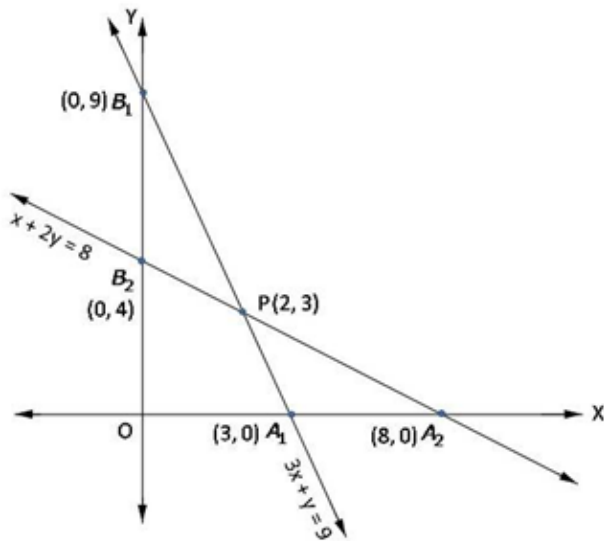
$$x, y \geq 0$$

[Since production of  $A$  and  $B$  can not be less than zero]

Region  $3x + y \leq 9$ : line  $3x + y = 9$  meets axes at  $A_1(3,0)$ ,  $B_1(0,9)$  respectively. Region containing origin represents  $3x + y \leq 9$  as  $(0,0)$  satisfies  $3x + y \geq 9$ .

Region  $x + 2y \leq 8$ : line  $x + 2y = 8$  meets axes at  $A_2(8,0)$ ,  $B_2(0,4)$  respectively. Region containing origin represents  $x + 2y \leq 8$  as  $(0,0)$  satisfies  $x + 2y \leq 8$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $O A_2 P B_2$  is the feasible region. Point  $P (2, 3)$  is obtained by solving  $3x + y = 9$  and  $x + 2y = 8$

The value of  $Z = 40x + 50y$  at

$$\begin{aligned}
 O (0, 0) &= 40(0) + 50(0) = 0 \\
 A_1 (3, 0) &= 40(3) + 50(0) = 120 \\
 P (2, 3) &= 40(2) + 50(3) = 230 \\
 B_2 (0, 4) &= 40(0) + 50(4) = 200
 \end{aligned}$$

Therefore maximum  $Z = 230$  at  $x = 2, Y = 3$

Hence,

Maximum profit = Rs 230 number of goods of type  $A = 2$ , type  $B = 3$

Let daily production of chairs and tables be  $x$  and  $y$  respectively.

Since, profits on each chair and table are Rs 3 and Rs 5. So, profits on  $x$  number of chairs and  $y$  number of tables are Rs  $3x$  and Rs  $5y$  respectively, Let  $Z$  be total profit on table and chair, so,

$$Z = 3x + 5y$$

Since, each chair and table require 2 hrs and 4 hrs on machine  $A$  respectively. so,  $x$  number of chair and  $y$  number of table require  $2x$  and  $4y$  hrs on machine  $A$  respectively but, maximum time available on machine  $A$  be 16 hrs, so,

$$2x + 4y \leq 16$$

$$x + 2y \leq 8 \quad \text{(first constraint)}$$

Since, each chair and table require 6 hrs and 2 hrs on machine  $B$ . so,  $x$  number of chair and  $y$  number of table require  $6x$  and  $2y$  hrs on machine  $B$  respectively but, maximum time available on machine  $B$  be 30 hrs, so,

$$6x + 2y \leq 30$$

$$3x + y \leq 15 \quad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 3x + 5y$$

subject to constraints,

$$x + 2y \leq 8$$

$$3x + y \leq 15$$

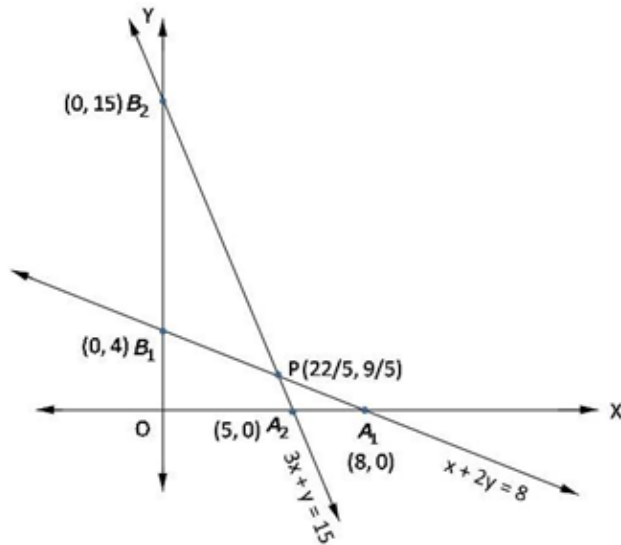
$$x, y \geq 0$$

[Since production of chair and table can not be less than zero]

Region  $x + 2y \leq 8$ : line  $x + 2y = 8$  meets axes at  $A_1(8, 0)$ ,  $B_1(0, 4)$  respectively. Region containing origin represents  $x + 2y \leq 8$  as  $(0, 0)$  satisfies  $x + 2y \leq 8$ .

Region  $3x + y \leq 15$ : line  $3x + y = 15$  meets axes at  $A_2(5, 0)$ ,  $B_2(0, 15)$  respectively. Region containing origin represents  $3x + y \leq 15$  as  $(0, 0)$  satisfies  $3x + y \leq 15$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $O A_2 P B_1$  represents a feasible region. Point  $P\left(\frac{22}{5}, \frac{9}{5}\right)$  is obtained by solving  $x + 2y = 8$  and  $3x + y = 15$

The value of  $Z = 3x + 5y$  at

$$O(0, 0) = 3(0) + 5(0) = 0$$

$$A_2(5, 0) = 3(5) + 5(0) = 15$$

$$P\left(\frac{22}{5}, \frac{9}{5}\right) = 3\left(\frac{22}{5}\right) + 5\left(\frac{9}{5}\right) = \frac{111}{5} = 22.2$$

$$B_1(0, 4) = 3(0) + 5(4) = 20$$

Maximum  $Z = 22.2$  at  $x = \frac{22}{5}, y = \frac{9}{5}$

Daily production of chair =  $\frac{22}{5}$ , table =  $\frac{9}{5}$

maximum profit = Rs 22.2

## Linear Programming Ex 30.4 Q11

Let required production of chairs and tables be  $x$  and  $y$ .

Since, profits on each chair and table are Rs 45 and Rs 80, So,  
profits on  $x$  number of chairs and  $y$  number of tables are Rs  $45x$  and Rs  $80y$ ,  
Let  $Z$  be total profit on tables and chairs, so,

$$Z = 45x + 80y$$

Since, each chair and table require 5 sq.ft. and 20 sq.ft. of wood respectively. so,  
 $x$  number of chair and  $y$  number of table require  $5x$  and  $20y$  sq.ft. of wood respectively but,  
400 sq.ft. of wood is available, so,

$$5x + 20y \leq 400$$

$$\Rightarrow x + 4y \leq 80 \quad (\text{first constraint})$$

Since, each chair and table require 10 and 25 men-hrs respectively. so,  
 $x$  number of chairs and  $y$  number of tables require  $10x$  and  $25y$  men-hrs  
respectively but, only 450 men-hrs are available, so,

$$10x + 25y \leq 450$$

$$\Rightarrow 2x + 5y \leq 90 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 45x + 80y$$

Subject to constraints,

$$x + 4y \leq 80$$

$$2x + 5y \leq 90$$

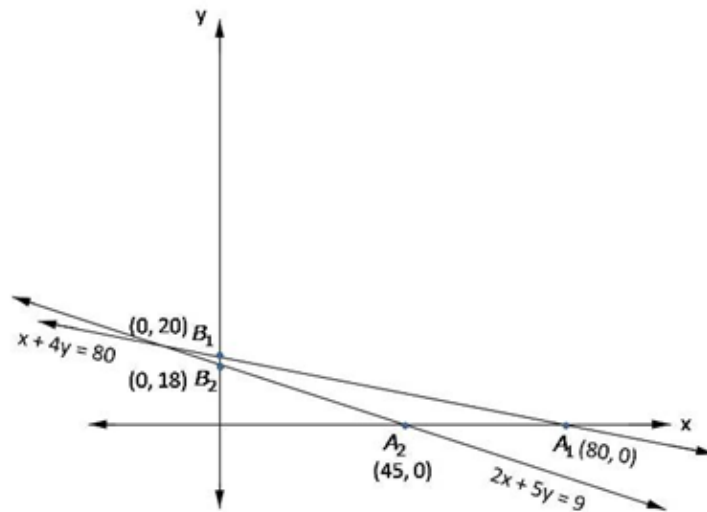
$$x, y \geq 0$$

[Since production of tabel and chair can not be less than zero]

Region  $x + 4y \leq 80$ : line  $x + 4y = 80$  meets axes at  $A_1(80,0)$ ,  $B_1(0,20)$  respectively. Region  
containing origin represents  $x + 4y \leq 80$  as  $(0,0)$  satisfies  $x + 4y \leq 80$ .

Region  $2x + 5y \leq 90$ : line  $2x + 5y = 90$  meets axes at  $A_2(45,0)$ ,  $B_2(0,18)$  respectively. Region  
containing origin represents  $2x + 5y \leq 90$  as  $(0,0)$  satisfies  $2x + 5y \leq 90$ .

Region  $x, y \geq 0$  : it represents first quadrant.



Shaded region  $O A_2 B_2$  is the feasible region.

The value of  $Z = 45x + 80y$  at

$$O (0, 0) = 45(0) + 80(0) = 0$$

$$A_2 (45, 0) = 45(45) + 80(0) = 2025$$

$$B_2 (0, 18) = 45(0) + 80(18) = 1440$$

Therefore,

Maximum  $Z = 2025$  at  $x = 45, y = 0$

Profit is maximum when number of chairs = 45, tables = 0

profit = Rs 2025

Let required production of product  $A$  and  $B$  be  $x$  and  $y$  respectively.

Since, profit on each product  $A$  and  $B$  are Rs 3 and Rs 4 respectively, So, profit on  $x$  product  $A$  and  $y$  product  $B$  are Rs  $3x$  and Rs  $4y$  respectively, Let  $Z$  be the total profit on product, so,

$$Z = 3x + 4y$$

Since, each product  $A$  and  $B$  requires 4 minutes each on machine  $M_1$ . so,  $x$  product  $A$  and  $y$  product  $B$  require  $4x$  and  $4y$  minutes on machine  $M_1$  respectively but maximum available time on machine  $M_1$  is 8 hrs 20 min.= 500 min. so,

$$4x + 4y \leq 500$$

$$\Rightarrow x + y \leq 125 \quad (\text{first constraint})$$

Since, each product  $A$  and  $B$  requires 8 minutes and 4 min. on machine  $M_2$  respectively. so,  $x$  product  $A$  and  $y$  product  $B$  require  $8x$  and  $4y$  min. respectively on machine  $M_2$  but, maximum available time on machine  $M_2$  is 10 hrs = 600 min. so,

$$8x + 4y \leq 600$$

$$\Rightarrow 2x + y \leq 150 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 3x + 4y$$

subject to constraints,

$$x + y \leq 125$$

$$2x + y \leq 150$$

$$x, y \geq 0$$

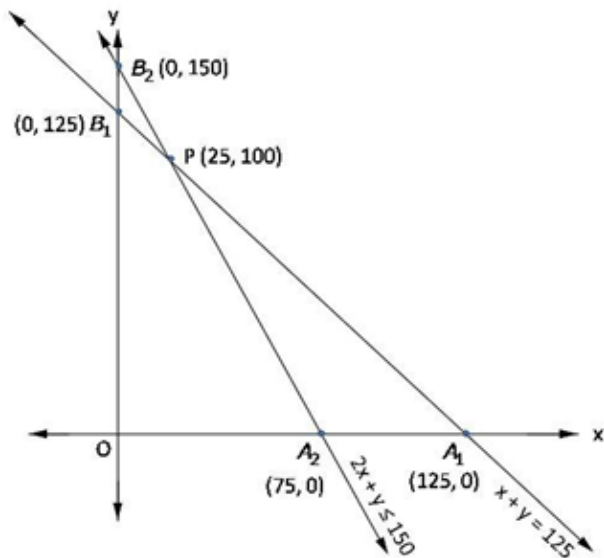
[Since number of product can not be less than zero]

Region  $x + y \leq 125$ : line  $x + y = 125$  meets axis at  $A_1(125, 0)$ ,  $B_1(0, 125)$  respectively. Region  $x + y \leq 125$  contains origin represents as  $(0, 0)$  satisfies  $x + y \leq 125$ .

Region  $2x + y \leq 150$ : line  $2x + y = 150$  meets axis at  $A_2(75, 0)$ ,  $B_2(0, 150)$  respectively. Region containing origin represents  $2x + y \leq 150$  as  $(0, 0)$  satisfies  $2x + y \leq 150$

Region  $x, y \geq 0$ : it represents first quadrant.

Shaded region  $OA_2PB_1$  is feasible region  $P(25, 100)$  is obtained by solving  $x + y = 125$  and  $2x + y = 150$



The value of  $Z = 3x + 4y$  at

$$O(0,0) = 3(0) + 4(0) = 0$$

$$A_2(75,0) = 3(75) + 4(0) = 225$$

$$P(25,100) = 3(25) + 4(100) = 475$$

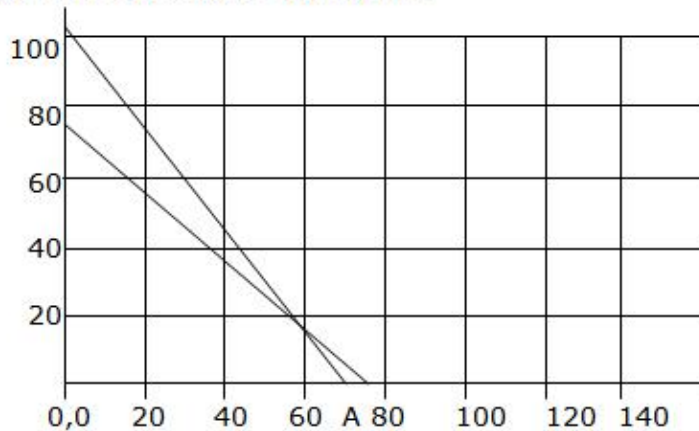
$$B_1(0,125) = 3(0) + 4(125) = 500$$

Maximum profit = Rs 500, product  $A = 0$   
 product  $B = 125$



	Item A	Item B	
	x	y	
Motors	3x	2y	$\leq 210$
Transformer	4x	4y	$\leq 300$
Profit Rs.	20x	30y	Maximize

The above LPP can be presented in a table above.  
 Aim is to find the values of x & y that maximize the function  $Z = 20x + 30y$ , subject to the conditions  
 $3x + 2y \leq 210$ ; gives  $x=0, y=105$  &  $y=0, x=70$   
 $4x + 4y \leq 300$ ; gives  $x=0, y=75$  &  $y=0, x=75$   
 $x, y \geq 0$ . Plotting the constraints,



The feasible region is 80-B-A-0,0  
 Tabulating the value of Z at the corner points

Corner point	Value of $Z = 20x + 30y$
0, 0	0
0, 75	2250
70, 0	1400
60, 15	1650

The maximum occur with the production of 0 units of Item A and 75 units of Item B, with a value of Rs. 2250/-

Let number of I product and II product produced are  $x$  and  $y$  respectively.

Since, profits on each unit of product I and product II are 2 and 3 monetary unit, So, profits on  $x$  units of product I and  $y$  units of product II are  $2x$  and  $3y$  monetary units respectively, Let  $Z$  be total profit, so,

$$Z = 2x + 3y$$

Since, each product I and II require 2 and 4 units of resources  $A$ , so,  $x$  units of product I and  $y$  units of product II require  $2x$  and  $4y$  units of resource  $A$  respectively, but maximum available quantity of resource  $A$  is 20 units. so,

$$2x + 4y \leq 20$$

$$\Rightarrow x + 2y \leq 10 \quad (\text{first constraint})$$

Since, each product I and II require 2 and 4 units of resource  $B$  each, so,  $x$  units of product I and  $y$  units of product II require  $2x$  and  $2y$  units of resource  $B$  respectively, but maximum available quantity of resource  $B$  is 12 units. so,

$$2x + 2y \leq 12$$

$$\Rightarrow x + y \leq 6 \quad (\text{second constraint})$$

Since, each units of product I require 4 units of resource  $C$ . It is not required by product II, so,  $x$  units of product I require  $4x$  units of resource  $C$ , but maximum available quantity of resource  $C$  is 16 units. so,

$$4x \leq 16$$

$$\Rightarrow x \leq 4 \quad (\text{Third constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \leq 10$$

$$x + y \leq 6$$

$$x \leq 4$$

$$x, y \geq 0$$

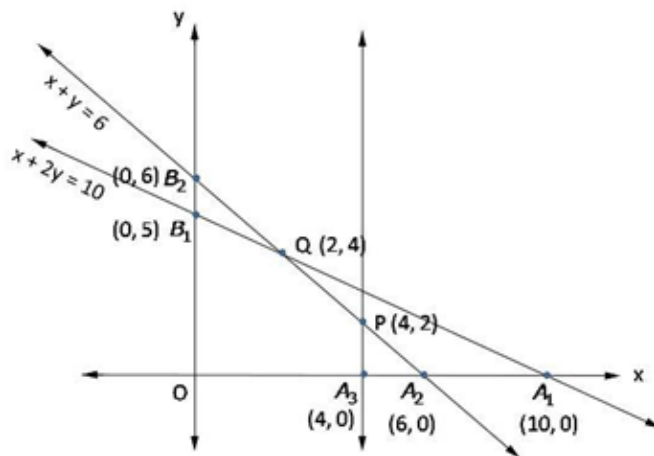
[Since production fo I and II can not be less than zero]

Region  $x + 2y \leq 10$ : line  $x + 2y = 10$  meets axes at  $A_1(10, 0)$ ,  $B_1(0, 5)$  respectively. Region containing origin represents  $x + 2y \leq 10$  as  $(0, 0)$  satisfies  $x + 2y \leq 10$ .

Region  $x + y \leq 6$ : line  $x + y = 6$  meets axes at  $A_2(6, 0)$ ,  $B_2(0, 6)$  respectively. Region containing origin represents  $x + y \leq 6$  as  $(0, 0)$  satisfies  $x + y \leq 6$ .

Region  $x \leq 4$ : line  $x = 4$  is parallel to  $y$ -axis and meets  $y$ -axis at  $A_3(4, 0)$ . Region containing origin represents  $x \leq 4$  as  $(0, 0)$  satisfies  $x \leq 4$

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $OA_3PQB_1$  represents feasible region  $P(4, 2)$  is obtained by solving  $x = 4$  and  $x + y = 6$ ,  $Q(2, 4)$  is obtained by solving  $x + y = 6$  and  $x + 2y = 10$ .

The value of  $Z = 2x + 3y$  at

$$\begin{aligned} O(0, 0) &= 2(0) + 3(0) = 0 \\ A_3(4, 0) &= 2(4) + 3(0) = 8 \\ P(4, 2) &= 2(4) + 3(2) = 14 \\ Q(2, 4) &= 2(2) + 3(4) = 16 \\ B_1(0, 5) &= 2(0) + 3(5) = 15 \end{aligned}$$

Maximum  $Z = 16$  at  $x = 2, y = 4$

First product = 2 units, second product = 4 unit

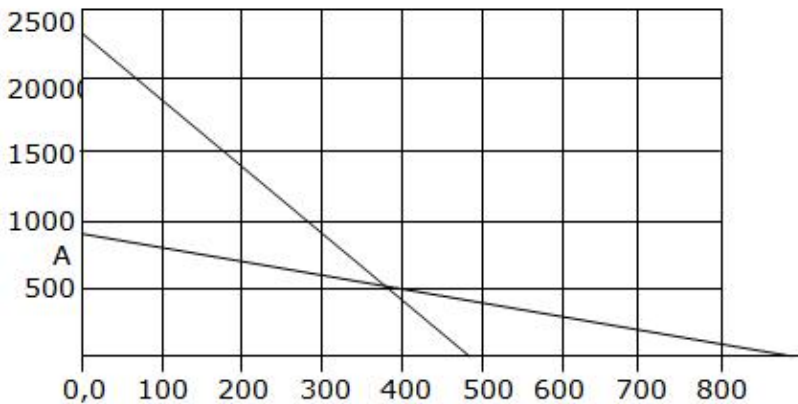
Maximum profit = 16 monetary units

## Linear Programming Ex 30.4 Q15

	Hardcover	Paperback	
	x	y	
Printing time	5x	5y	$\leq 4800$
Binding time	10x	2y	$\leq 4800$
Selling price Rs.	72x	40y	Maximize

The above LPP can be presented in a table above.

Aim is to find the values of x & y that maximize the function  $Z = 72x + 40y$ , subject to the conditions  $5x + 5y \leq 4800$ ; gives  $x=0, y= 960$  &  $y=0, x=960$   
 $10x + 2y \leq 4800$ ; gives  $x=0, y=2400$  &  $y=0, x=480$   
 $x, y \geq 0$ . Plotting the constraints,



The feasible region is A-B-480-0,0

Tabulating the value of Z at the corner points

Corner point	Value of $Z = 72x + 40y$
0, 0	0
0, 480	19200
360, 600	49920
480, 0	34560

The maximum occurs with the production of 360 units of Hardcover books and 600 units of Paperback books, with a value of Rs. 49920/-. This the selling price.

Cost price = fixed cost + variable cost  
 $= 9600 + 56 \times 360 + 28 \times 600 = 46560$

Profit = Selling price - cost price =  $49920 - 46560$   
 $= \text{Rs. } 3360$

	Pill size A	Pill size B	
	x	y	
Aspirin	2x	1.y	$\geq 12$
Bicarbonate	5x	8y	$\geq 7.4$
Codeine	1.x	66y	$\geq 24$
Relief	x	y	Minimize

The above LPP can be presented in a table above.

Aim is to find the values of x & y that minimize the

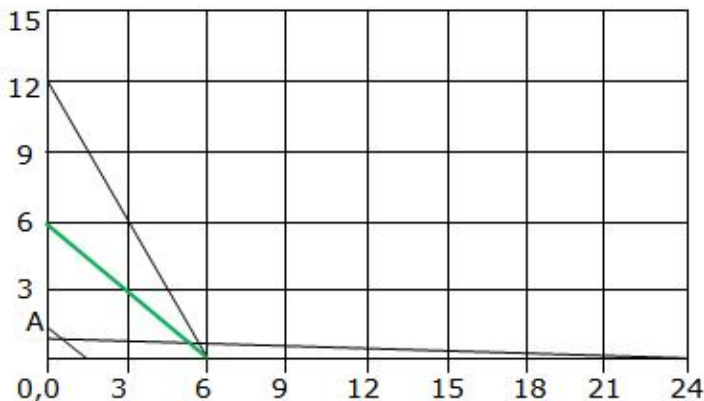
function  $Z = x + y$ , subject to the conditions

$2x + y \geq 12$ ; gives  $x=0, y=12$  &  $y=0, x=6$

$5x + 8y \geq 7.4$ ; gives  $x=0, y=7.4/8$  &  $y=0, x=7.4/5$

$x + 66y \geq 24$ ; gives  $x=0, y=4/11$  &  $y=0, x=24$

$x, y \geq 0$ . Plotting the constraints,



The feasible region is 12-C-24

Tabulating the value of Z at the corner points

Corner point	Value of $Z = x + y$
0, 12	12
24, 0	24
5.86, 0.27	6.13

The minimum occurs with 5.86 pills of size A and 0.27 pills of size B. since the feasible region is unbounded plot  $x+y < 6.13$ . the green line shows here are no common points with the unbounded feasible region so the obtained point is the point that gives minimum pills to be consumed.

Let required quantity of compound  $A$  and  $B$  are  $x$  and  $y$  kg.

Since, cost of one kg of compound  $A$  and  $B$  are Rs 4 and Rs 6 per kg. So, cost of  $x$  kg. of compound  $A$  and  $y$  kg. of compound  $B$  are Rs  $4x$  and Rs  $6y$  respectively, Let  $Z$  be the total cost of compounds, so,

$$Z = 4x + 6y$$

Since, compound  $A$  and  $B$  contain 1 and 2 units of ingredient  $C$  per kg. respectively, so,  $x$  kg. of compound  $A$  and  $y$  kg. of compound  $B$  contain  $x$  and  $2y$  units of ingredient  $C$  respectively but minimum requirement of ingredient  $C$  is 80 units, so,

$$x + 2y \geq 80 \quad (\text{first constraint})$$

Since, compound  $A$  and  $B$  contain 3 and 1 unit of ingredient  $D$  per kg. respectively, so,  $x$  kg. of compound  $A$  and  $y$  kg. of compound  $B$  contain  $3x$  and  $y$  units of ingredient  $D$  respectively but minimum requirement of ingredient  $D$  is 75 units, so,

$$3x + y \geq 75 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which minimize

$$Z = 4x + 6y$$

Subject to constraints,

$$x + 2y \geq 80$$

$$3x + y \geq 75$$

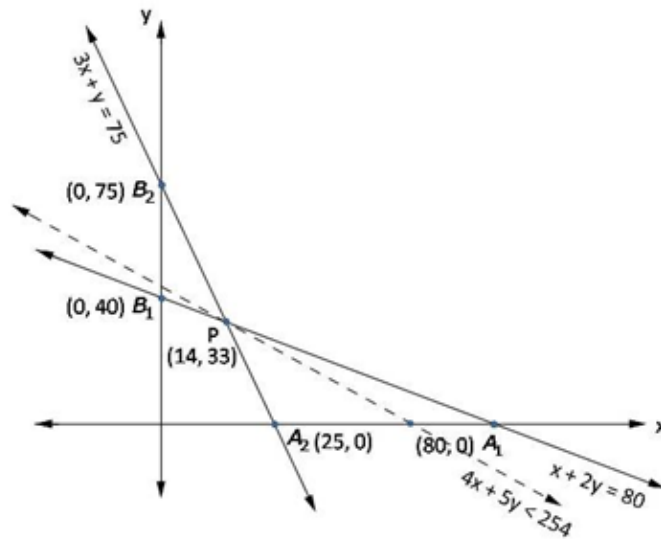
$$x, y \geq 0$$

[Since production can not be less than zero]

Region  $x + 2y \geq 80$ : line  $x + 2y = 80$  meets axes at  $A_1(80, 0)$ ,  $B_1(0, 40)$  respectively. Region not containing origin represents  $x + 2y \geq 80$  as  $(0, 0)$  does not satisfy  $x + 2y \geq 80$ .

Region  $3x + y \geq 75$ : line  $3x + y = 75$  meets axes at  $A_2(25, 0)$ ,  $B_2(0, 75)$  respectively. Region not containing origin represents  $3x + y \geq 75$  as  $(0, 0)$  does not satisfy  $3x + y \geq 75$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Unbounded shaded region  $A_1P B_2$  represents feasible region. point  $P$  is obtained by solving  $x + 2y = 80$  and  $3x + y = 75$

The value of  $Z = 4x + 6y$  at

$$\begin{aligned} A_1(80, 0) &= 4(80) + 6(0) = 320 \\ P(14, 33) &= 4(14) + 6(33) = 254 \\ B_2(0, 75) &= 4(0) + 6(75) = 450 \end{aligned}$$

Smallest value of  $Z = 254$  open half plane  $4x + 6y < 254$  has no point in common with feasible region. so,

Smallest value is the minimum value.

Minimum cost = Rs 254  
 quantity of  $A = 14$  kg  
 quantity of  $B = 33$  kg

Let the company manufacture  $x$  souvenirs of type A and  $y$  souvenirs of type B. Therefore,

$$x \geq 0 \text{ and } y \geq 0$$

The given information can be compiled in a table as follows.

	Type A	Type B	Availability
<b>Cutting (min)</b>	5	8	$3 \times 60 + 20 = 200$
<b>Assembling (min)</b>	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are

$$5x + 8y \leq 200$$

$$10x + 8y \leq 240 \text{ i.e., } 5x + 4y \leq 120$$

$$\text{Total profit, } Z = 5x + 6y$$

The mathematical formulation of the given problem is

$$\text{Maximize } Z = 5x + 6y \dots (1)$$

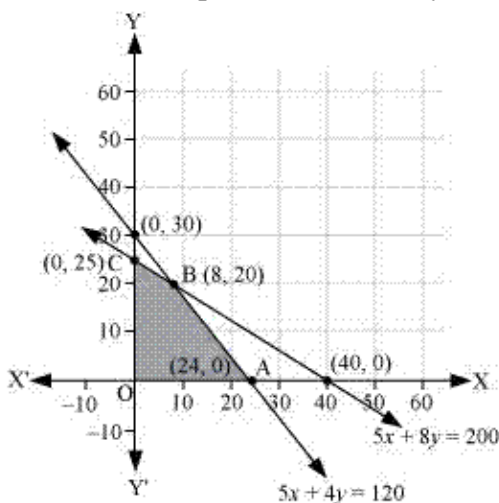
subject to the constraints,

$$5x + 8y \leq 200 \dots (2)$$

$$5x + 4y \leq 120 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of  $Z$  at these corner points are as follows.

Corner point	$Z = 5x + 6y$	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of  $Z$  is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.



## Linear Programming Ex 30.4 Q19

Let required number of product  $A$  and  $B$  be  $x$  and  $y$  respectively.

Since, profit on each product  $A$  and  $B$  are Rs 20 and Rs 30 respectively. So,  $x$  number of product  $A$  and  $y$  number of product  $B$  gain profits of Rs  $20x$  and Rs  $30y$  respectively, Let  $Z$  be total profit then,

$$Z = 20x + 30y$$

Since, selling prices of each product  $A$  and  $B$  are Rs 200 and Rs 300 respectively, so, revenues earned by selling  $x$  units of product  $A$  and  $y$  units of product  $B$  are  $200x$  and  $300y$  respectively but weekly turnover must not be less than Rs 10000, so,

$$200x + 300y \geq 10000$$

$$2x + 3y \geq 100 \quad (\text{first constraint})$$

Since, each product  $A$  and  $B$  require  $\frac{1}{2}$  and 1 hr. to make so,  $x$  units of product  $A$  and  $y$  units of product  $B$  are  $\frac{1}{2}x$  and  $y$  hrs. to make respectively but working time available is 40 hrs maximum, so,

$$\frac{1}{2}x + y \leq 40$$

$$x + 2y \leq 80 \quad (\text{second constraint})$$

There is a permanent order of 14 and 16 of product  $A$  and  $B$  respectively, so,

$$x \geq 14$$

$$y \geq 16 \quad (\text{third and fourth constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 20x + 30y$$

Subject to constraints,

$$2x + 3y \geq 100$$

$$x + 2y \leq 80$$

$$x \geq 14$$

$$y \geq 16$$

$$x, y \geq 0$$

[Since production can not be less than zero]

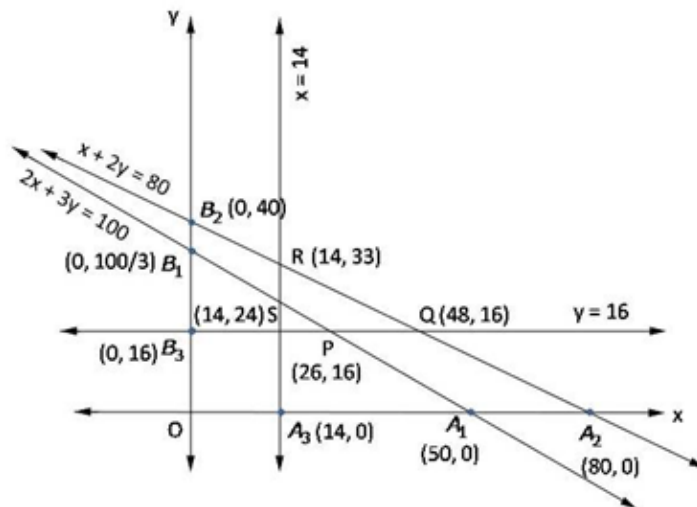
Region  $2x + 3y \geq 100$ : line  $2x + 3y = 100$  meets axes at  $A_1(50, 0)$ ,  $B_1(0, \frac{100}{3})$  respectively. Region not containing origin represents  $2x + 3y \geq 100$  as  $(0, 0)$  does not satisfy  $2x + 3y \geq 100$ .

Region  $x + 2y \leq 80$ : line  $x + 2y = 80$  meets axes at  $A_2(80, 0)$ ,  $B_2(0, 40)$  respectively. Region not containing origin represents  $x + 2y \leq 80$  as  $(0, 0)$  satisfies  $x + 2y \leq 80$ .

Region  $x \geq 14$ : line  $x = 14$  is parallel to  $y$ -axis and meets  $x$ -axis at  $A_3(14, 0)$ . Region not containing origin represents  $x \geq 14$  as  $(0, 0)$  does not satisfy  $x \geq 14$ .

Region  $y \geq 16$ : line  $y = 16$  is parallel to  $x$ -axis and meets  $y$ -axis at  $B_3(0, 16)$ . Region not containing origin represents  $y \geq 16$  as  $(0, 0)$  does not satisfy  $y \geq 16$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $PQRS$  represents feasible region. Point  $P(26, 16)$  is obtained by solving  $y = 16$  and  $2x + 3y = 100$ ,  $Q(48, 16)$  is obtained by solving  $y = 16$  and  $x + 2y = 80$ ,  $R(14, 33)$  is obtained by solving  $x = 14$  and  $x + 2y = 80$ ,  $S(14, 24)$  is obtained by solving  $x = 14$  and  $2x + 3y = 100$

The value of  $Z = 20x + 30y$  at

$$\begin{aligned} P(26, 16) &= 20(26) + 30(16) = 1000 \\ Q(48, 16) &= 20(48) + 30(16) = 1440 \\ R(14, 33) &= 20(14) + 30(33) = 1270 \\ S(14, 24) &= 20(14) + 30(24) = 1000 \end{aligned}$$

maximum  $Z = 1440$  at  $x = 48, y = 16$

Number product  $A = 48$ , product  $B = 16$

maximum profit = Rs 1440

## Linear Programming Ex 30.4 Q20

Let required number of trunk I and trunk II be  $x$  and  $y$  respectively.

Since, profit on each trunk I and trunk II are Rs 30 and Rs 25 respectively. So, profit on  $x$  trunk of type I and  $y$  trunk of type II are Rs  $30x$  and Rs  $25y$  respectively, Let total profit on trunks be  $Z$ , so,

$$Z = 30x + 25y$$

Since, each trunk I and trunk II is required to work 3 hrs each on machine A, so,  $x$  trunk I and  $y$  trunk II is required  $3x$  and  $3y$  hrs respectively to work on machine A but machine A can work for at most 18 hrs, so,

$$3x + 3y \leq 18$$

$$\Rightarrow x + y \leq 6 \quad (\text{first constraint})$$

Since, each trunk I and II is required to work 3 hrs and 2 hrs on machine B, so,  $x$  trunk I and  $y$  trunk II is required  $3x$  and  $2y$  hrs to work respectively on machine B but machine B can work for at most 15 hrs, so,

$$3x + 2y \leq 15 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is, Find  $x$  and  $y$  which maximize

$$Z = 30x + 25y$$

Subject to constraints,

$$x + y \leq 6$$

$$3x + 2y \leq 15$$

$$x, y \geq 0$$

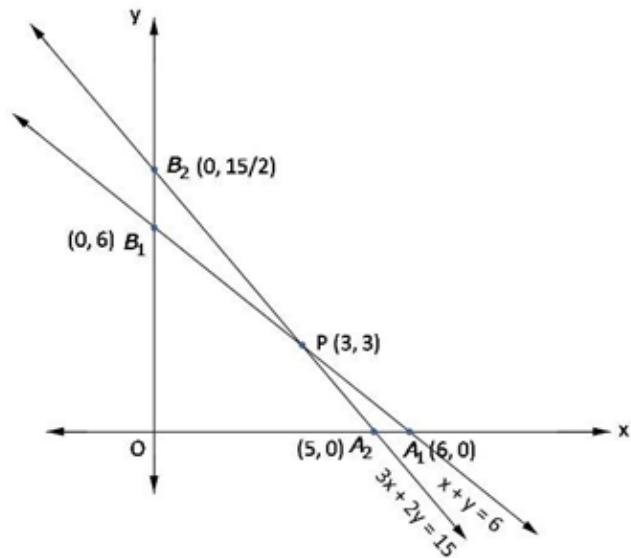
[Since production of trunk can not be less than zero]

Region  $x + y \leq 6$ : line  $x + y = 6$  meets axes at  $A_1(6, 0)$ ,  $B_1(0, 6)$  respectively. Region containing origin represents  $x + y \leq 6$  as  $(0, 0)$  satisfies  $x + y \leq 6$ .

Region  $3x + 2y \leq 15$ : line  $3x + 2y = 15$  meets axes at  $A_2(5, 0)$ ,  $B_2(0, \frac{15}{2})$  respectively. Region containing origin represents  $3x + 2y \leq 15$  as  $(0, 0)$  satisfies  $3x + 2y \leq 15$ .

Region  $x, y \geq 0$ : it represents first quadrant.

Shaded region  $A_2PB_1$  represents feasible region. Point  $P(3, 3)$  is obtained by solving  $x + y = 6$  and  $3x + 2y = 15$ ,



The value of  $Z = 30x + 25y$  at

$$A_2(5,0) = 30(5) + 25(0) = 150$$

$$P(3,3) = 30(3) + 25(3) = 165$$

$$B_1(0,6) = 30(0) + 25(6) = 150$$

$$O(0,0) = 30(0) + 25(0) = 0$$

maximum  $Z = 165$  at  $x = 3, y = 3$

Trunk of type  $A = 3$ , type  $B = 3$

maximum profit = Rs 165

Let production of each bottle of  $A$  and  $B$  are  $x$  and  $y$  respectively.

Since, profits on each bottle of  $A$  and  $B$  are Rs 8 and Rs 7 per bottle respectively. So, profit on  $x$  bottles of  $A$  and  $y$  bottles of  $B$  are  $8x$  and  $7y$  respectively, Let  $Z$  be total profit on bottles so,

$$Z = 8x + 7y$$

Since, it takes 3 hrs and 1 hr to prepare enough material to fill 1000 bottles of type  $A$  and  $B$  respectively, so,  $x$  bottles of  $A$  and  $y$  bottles of  $B$  are preparing is  $\frac{3x}{1000}$  hrs and  $\frac{y}{100}$  hrs respectively but total 66 hrs are available, so,

$$\frac{3x}{1000} + \frac{y}{100} \leq 66$$

$$\Rightarrow 3x + y \leq 66000 \quad (\text{first constraint})$$

Since, raw material available to make 2000 bottles of  $A$  and 4000 bottles of  $B$  but there are 45000 bottles into which either of medicines can be put so,

$$\Rightarrow x \leq 20000 \quad (\text{second constraint})$$

$$y \leq 40000 \quad (\text{third constraint})$$

$$x + y \leq 45000 \quad (\text{fourth constraint})$$

$$x, y \geq 0$$

[Since production of bottles can not be less than zero]

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 8x + 7y$$

Subject to constraints,

$$3x + y \leq 66000$$

$$x \leq 20000$$

$$y \leq 40000$$

$$x + y \leq 45000$$

$$x, y \geq 0$$

Region  $3x + y \leq 66000$ : line  $3x + y = 66000$  meets axes at  $A_1(22000, 0)$ ,  $B_1(0, 66000)$

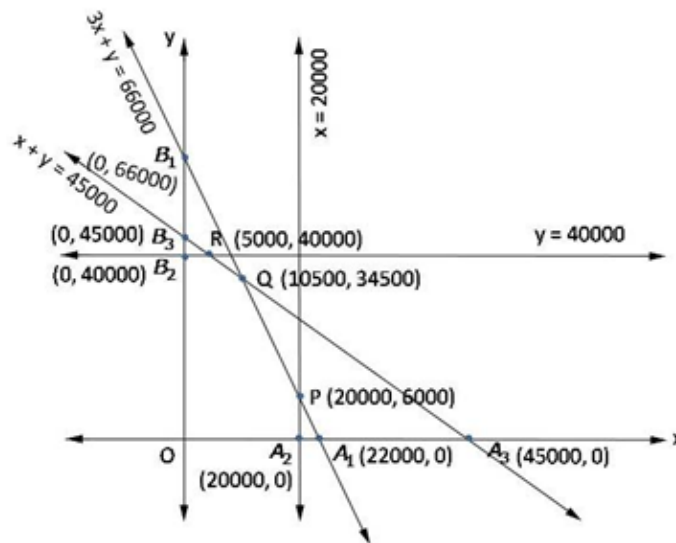
respectively. Region containing origin represents  $3x + y \leq 66000$  as  $(0, 0)$  satisfies  $3x + y \leq 66000$ .

Region  $x \leq 20000$ : line  $x = 20000$  is parallel to  $y$ -axis and meets  $x$ -axis at  $A_2(20000, 0)$ . Region containing origin represents  $x \leq 20000$  as  $(0,0)$  satisfies  $x \leq 20000$ .

Region  $y \leq 40000$ : line  $y = 40000$  is parallel to  $x$ -axis and meets  $y$ -axis at  $B_2(0, 40000)$ . Region containing origin represents  $y \leq 40000$  as  $(0,0)$  satisfies  $y \leq 40000$ .

Region  $x + y \leq 45000$ : line  $x + y = 45000$  meets axes at  $A_3(45000, 0), B_3(0, 45000)$  respectively. Region containing origin represents  $x + y \leq 45000$  as  $(0,0)$  satisfies  $x + y \leq 45000$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $OA_2PRB_2$  represents feasible region. Point  $P(20000, 6000)$  is obtained by solving  $x = 20000$  and  $3x + y = 66000$ ,  $Q(10500, 34500)$  is obtained by solving  $x + y = 45000$  and  $3x + y = 66000$ ,  $R(15000, 40000)$  is obtained by solving  $x + y = 45000$ ,  $y = 40000$

The value of  $Z = 8x + 7y$  at

$O(0, 0)$	$= 8(0) + 7(0) = 0$
$A_2(20000, 0)$	$= 8(20000) + 7(0) = 160000$
$P(20000, 6000)$	$= 8(20000) + 7(6000) = 202000$
$Q(10500, 34500)$	$= 8(10500) + 7(34500) = 325500$
$R(5000, 40000)$	$= 8(5000) + 7(40000) = 320000$
$B_2(0, 40000)$	$= 8(0) + 7(40000) = 280000$

maximum  $Z = 325500$  at  $x = 10500, y = 34500$

Number bottles  $A$  type = 10500,  $B$  type = 34500

maximum profit = Rs 325500

## Linear Programming Ex 30.4 Q22

Let required number of first class and economy class tickets be  $x$  and  $y$  respectively.

Each ticket of first class and economy class make profit of Rs 400 and Rs 600 respectively. So,  $x$  ticket of first class and  $y$  tickets of economy class make profits of Rs  $400x$  and Rs  $600y$  respectively, Let total profit be  $Z$ , so,

$$Z = 400x + 600y$$

Given, aeroplane can carry a maximum of 200 passengers, so,

$$\Rightarrow x + y \leq 200 \quad (\text{first constraint})$$

Given, airline reserves at least 20 seats for first class, so,

$$\Rightarrow x \geq 20 \quad (\text{second constraint})$$

Given, at least 4 times as many passengers prefer to travel by economy class to the first class, so,

$$y \geq 4x$$

$$\Rightarrow 4x - y \leq 0 \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which maximize  $Z = 400x + 600y$

Subject to constraints,

$$x + y \leq 200$$

$$x \geq 20$$

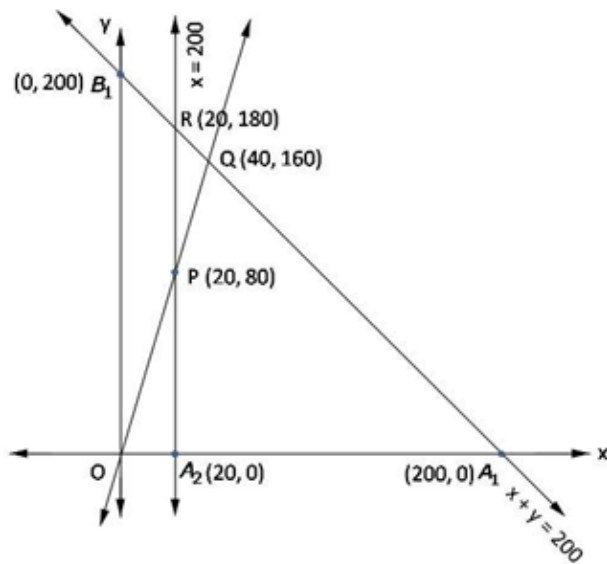
$$4x - y \leq 0$$

$$x, y \geq 0$$

[Since seats of both the classes can not be less than zero]

Region  $x + y \leq 200$ : line  $x + y = 200$  meets axes at  $A_1 (200, 0)$ ,  $B_1 (0, 200)$  respectively.

Region containing origin represents  $x + y \leq 200$  as  $(0, 0)$  satisfies  $x + y \leq 200$ .



Shaded region  $PQR$  represents feasible region.  $Q(40, 160)$  is obtained by solving  $x + y = 200$  and  $4x - y = 0$ ,  $R(20, 180)$  is obtained by solving  $x = 20$  and  $x + y = 200$

The value of  $Z = 400x + 600y$  at

$$P(20, 80) = 400(20) + 600(80) = 56000$$

$$Q(40, 160) = 400(40) + 600(160) = 112000$$

$$R(20, 180) = 400(20) + 600(180) = 116000$$

so,

maximum  $Z = \text{Rs } 116000$  at  $x = 20, y = 180$

Number of first class ticket = 20,

Number of economy class ticket = 180

maximum profit = Rs 116000



	Type I	Type II	
	x	y	
Nitrogen	0.1x	0.05y	$\geq 14$
Bicarbonate	0.06x	0.1y	$\geq 14$
Cost	0.6x	0.4y	Minimize

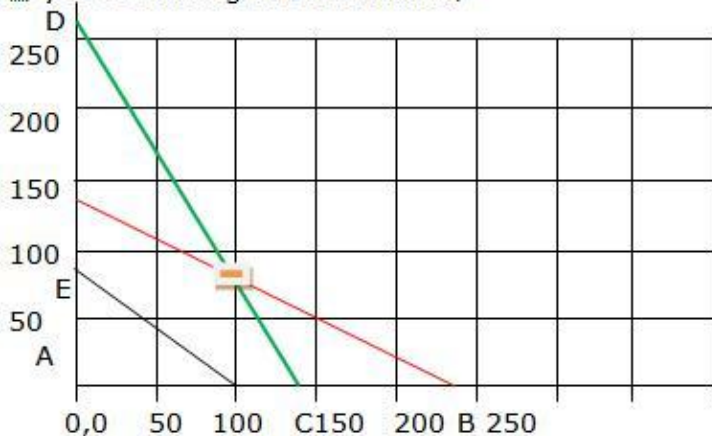
The above LPP can be presented in a table above.

Aim is to find the values of x & y that minimize the function  $Z = 0.6x + 0.4y$ , subject to the conditions

$0.1x + 0.05y \geq 14$ ; gives  $x=0, y=280$  &  $y=0, x=140$

$0.06x + 0.1y \geq 14$ ; gives  $x=0, y=140$  &  $y=0, x=233.33$

$x, y \geq 0$ . Plotting the constraints,



The feasible region is the unbounded region D-C-B

Corner point	Value of $Z = 0.6x + 0.4y$
0, 280	112
233.33, 0	140
100, 80	92

The minimum occurs at  $x=100, y=80$  with a value of 92

Since the region is unbounded plot  $0.6x + 0.4y \leq 92$

Plotting the points, we get line E-100.

There are no common points so  $x=100, y=80$  with a value of 92 is the optimal minimum.

## Linear Programming Ex 30.4 Q24

Let he invests Rs  $x$  and Rs  $y$  in saving certificate (sc) and National saving bond (NSB) respectively.

Since, rate of interest on SC is 8% annual and on NSB is 10% annual, So, interest on Rs  $x$  of SC is  $\frac{8x}{100}$  and Rs  $y$  of NSB is  $\frac{10x}{100}$  per annum.

Let  $Z$  be total interest earned so,

$$Z = \frac{8x}{100} + \frac{10y}{100}$$

Given he wants to invest Rs 12000 is total

$$x + y \leq 12000 \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = \frac{8x}{100} + \frac{10y}{100}$$

Subject to constraints,

$$x \geq 2000$$

$$y \geq 4000$$

$$x + y \leq 12000$$

$$x, y \geq 0$$

[Since investment can not be less than zero]

Region  $x \geq 2000$ : line  $x = 2000$  is parallel to  $y$ -axis and meets  $x$ -axis at  $A_1(2000, 0)$ .

Region not containing origin represents  $x \geq 2000$  as  $(0, 0)$  does not satisfy  $x \geq 2000$

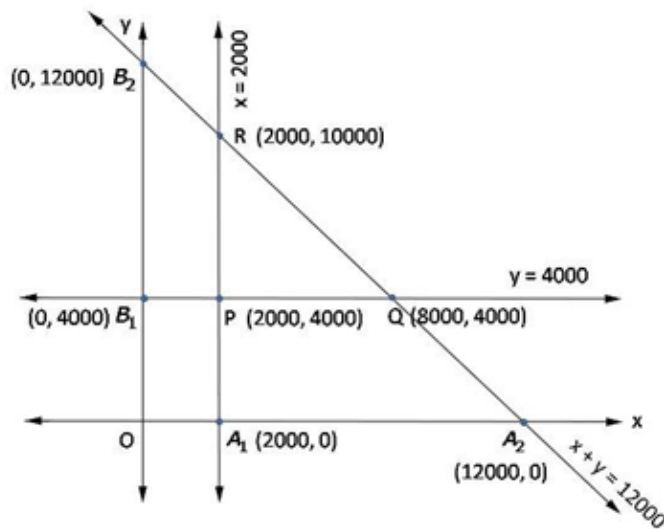
Region  $y \geq 4000$ : line  $y = 4000$  is parallel to  $x$ -axis and meets  $y$ -axis at  $B_1(0, 4000)$ . Region

not containing origin represents  $y \geq 4000$  as  $(0, 0)$  does not satisfy  $y \geq 4000$ .

Region  $x + y \leq 12000$ : line  $x + y = 12000$  meets axes at  $A_2(12000, 0)$ ,  $b_2(0, 1200)$  respectively.

Region containing represents  $x + y \leq 12000$  as  $(0, 0)$  satisfies  $x + y \leq 12000$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $PQR$  represents feasible region.  $P(2000, 4000)$  is obtained by solving  $x = 2000$  and  $y = 4000$ ,  $Q(8000, 4000)$  is obtained by solving  $x + y = 12000$  and  $y = 4000$   $R(2000, 10000)$  is obtained by solving  $x = 2000$  and  $y + x = 12000$

The value of  $Z = \frac{8x}{100} + \frac{10y}{100}$  at

$$P(2000, 4000) = \frac{8}{100}(2000) + \frac{10}{100}(4000) = 560$$

$$Q(8000, 4000) = \frac{8}{100}(8000) + \frac{10}{100}(4000) = 1040$$

$$R(2000, 10000) = \frac{8}{1000}(2000) + \frac{10}{100}(10000) = 1160$$

so,

maximum  $Z = \text{Rs } 1160$  at  $x = 2000, y = 10000$

He should invest Rs 2000 in Saving Certificates and 1000 in National Saving scheme, maximum Interest = Rs 1160

Let required number of trees of type  $A$  and  $B$  be Rs  $x$  and Rs  $y$  respectively.

Since, selling price of 1 kg of type  $A$  is Rs 2 and growth is 20 kg per tree, so, revenue from type  $A$  is Rs  $40x$ , selling price of 1 kg of type  $B$  is Rs 1.5 and growth 40 kg per tree, so, revenue from type  $B$  is Rs  $60y$ . Total revenue is  $(40x + 60y)$ . Costs of each tree of type  $A$  and  $B$  are Rs 20 and Rs 25, so, costs of  $x$  trees of type  $A$  and  $y$  trees of type  $B$  are Rs  $20x$  and  $25y$  respectively. Total cost is Rs  $(20x + 25y)$

Let  $Z$  be total profit so,

$$Z = (40x + 60y) - (20x + 25y)$$

$$Z = 20x + 35y$$

Since he has Rs 1400 to invest so,

$$\text{cost} \leq 1400$$

$$\Rightarrow 20x + 35y \leq 1400$$

$$\Rightarrow 4x + 5y \leq 280 \quad (\text{first constraint})$$

Since each tree of type  $A$  and  $B$  needs 10 sq. m and 20 sq. m of ground respectively so,  $x$  trees of type  $A$  and  $y$  trees of type  $B$  need  $10x$  sq. m and  $20y$  sq. m of ground respectively. but total ground available is 1000 sq. m so,

$$10x + 20y \leq 1000$$

$$\Rightarrow x + 2y \leq 100 \quad (\text{second constraint})$$

$$x, y \geq 0$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 20x + 35y$$

Subject to constraints,

$$4x + 5y \leq 280$$

$$\Rightarrow x + 2y \leq 100$$

$$x, y \geq 0$$

[Since number of trees can not be less than zero]

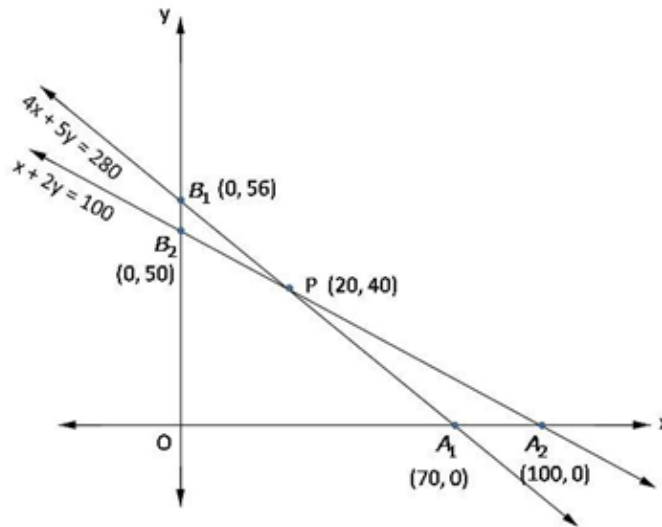
Region  $4x + 5y \leq 280$ : line  $4x + 5y = 280$  meets axes at  $A_1 (70,0)$ ,  $B_1 (0,56)$  respectively.

Region containing origin represents  $4x + 5y \leq 280$  as  $(0,0)$  satisfies  $4x + 5y \leq 280$ .

Region  $x + 2y \leq 100$ : line  $x + 2y = 100$  meets axes at  $A_2 (100,0)$ ,  $B_2 (0,50)$  respectively.

Region containing origin represents  $x + 2y \leq 100$  as  $(0,0)$  satisfies  $x + 2y \leq 100$ .

Region  $x, y \geq 0$ : it represents first quadrant.



Shaded region  $OA_1PB_2$  the feasible region.  $P (20, 40)$  is obtained

by solving  $x + 2y = 100$  and  $4x + 5y = 280$ ,

The value of  $Z = 20x + 35y$  at

$$O (0, 0) = 20(0) + 35(0) = 0$$

$$A_1 (70, 0) = 20(70) + 35(0) = 1400$$

$$P (20, 40) = 20(20) + 35(40) = 1800$$

$$B_2 (0, 50) = 20(0) + 35(50) = 1750$$

maximum  $Z = 1800$  at  $x = 20, y = 40$

20 trees of type A, 40 trees of type B, profit = Rs 1800

Let the cottage industry manufacture  $x$  pedestal lamps and  $y$  wooden shades.  
Therefore,

$$x \geq 0 \text{ and } y \geq 0$$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$$2x + y \leq 12$$

$$3x + 2y \leq 20$$

$$\text{Total profit, } Z = 5x + 3y$$

The mathematical formulation of the given problem is

$$\text{Maximize } Z = 5x + 3y \dots (1)$$

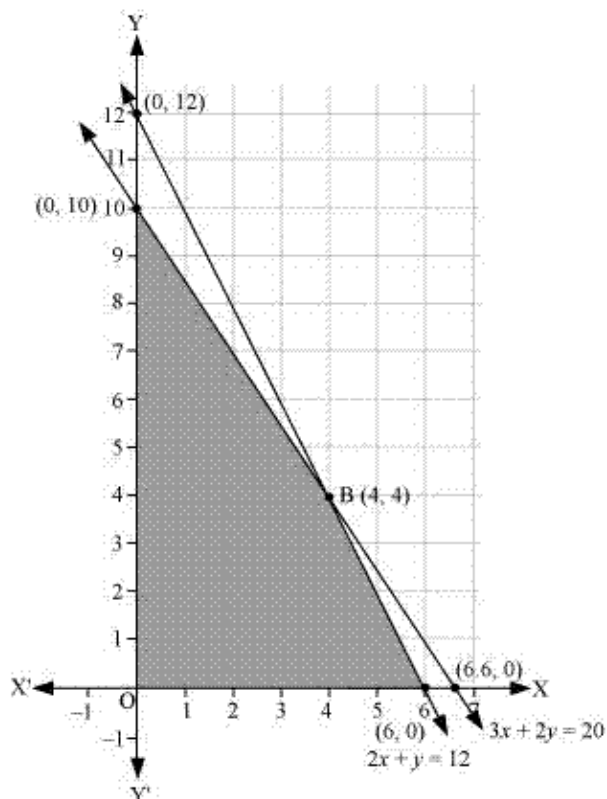
subject to the constraints,

$$2x + y \leq 12 \dots (2)$$

$$3x + 2y \leq 20 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of  $Z$  at these corner points are as follows

Corner point	$Z = 5x + 3y$	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of  $Z$  is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

### Linear Programming Ex 30.4 Q27

Let required number of goods of type  $x$  and  $y$  be  $x_1$  and  $x_2$  respectively.

Since, selling prices of each goods of type  $x$  and  $y$  are Rs 100 and Rs 120 respectively, so, selling price of  $x_1$  units of goods of type  $x$  and  $x_2$  units of goods of type  $y$  are Rs  $100x_1$  and Rs  $120x_2$  respectively respectively

Let  $Z$  be total revenue, so

$$Z = 100x_1 + 120x_2.$$

Since each unit of goods  $x$  and  $y$  require 2 and 3 units of labour, so,  $x_1$  unit of  $x$  and  $x_2$  unit of  $y$  require  $2x_1$  and  $3x_2$  units of labour units but maximum labour units available is 30 units, so,

$$2x_1 + 3x_2 \leq 30 \quad (\text{first constraint})$$

Since each unit of goods  $x$  and  $y$  require 3 and 1 unit of capital so,  $x_1$  unit of  $x$  and  $x_2$  unit of  $y$  require  $3x_1$  and  $x_2$  units of capital respectively but maximum units available for capital is 17, so,

$$3x_1 + x_2 \leq 17 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which maximize  $Z = 20x + 35y$

Subject to constraints,

$$2x_1 + 3x_2 \leq 30$$

$$\Rightarrow 3x_1 + x_2 \leq 17$$

$$x_1, x_2 \geq 0$$

[Since production of goods can not be less than zero]

Region  $2x_1 + 3x_2 \leq 30$ : line  $2x_1 + 3x_2 = 30$  meets axes at  $A_1(15,0)$ ,  $B_1(0,10)$  respectively.

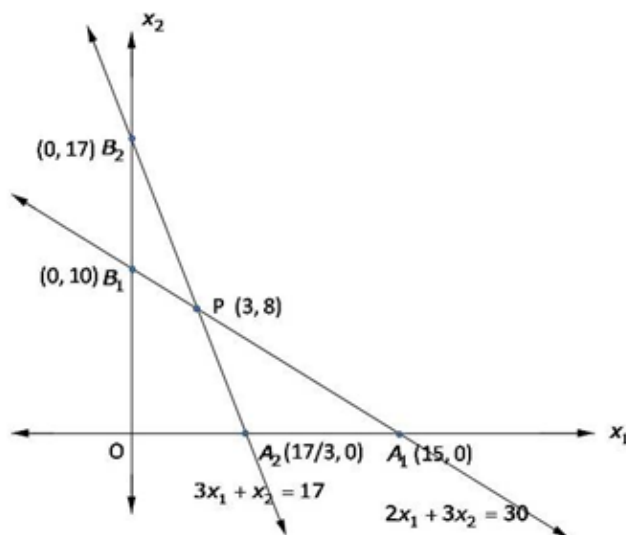
Region containing origin represents  $2x_1 + 3x_2 \leq 30$  as  $(0,0)$  satisfies  $2x_1 + 3x_2 = 30$ .

Region  $3x_1 + x_2 \leq 17$ : line  $3x_1 + x_2 = 17$  meets axes at  $A_2\left(\frac{17}{3},0\right)$ ,  $B_2(0,17)$  respectively.

Region containing origin represents  $3x_1 + x_2 \leq 17$  as  $(0,0)$  satisfies  $3x_1 + x_2 \leq 17$ .

Region  $x_1, x_2 \geq 0$ : it represent first quadrant shaded region  $OA_2PB_1$  represents feasible region. Point  $P(3,8)$  is obtained by solving

$$2x_1 + 3x_2 = 30 \text{ and } 3x_1 + x_2 = 17$$



The value of  $Z = 100x_1 + 120x_2$  at

$$O(0,0) = 100(0) + 120(0) = 0$$

$$A_2\left(\frac{17}{3},0\right) = 100\left(\frac{17}{3}\right) + 120(0) = \frac{1700}{3} = 566\frac{2}{3}$$

$$P(3,8) = 100(3) + 120(8) = 1260$$

$$B_1(0,10) = 100(0) + 120(10) = 1200$$

maximum  $Z = 1260$  at  $x = 3, y = 8$

goods of type  $x = 3$ , type  $y = 8$

maximum profit = Rs 12160



Let required number of product  $A$  and  $B$  be  $x$  and  $y$  respectively.

Since, profit on each product  $A$  and  $B$  are Rs 5 and Rs 3 respectively, so, profits on  $x$  product  $A$  and  $y$  product  $B$  are Rs  $5x$  and Rs  $3y$  respectively

Let  $Z$  be total profit so

$$Z = 5x + 3y$$

Since each unit of product  $A$  and  $B$  require one min. each on machine  $M_1$ , so,  $x$  unit of product  $A$  and  $y$  units of product  $B$  require  $x$  and  $y$  min. respectively on machine  $M_1$  but  $M_1$  can work at most  $5 \times 60 = 300$  min., so

$$x + y \leq 300 \quad (\text{first constraint})$$

Since each unit of product  $A$  and  $B$  require 2 and one min. respectively on machine  $M_2$ , so,  $x$  unit of product  $A$  and  $y$  units of product  $B$  require  $2x$  and  $y$  min. respectively on machine  $M_2$  but  $M_2$  can work at most  $6 \times 60 = 360$  min., so

$$2x + y \leq 360 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which maximize  $Z = 5x + 3y$

Subject to constraints,

$$x + y \leq 300$$

$$\Rightarrow 2x + y \leq 360$$

$$x, y \geq 0$$

[Since production can not be less than zero]

Region  $x + y \leq 300$ : line  $x + y = 300$  meets axes at  $A_1(300,0)$ ,  $B_1(0,300)$  respectively.

Region containing origin represents  $x + y \leq 300$  as  $(0,0)$  satisfies  $x + y = 300$ .

Region  $2x + y \leq 360$ : line  $2x + y = 360$  meets axes at  $A_2(180,0)$ ,  $B_2(0,360)$  respectively.

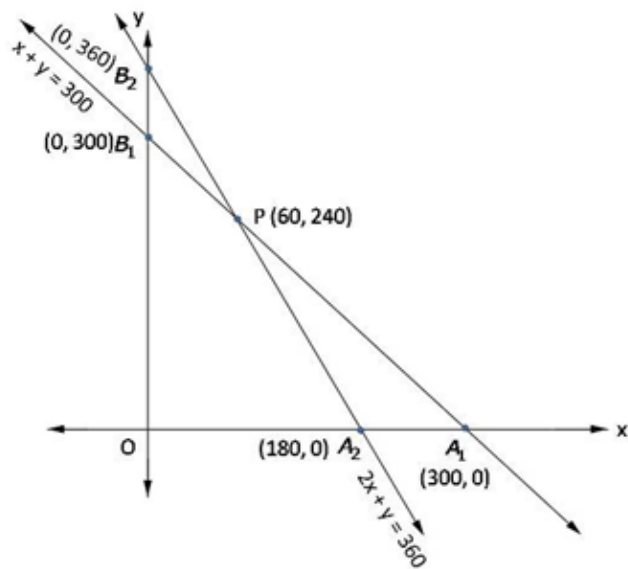
Region containing origin represents  $2x + y \leq 360$  as  $(0,0)$  satisfies  $2x + y \leq 360$ .

Region  $x, y \geq 0$ : it represent first quadrant

Shaded region  $OA_2PB_1$  represents feasible region.

Point  $P(60,240)$  is obtained by solving

$$x + y = 300 \text{ and } 2x + y = 360$$



The value of  $Z = 5x + 3y$  at

$$O(0, 0) = 5(0) + 3(0) = 0$$

$$A_2(180, 0) = 5(180) + 3(0) = 900$$

$$P(60, 240) = 5(60) + 3(240) = 1020$$

$$B_1(0, 300) = 5(0) + 3(300) = 900$$

maximum  $Z = 1020$  at  $x = 60, y = 240$

Number of product  $A = 60$ , product  $B = 240$

maximum profit = Rs 1020

Let required quantity of item  $A$  and  $B$  produced be  $x$  and  $y$  respectively.

Since, profits on each item  $A$  and  $B$  are Rs 300 and Rs 160 respectively, so, profits on  $x$  unit of item  $A$  and  $y$  units of item  $B$  are Rs  $300x$  and Rs  $160y$  respectively

Let  $Z$  be total profit so

$$Z = 300x + 160y$$

Since one unit of item  $A$  and  $B$  require one and  $\frac{1}{2}$  hr respectively, so,  $x$  units of item  $A$  and  $y$  units of item  $B$  require  $x$  and  $\frac{1}{2}y$  hr. respectively but maximum time available is 16 hours.,so

$$x + \frac{1}{2}y \leq 16$$

$$\Rightarrow 2x + y \leq 32 \quad (\text{first constraint})$$

Given, manufacturer can produce at most 24 items, so,

$$\Rightarrow x + y \leq 24 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which maximize  $Z = 300x + 160y$

Subject to constraints,

$$2x + y \leq 32$$

$$x + y \leq 24$$

$$x, y \geq 0$$

[Since production can not be less than zero]

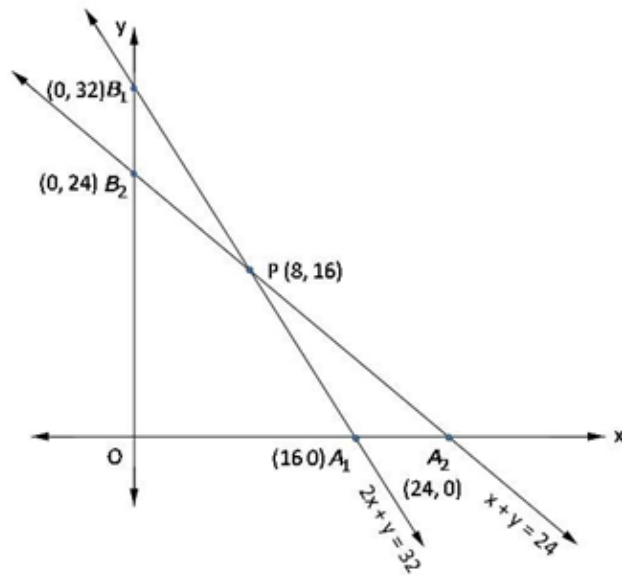
Region  $2x + y \leq 32$ : line  $2x + y = 32$  meets axes at  $A_1(16,0)$ ,  $B_1(0,32)$  respectively.

Region containing origin represents  $2x + y \leq 32$  as  $(0,0)$  satisfies  $2x + y \leq 32$ .

Region  $x + y \leq 24$ : line  $x + y = 24$  meets axes at  $A_2(24,0)$ ,  $B_2(0,24)$  respectively.

Region containing origin represents  $x + y \leq 24$  as  $(0,0)$  satisfies  $x + y \leq 24$ .

Region  $x, y \geq 0$ : it represent first quadrant



Shaded region  $OA_1PB_2$  represents feasible region.

Point  $P$  is obtained by solving

$$x + y = 24 \text{ and } 2x + y = 32$$

The value of  $Z = 300x + 160y$  at

$$O(0,0) = 300(0) + 160(0) = 0$$

$$A_1(16,0) = 300(16) + 160(0) = 4800$$

$$P(8,16) = 300(8) + 160(16) = 4960$$

$$B_2(0,24) = 300(0) + 160(24) = 3840$$

maximum  $Z = 4960$

Number of item  $A = 8$ , item  $B = 16$

maximum profit = Rs 4960

Let number of toys of type  $A$  and  $B$  produced are  $x$  and  $y$  respectively.

Since, profits on each unit of toys  $A$  and  $B$  are Rs 50 and Rs 60 respectively, so, profits on  $x$  units of toys  $A$  and  $y$  units of toy  $B$  are Rs  $50x$  and Rs  $60y$  respectively  
Let  $Z$  be total profit so

$$Z = 50x + 60y$$

Since each unit of toy  $A$  and toy  $B$  require 5 min. and 8 min. on cutting, so,  $x$  units of toy  $A$  and  $y$  units of toy  $B$  require  $5x$  and  $8y$  min. respectively but maximum time available for cutting  $3 \times 60 = 180$  min., so

$$5x + 8y \leq 180 \quad (\text{first constraint})$$

Since each unit of toy  $A$  and toy  $B$  require 10 min. and 8 min. for assembling, so,  $x$  units of toy  $A$  and  $y$  units of toy  $B$  require  $10x$  and  $8y$  min. for assembling respectively but maximum time available for assembling is  $4 \times 60 = 240$  min., so

$$10x + 8y \leq 240$$

$$\Rightarrow 5x + 4y \leq 120 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which maximize  $Z = 50x + 60y$

Subject to constraints,

$$5x + 8y \leq 180$$

$$5x + 4y \leq 120$$

$$x, y \geq 0$$

[Since production can not be less than zero]

Region  $5x + 8y \leq 180$ : line  $5x + 8y = 180$  meets axes at  $A_1(36,0)$ ,  $B_1\left(0, \frac{45}{2}\right)$  respectively.

Region containing origin represents  $5x + 8y \leq 180$  as  $(0,0)$  satisfies  $5x + 8y \leq 180$ .

Region  $5x + 4y \leq 120$ : line  $5x + 4y = 120$  meets axes at  $A_2(24,0)$ ,  $B_2(0,30)$  respectively.

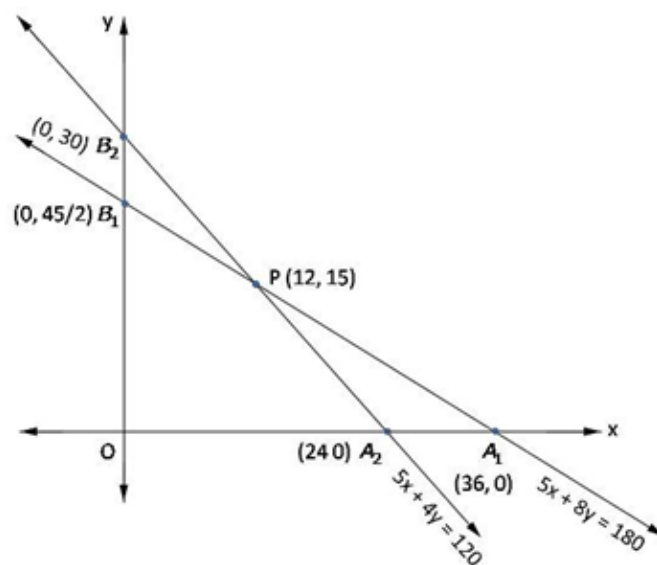
Region containing origin represents  $5x + 4y \leq 120$  as  $(0,0)$  satisfies  $5x + 4y \leq 120$ .

Region  $x, y \geq 0$ : it represent first quadrant

Shaded region  $OA_2PB_1$  represents feasible region.

Point  $P(12, 15)$  is obtained by solving

$$5x + 8y = 180 \text{ and } 5x + 4y = 120$$



The value of  $Z = 50x + 60y$  at

$$O(0, 0) = 50(0) + 60(0) = 0$$

$$A_2(24, 0) = 50(24) + 60(0) = 1200$$

$$P(12, 15) = 50(12) + 60(15) = 1500$$

$$B_1\left(0, \frac{45}{2}\right) = 50(0) + 60\left(\frac{45}{2}\right) = 1350$$

Maximum  $Z = 1500$  at  $x = 12$ ,  $y = 15$

Number of toys  $A = 12$ , toys  $B = 15$

maximum profit = Rs 1500

Let required number of product  $A$  and  $B$  are  $x$  and  $y$  respectively.

Since, profits on each unit of product  $A$  and product  $B$  are Rs 6 and Rs 8 respectively, so, profits on  $x$  units of product  $A$  and  $y$  units of product  $B$  are Rs  $6x$  and Rs  $8y$  respectively

Let  $Z$  be total profit so

$$Z = 6x + 8y$$

Since each unit of product  $A$  and  $B$  require 4 and 2 hrs for assembling respectively, so,  $x$  units of product  $A$  and  $y$  units of product  $B$  require  $4x$  and  $2y$  hrs for assembling respectively but maximum time available for assembling is 60 hrs.,so

$$4x + 2y \leq 60$$

$$2x + y \leq 30 \quad \text{(first constraint)}$$

Since each unit of product  $A$  and  $B$  require 2 and 4 hrs for finishing, so,  $x$  units of product  $A$  and  $y$  units of product  $B$  require  $2x$  and  $4y$  hrs for finishing respectively but maximum time available for finishing is 48 hrs.,so

$$2x + 4y \leq 48$$

$$x + 2y \leq 24 \quad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 6x + 8y$$

Subject to constraints,

$$2x + y \leq 30$$

$$x + 2y \leq 24$$

$$x, y \geq 0$$

[Since production of both can not be less than zero]

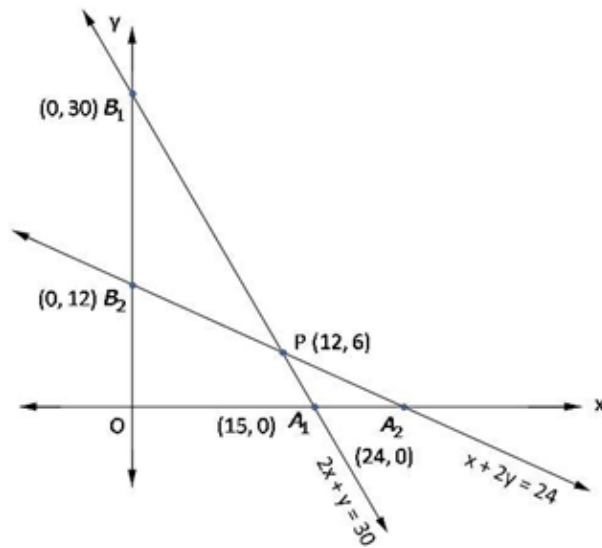
Region  $2x + y \leq 30$ : line  $2x + y = 24$  meets axes at  $A_1(15,0)$ ,  $B_1(0,30)$  respectively.

Region containing origin represents  $2x + y \leq 30$  as  $(0,0)$  satisfies  $2x + y \leq 30$ .

Region  $x + 2y \leq 24$ : line  $x + 2y = 24$  meets axes at  $A_2(24,0)$ ,  $B_2(0,12)$  respectively.

Region containing origin represents  $x + 2y \leq 24$  as  $(0,0)$  satisfies  $x + 2y \leq 24$ .

Region  $x, y \geq 0$ : it represent first quadrant



Shaded region  $OA_1PB_2$  represents feasible region.

Point  $P(12, 6)$  is obtained by solving

$$x + 2y = 24 \text{ and } x + 2y = 30$$

The value of  $Z = 6x + 8y$  at

$$O(0, 0) = 6(0) + 8(0) = 0$$

$$A_1(15, 0) = 6(15) + 8(0) = 90$$

$$P(12, 6) = 6(12) + 8(6) = 120$$

$$B_2(0, 12) = 6(0) + 8(12) = 96$$

maximum  $Z = 120$  at  $x = 12$ ,  $y = 6$

Number of product  $A = 12$ , product  $B = 6$

maximum profit = Rs 120



Let  $x$  &  $y$  be the No. of items of A & B respectively.

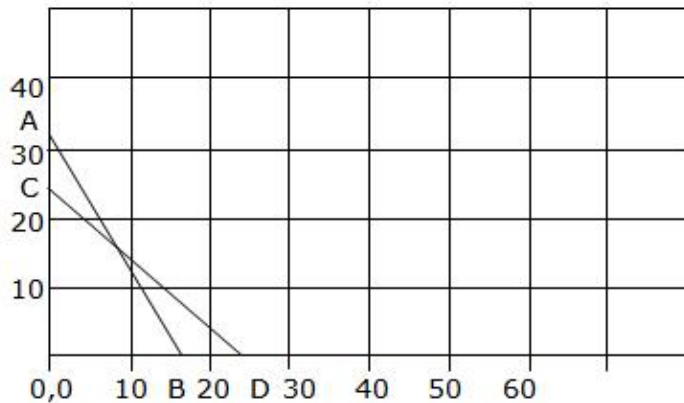
$$x + y = 24 \quad (\text{total No. of items constraint})$$

$$x + 0.5y \leq 16 \quad (\text{time constraint})$$

$$x, y \geq 0$$

$$Z = 300x + 160y \quad (\text{profit function to be maximized})$$

Plotting the inequalities gives,



The feasible region is 0,0-C-F-B

Corner point	Value of $Z = 300x + 160y$
0, 0	0
0, 24	3840
16, 0	4800
8, 16	4960

The firm must produce 8 items of A and 16 items of B to maximize the profit at Rs. 4960/-

### Linear Programming Ex 30.4 Q33

Let required number of product A and B are  $x$  and  $y$  respectively.

Since, profits on each unit of product A and product B are Rs 20 and Rs 15 respectively, so,  $x$  units of product A and  $y$  units of product B give profit of Rs  $20x$  and Rs  $15y$  respectively

Let  $Z$  be total profit so

$$Z = 20x + 15y$$

Since each unit of product A and B require 5 and 3 man-hrs respectively, so,  $x$  units of product A and  $y$  units of product B require  $5x$  and  $3y$  man-hrs respectively but maximum time available for is 500 man-hrs., so

$$5x + 3y \leq 500 \quad (\text{first constraint})$$

Since maximum number that product A and B can be sold is 70 and 125 respectively, so,

$$x \leq 70 \quad (\text{second constraint})$$

$$y \leq 125 \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 20x + 15y$$

Subject to constraints,

$$5x + 3y \leq 500$$

$$x \leq 70$$

$$y \leq 125$$

$$x, y \geq 0$$

[Since production of both can not be less than zero]

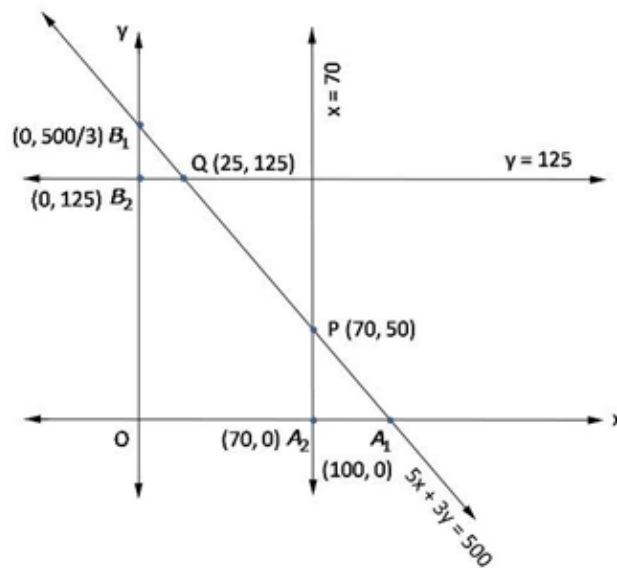
Region  $5x + 3y \leq 500$ : line  $5x + 3y = 500$  meets axes at  $A_1(100,0)$ ,  $B_1\left(0, \frac{500}{3}\right)$  respectively.

Region containing origin represents  $5x + 3y \leq 500$  as  $(0,0)$  satisfies  $5x + 3y \leq 500$ .

Region  $x \leq 70$ : line  $x = 70$  is parallel to  $y$ -axis meets  $x$ -axes at  $A_2(70,0)$ . Region containing origin represents  $x \leq 70$  as  $(0,0)$  satisfies  $x \leq 70$ .

Region  $y \leq 125$ : line  $y = 125$  is parallel to  $x$ -axis meets  $y$ -axes at  $B_2(0,125)$ . with  $y$ -axis. Region containing origin represents  $y \leq 125$  as  $(0,0)$  satisfies  $y \leq 125$ .

Region  $x, y \geq 0$ : it represent first quadrant.



Shaded region  $OA_2PQB_2$  represents feasible region.

Point  $P(70,50)$  is obtained by solving  $x = 70$

Point  $Q(25,125)$  is obtained by solving  $y = 125$  and  $5x + 3y = 500$ .

The value of  $Z = 20x + 15y$  at

$$O(0,0) = 20(0) + 15(0) = 0$$

$$A_2(70,0) = 20(70) + 15(0) = 1400$$

$$P(70,50) = 20(70) + 15(50) = 2150$$

$$Q(25,125) = 20(25) + 15(125) = 2375$$

$$B_2(0,125) = 20(0) + 15(125) = 1875$$

maximum  $Z = 2375$  at  $x = 25$ ,  $y = 125$

Number of product  $A = 25$ , product  $B = 125$

maximum profit = Rs 2375

## Linear Programming Ex 30.4 Q34

Let required quantity of large and small boxes are  $x$  and  $y$  respectively.

Since, profits on each unit of large and small boxes are Rs 3 and Rs 2 respectively, so, profit on  $x$  units of large and  $y$  units of small boxes are Rs  $3x$  and Rs  $2y$  respectively

Let  $Z$  be total profit so

$$Z = 3x + 2y$$

Since each large and small box require 4 sq. m. and 3 sq. m. cardboard respectively, so,  $x$  units of large and  $y$  units of small boxes require  $4x$  and  $3y$  sq.m. cardboard respectively but only 60 sq. m. of cardboard is available, so

$$4x + 3y \leq 60 \quad (\text{first constraint})$$

Since manufacturer is required to make at least three large boxes, so,

$$x \geq 3 \quad (\text{second constraint})$$

Since manufacturer is required to make at least twice as many small boxes as large boxes, so,

$$y \geq 2x \quad (\text{third constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 20x + 15y$$

Subject to constraints,

$$4x + 3y \leq 60$$

$$x \geq 3$$

$$y \geq 2x$$

$$x, y \geq 0 \quad [\text{Since production can not be less than zero}]$$

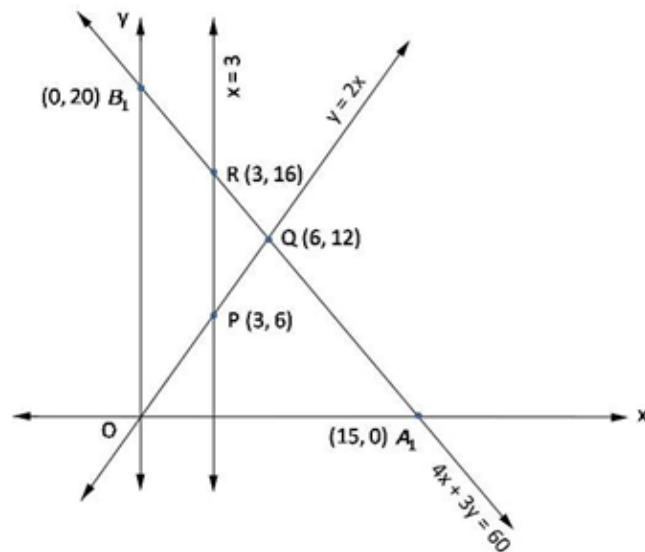
Region  $4x + 3y \leq 60$ : line  $4x + 3y = 60$  meets axes at  $A_1(15,0)$ ,  $B_1(0,20)$  respectively.

Region containing origin represents  $4x + 3y \leq 60$  as  $(0,0)$  satisfies  $4x + 3y \leq 60$ .

Region  $x \geq 3$ : line  $x = 3$  is parallel to  $y$ -axis meets  $x$ -axes at  $A_2(3,0)$ . Region containing origin represents  $x \geq 3$  as  $(0,0)$  satisfies  $x \geq 3$ .

Region  $y \geq 2x$ : line  $y = 2x$  is passes through origin and  $P(3,6)$ . Region containing  $B_1(0,20)$  represents  $y \geq 2x$  as  $(0,20)$  satisfies  $y \geq 2x$ .

Region  $x, y \geq 0$ : it represent first quadrant.



Shaded region  $PQR$  represents feasible region.

Point  $Q(6,12)$  is obtained by solving  $y = 2x$  and  $4x + 3y = 60$

Point  $R(3,16)$  is obtained by solving  $x = 3$  and  $4x + 3y = 60$ .

The value of  $Z = 3x + 2y$  at

$$P(3,6) = 3(3) + 2(6) = 21$$

$$Q(6,12) = 3(6) + 2(12) = 42$$

$$R(3,16) = 3(3) + 2(16) = 41$$

maximum  $Z = 42$  at  $x = 6$ ,  $y = 12$

Number of large box = 6, small box = 12

maximum profit = Rs 42

The given data can be written in the tabular form as follows:

Product	A	B	Working week	Turn over
Time	0.5	1	40	
Prise	200	300		10000
Profit	20	30		
Permanent order	14	16		

Let  $x$  be the number of units of A and  $y$  be the number of units of B produced to earn the maximum profit.

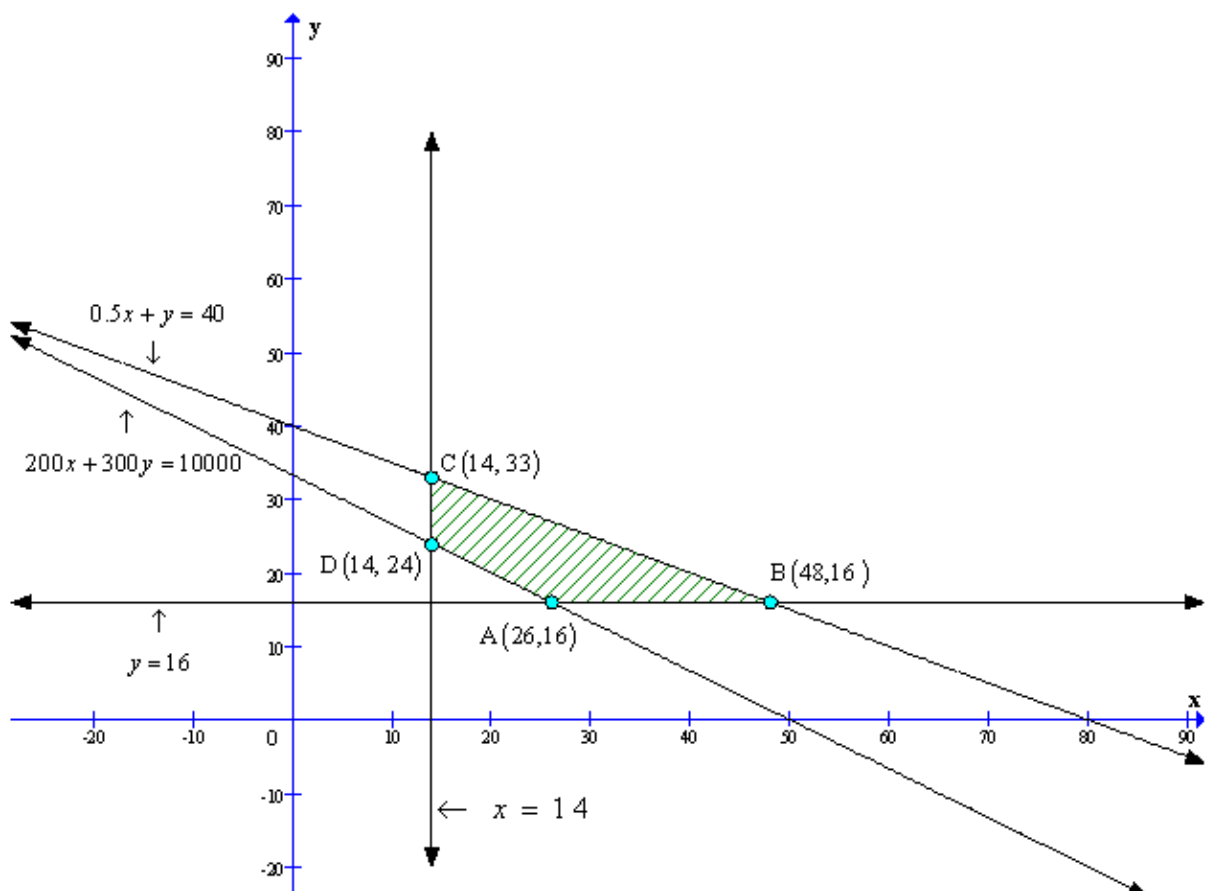
Then the mathematical model of the LPP is as follows:

$$\begin{aligned} \text{Maximize } Z &= 20x + 30y \\ \text{Subject to } 0.5x + y &\leq 40, \\ 200x + 300y &\geq 10000 \end{aligned}$$

$$\text{and } x \geq 14, y \geq 16$$

To solve the LPP we draw the lines,

$$\begin{aligned} 0.5x + y &= 40, \\ 200x + 300y &= 10000 \\ x &= 14 \\ y &= 16 \end{aligned}$$



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(26, 16), B(48, 16), C(14, 33) and D(14, 24).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 20x + 30y$
A(26, 16)	$Z = 1000$
B(48, 16)	$Z = 1440$
C(14, 33)	$Z = 1270$
D(14, 24)	$Z = 600$

48 units of product A and 16 units of product B should be produced to earn the maximum profit of Rs. 1440.

### Linear Programming Ex 30.4 Q36

Let the distance covered with the speed of 25 km/hr be  $x$ .

Let the distance covered with the speed of 40 km/hr be  $y$ .

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = x + y$$

$$\text{Subject to } 2x + 5y \leq 100,$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

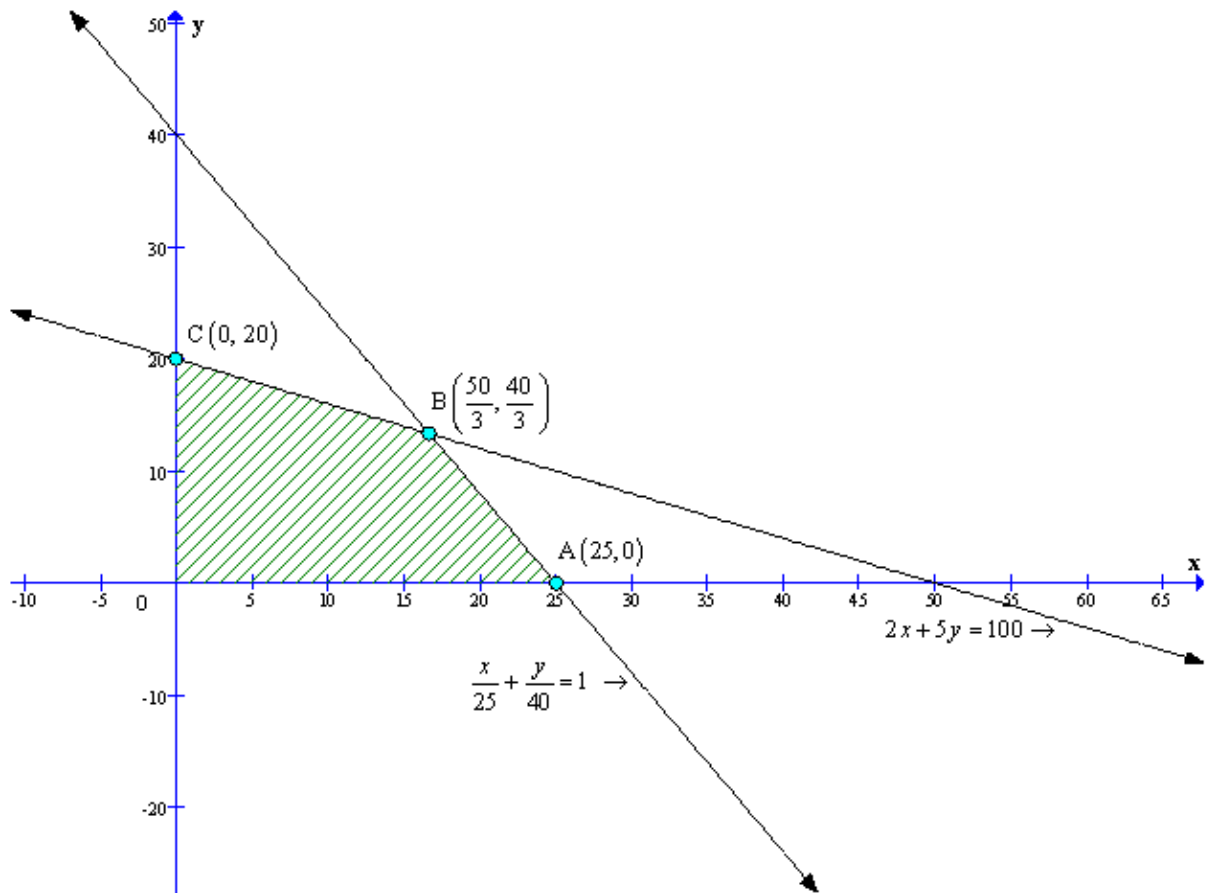
$$\text{and } x \geq 0, y \geq 0$$

To solve the LPP we draw the lines,

$$2x + 5y = 100,$$

$$\frac{x}{25} + \frac{y}{40} = 1$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(25, 0), B( $\frac{50}{3}$ ,  $\frac{40}{3}$ ) and C(0, 20).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = x + y$
A(25, 0)	$Z = 25$
B( $\frac{50}{3}$ , $\frac{40}{3}$ )	$Z = 30$
C(0, 20)	$Z = 20$

The distance covered at the speed of 25km/hr is  $\frac{50}{3}$  km and

The distance covered at the speed of 40km/hr is  $\frac{40}{3}$  km.

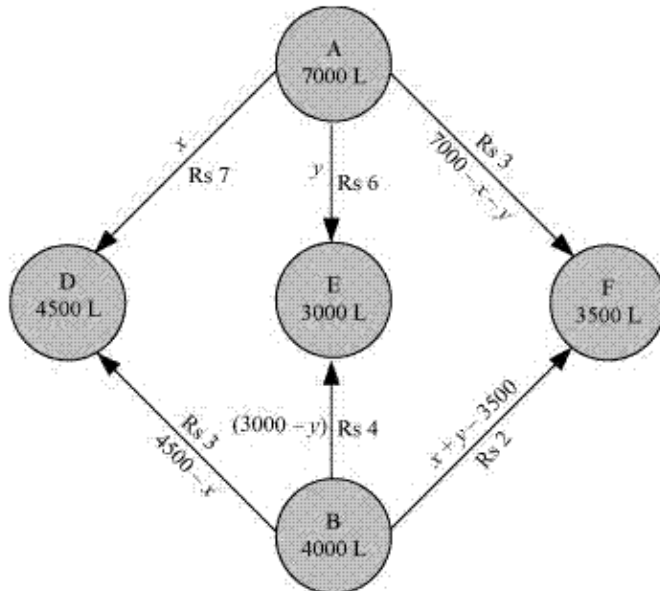
Maximum distance travelled is 30 km.

Let  $x$  and  $y$  litres of oil be supplied from A to the petrol pumps, D and E. Then,  $(7000 - x - y)$  will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500 L. Since  $x$  L are transported from depot A, the remaining  $(4500 - x)$  L will be transported from petrol pump B.

Similarly,  $(3000 - y)$  L and  $3500 - (7000 - x - y) = (x + y - 3500)$  L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows.



$$x \geq 0, y \geq 0, \text{ and } (7000 - x - y) \geq 0$$

$$\Rightarrow x \geq 0, y \geq 0, \text{ and } x + y \leq 7000$$

$$4500 - x \geq 0, 3000 - y \geq 0, \text{ and } x + y - 3500 \geq 0$$

$$\Rightarrow x \leq 4500, y \leq 3000, \text{ and } x + y \geq 3500$$

Cost of transporting 10 L of petrol = Re 1

$$\text{Cost of transporting 1 L of petrol} = \text{Rs } \frac{1}{10}$$

Therefore, total transportation cost is given by,

$$\begin{aligned} z &= \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10} (7000 - x - y) + \frac{3}{10} (4500 - x) + \frac{4}{10} (3000 - y) + \frac{2}{10} (x + y - 3500) \\ &= 0.3x + 0.1y + 3950 \end{aligned}$$

The problem can be formulated as follows.

$$\text{Minimize } z = 0.3x + 0.1y + 3950 \dots (1)$$

subject to the constraints,

$$x + y \leq 7000 \dots (2)$$

$$x \leq 4500 \dots (3)$$

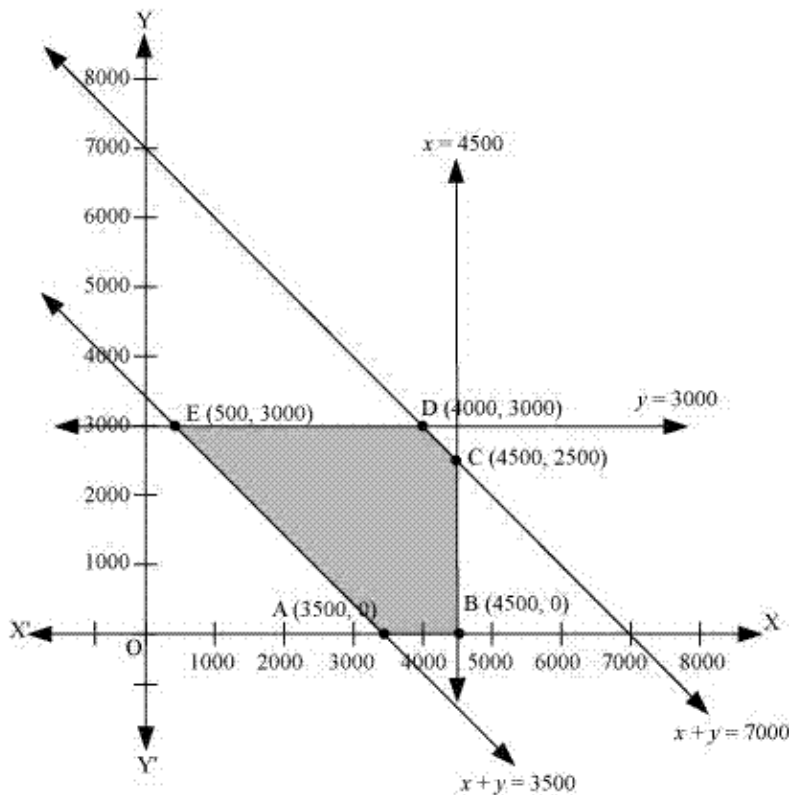
$$y \leq 3000 \dots (4)$$

$$x + y \geq 3500 \dots (5)$$

$$x, y \geq 0 \dots (6)$$

The feasible region determined by the constraints is as follows.





The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000).

The values of  $z$  at these corner points are as follows.

Corner point	$z = 0.3x + 0.1y + 3950$	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	→ Minimum

The minimum value of  $z$  is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Let required number of gold rings and chains are  $x$  and  $y$  respectively.

Since, profits on each ring and chains are Rs 300 and Rs 190 respectively, so, profit on  $x$  units of ring and  $y$  units of chains are Rs  $300x$  and Rs  $190y$  respectively

Let  $Z$  be total profit so

$$Z = 300x + 190y$$

Since each unit of ring and chain require 1 hr and 30 min. to make respectively, so,  $x$  units of rings and  $y$  units of rings require  $60x$  and  $30y$  min. to make respectively, but total time available to make is  $16 \times 60 = 960$ , so

$$60x + 30y \leq 960$$

$$\Rightarrow 2x + y \leq 32 \quad (\text{first constraint})$$

Given, total number of rings and chains manufactured is at most 24, so,

$$x + y \leq 24 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = 300x + 160y$$

Subject to constraints,

$$2x + y \leq 32$$

$$x + y \leq 24$$

$$x, y \geq 0$$

[Since production can not be less than zero]

Region  $2x + y \leq 32$ : line  $2x + y = 32$  meets axes at  $A_1(16,0)$ ,  $B_1(0,32)$  respectively.

Region containing origin represents  $2x + y \leq 32$  as  $(0,0)$  satisfies  $2x + y \leq 32$ .

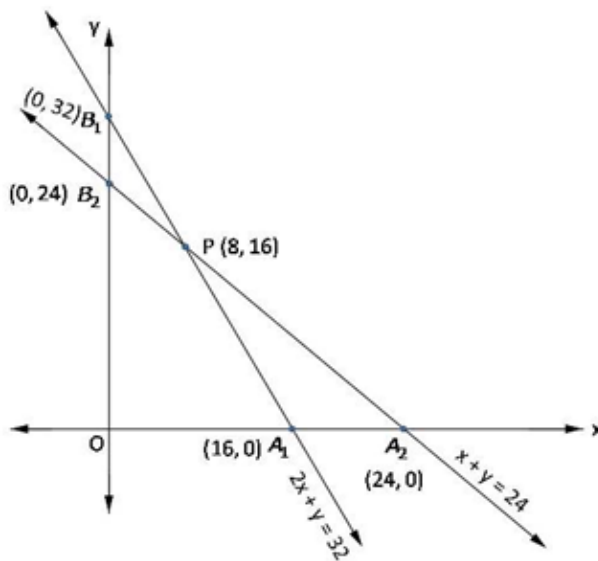
Region  $x + y \leq 24$ : line  $x + y = 24$  meets axes at  $A_2(24,0)$ ,  $B_2(0,24)$  respectively.

Region containing origin represents  $x + y \leq 24$  as  $(0,0)$  satisfies  $x + y \leq 24$ .

Region  $x, y \geq 0$ : it represent first quadrant

Shaded region  $OA_1PB_2$  represents feasible region.

Point  $P(8,16)$  is obtained by solving  $2x + y = 32$  and  $x + y = 24$ .



The value of  $Z = 300x + 160y$  at

$$\begin{aligned}
 O(0,0) &= 300(0) + 160(0) = 0 \\
 A_1(16,0) &= 300(16) + 160(0) = 4800 \\
 P(8,16) &= 300(8) + 160(16) = 4960 \\
 B_2(0,24) &= 300(0) + 160(24) = 3840
 \end{aligned}$$

maximum  $Z = 4960$  at  $x = 8$ ,  $y = 16$

Number of rings = 8, chains = 16

maximum profit = Rs 4960

### Linear Programming Ex 30.4 Q39

Let required number of books of type I and II be  $x$  and  $y$  respectively.

Let  $Z$  be total number of books in the shelf ,so,

$$Z = x + y$$

Since 1 book of type I and II 6 cm and 4 cm. thick respectively, so,  $x$  books of type I and  $y$  books of type II has thickness of  $6x$  and  $4y$  cm. respectivley, but shelf is 96 cm. long ,so

$$6x + 4y \leq 96$$

$$\Rightarrow 3x + 2y \leq 48 \quad (\text{first constraint})$$

Since 1 book of type I and II weight 1 kg and  $1\frac{1}{2}$  kg respectively, so,  $x$  books of type I and  $y$  books of type II weight  $x$  kg and  $\frac{3}{2}y$  kg respectivley, but shelf can support at most 21 kg,so

$$x + \frac{3}{2}y \leq 21$$

$$\Rightarrow 2x + 3y \leq 42 \quad (\text{second constraint})$$

Hence, mathematical formulation of LPP is find  $x$  and  $y$  which

$$\text{maximize } Z = x + y$$

Subject to constriants,

$$3x + 2y \leq 48$$

$$2x + 3y \leq 42$$

$$x, y \geq 0 \quad [\text{Since number of books can not be less than zero}]$$

Region  $3x + 2y \leq 48$ : line  $3x + 2y = 48$  meets axes at  $A_1(16,0)$ ,  $B_1(0,24)$  respectively.

Region containing origin represents  $3x + 2y \leq 48$  as  $(0,0)$  satisfies  $3x + 2y \leq 48$ .

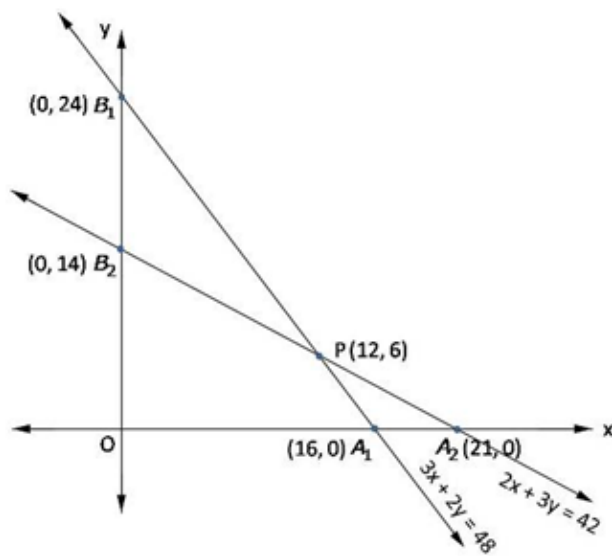
Region  $2x + 3y \leq 42$ : line  $2x + 3y = 42$  meets axes at  $A_2(21,0)$ ,  $B_2(0,14)$  respectively.

Region containing origin represents  $2x + 3y \leq 42$  as  $(0,0)$  satisfies  $2x + 3y \leq 42$ .

Region  $x, y \geq 0$ : it represent first quadrant

Shaded region  $OA_1PB_2$  represents feasible region.

Point  $P(12,6)$  is obtained by solving  $2x + 3y = 42$  and  $3x + 2y = 48$



The value of  $Z = x + y$  at

$O(0, 0)$	$= 0 + 0 = 0$
$A_1(16, 0)$	$= 16 + 0 = 16$
$P(12, 6)$	$= 12 + 6 = 18$
$B_2(0, 14)$	$= 0 + 14 = 14$

maximum  $Z = 18$  at  $x = 12$ ,  $y = 6$

Number of books of type I = 12, type II = 6

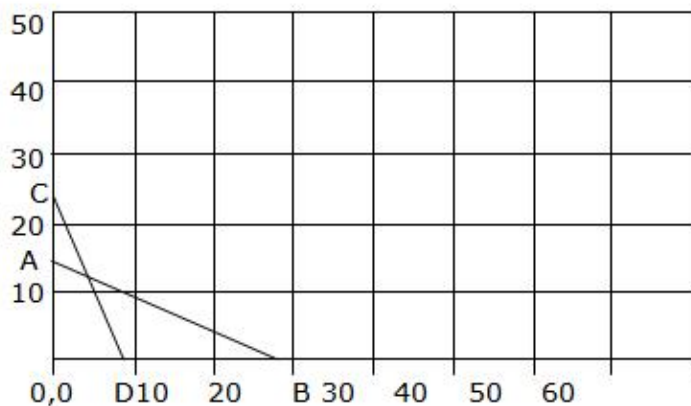
### Linear Programming Ex 30.4 Q40

Let  $x$  &  $y$  be the No. of tennis rackets and cricket bats produced.

- $1.5x + 3y \leq 42$  (constraint on machine time)
- $3x + y \leq 24$  (constraint on craftsman's time)
- $Z = 20x + 10y$  (Maximize profit)
- $x, y \geq 0$

plotting the inequalities we have,

when  $x=0$ ,  $y= 14$  and when  $y=0$ ,  $x=28$  and  
 when  $x=0$ ,  $y= 24$  and when  $y=0$ ,  $x=8$



The feasible region is given by  $O, 0-A-F-D$   
 Tabulating  $Z$  and corner points we have

Corner point	Value of $Z = 20x + 10y$
0, 0	0
0, 14	140
4, 12	200
8, 0	160

The factory must manufacture 4 tennis rackets and 12 cricket bats to earn the maximum profit of Rs. 200/-

### Linear Programming Ex 30.4 Q41

Let  $x$  &  $y$  be the No. of desktop model and portable model of personal computers stocked.

$x + y \leq 250$  (constraint on total demand of computers)

$25000x + 40000y \leq 70,00,000$  (constraint on cost)

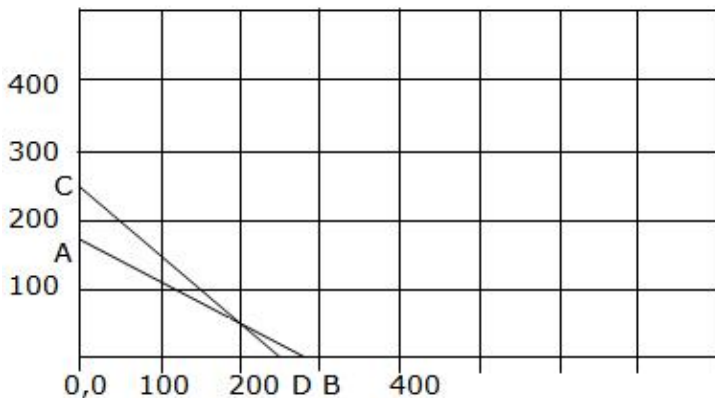
$Z = 4500x + 5000y$  (Maximize profit)

$x, y \geq 0$

plotting the inequalities we have,

when  $x=0$ ,  $y= 250$  and when  $y=0$ ,  $x=250$  and line CD

when  $x=0$ ,  $y= 175$  and when  $y=0$ ,  $x=280$



The feasible region is given by 0,0-A-E-D-0,0

Tabulating  $Z$  and corner points we have

Corner point	Value of $Z = 4500x + 5000y$
0, 0	0
0, 175	8,75,000
250, 0	11,25,000
200, 50	11,50,000

The merchant must stock 200 desktop models and 50 portable models to earn a maximum profit of Rs. 11,50,000/-

### Linear Programming Ex 30.4 Q42

Let  $x$  hectares of land grows crop X.

Let  $y$  hectares of land grows crop Y.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 10,500x + 9,000y$$

$$\text{Subject to } x + y \leq 50,$$

$$20x + 10y \leq 800$$

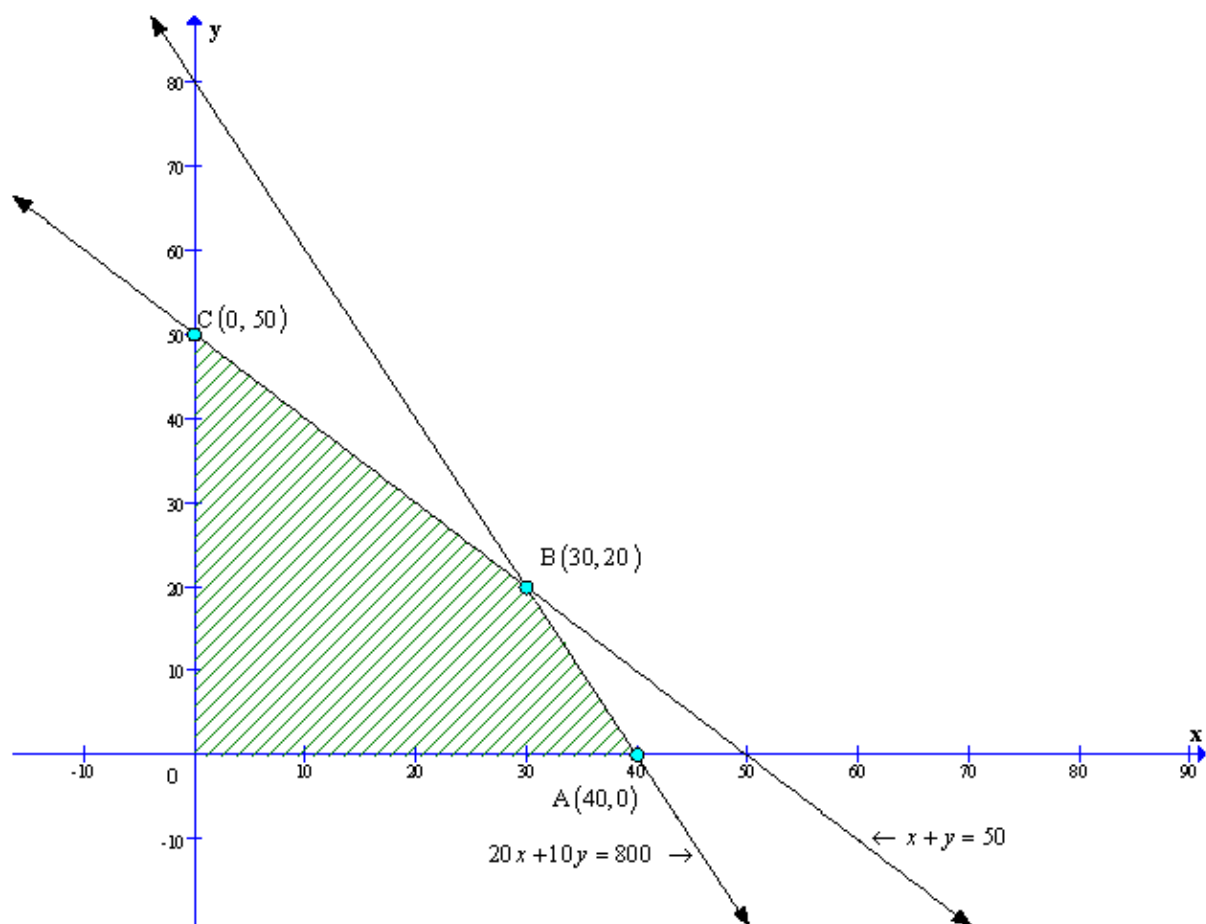
$$\text{and } x \geq 0, y \geq 0$$

To solve the LPP we draw the lines,

$$x + y = 50,$$

$$20x + 10y = 800$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 0), B(30, 20) and C(0, 50).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 10,500x + 9,000y$
A(40, 0)	$Z = 4,20,000$
B(30, 20)	$Z = 4,95,000$
C(0, 50)	$Z = 4,50,000$

30 hectors of land should be allocated to crop X and  
 20 hectors of land should be allocated to crop Y to maximize the profit.  
 The maximum profit that can be eared is Rs. 4,95,000.

### Linear Programming Ex 30.4 Q43

The given data can be written in the tabular form as follows:

Model	A	B	Maximum hours
Fabricating	9	12	180
Finishing	1	3	30
Profit	8000	12000	

Let  $x$  be the number of pieces of A and  $y$  be the number of pieces of B manufactured to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 8000x + 12000y$$

$$\text{Subject to } 9x + 12y \leq 180,$$

$$x + 3y \leq 30$$

$$\text{and } x \geq 0, y \geq 0$$

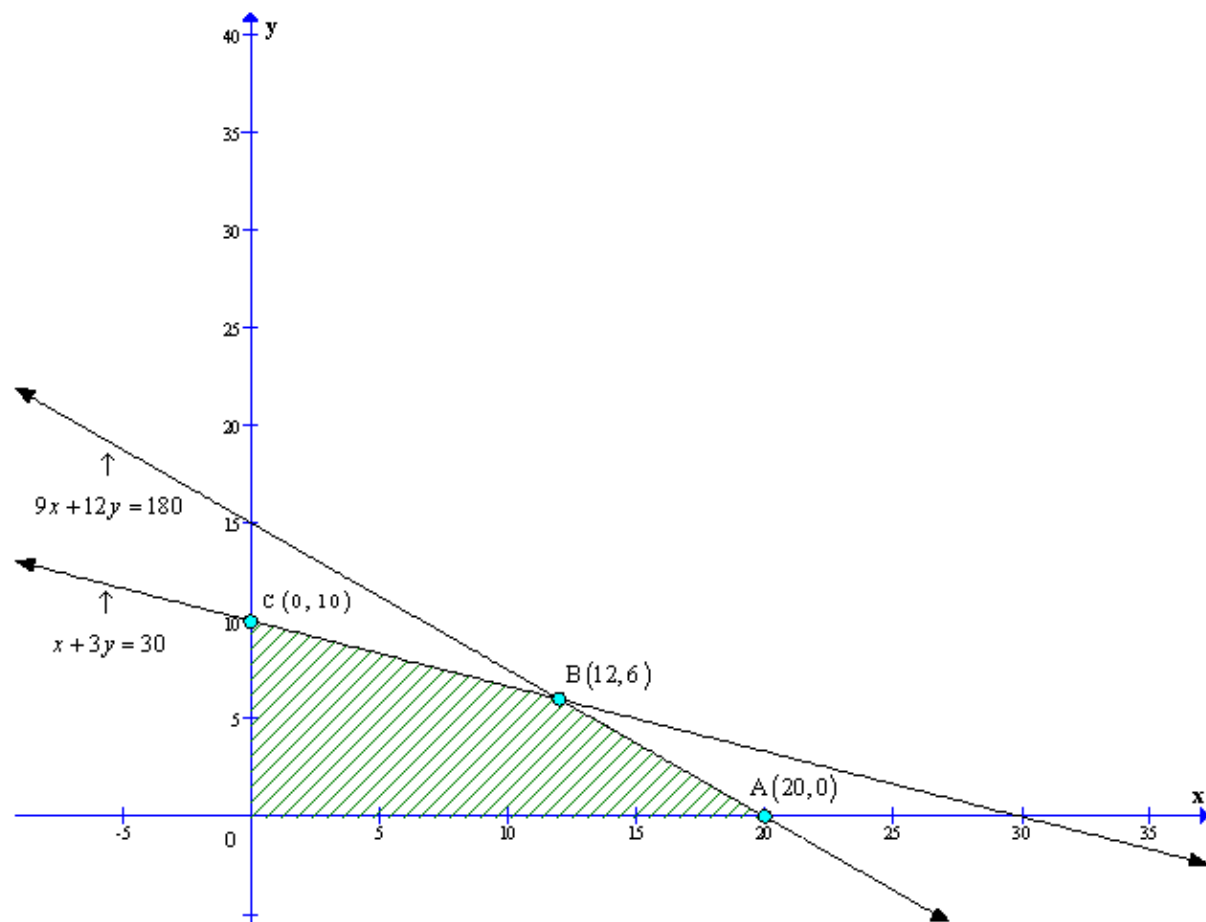
To solve the LPP we draw the lines,

$$9x + 12y = 180,$$

$$x + 3y = 30$$

The feasible region of the LPP is shaded in graph.





The coordinates of the vertices (Corner - points) of shaded feasible region ABC are  $A(20, 0)$ ,  $B(12, 6)$  and  $C(0, 10)$ .

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 8,000x + 12,000y$
$A(20, 0)$	$Z = 1,60,000$
$B(12, 6)$	$Z = 1,68,000$
$C(0, 10)$	$Z = 1,20,000$

12 pieces of Model A and 6 pieces of Model B should be eaned maximize the profit.

The maximum profit that can be eaned is Rs. 1,68,000.

The given data can be written in the tabular form as follows:

Product	Racket	Bat	Maximum hours
Machine	1.5	3	42
Craftman	3	1	24
Profit	20	10	

Let  $x$  be the number of rackets and  $y$  be the number of bats made to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 20x + 10y$$

$$\text{Subject to } 1.5x + 3y \leq 42,$$

$$3x + y \leq 24$$

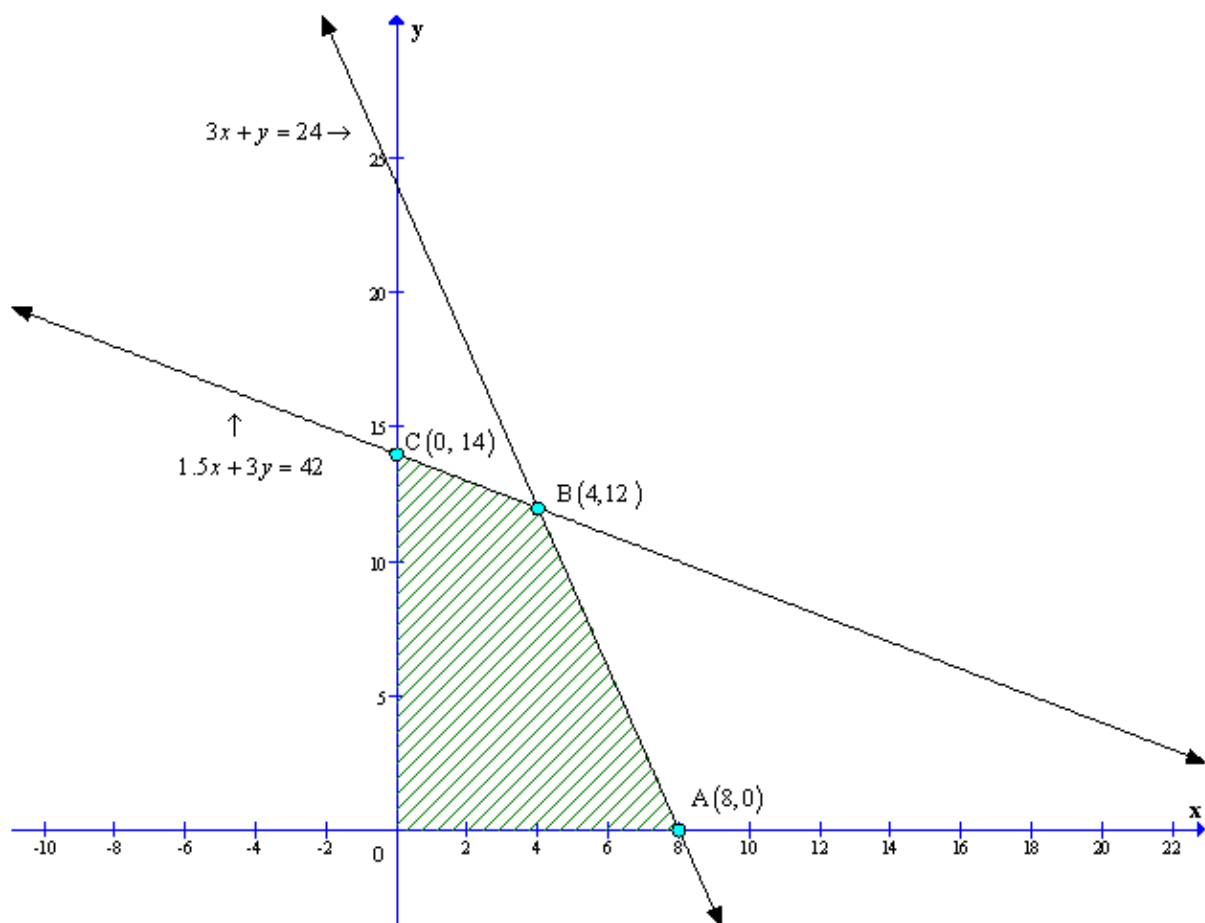
$$\text{and } x \geq 0, y \geq 0$$

To solve the LPP we draw the lines,

$$1.5x + 3y = 42,$$

$$3x + y = 24$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(8, 0), B(4, 12) and C(0, 14).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 20x + 10y$
A(8, 0)	$Z = 160$
B(4, 12)	$Z = 200$
C(0, 14)	$Z = 140$

4 rackets and 12 bats must be made if the factory is to work at full capacity.  
The maximum profit that can be eared is Rs. 200.

### Linear Programming Ex 30.4 Q45

Let  $x$  be the number of desktop computers and  $y$  be the number of portable computers which merchant should stock to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 4500x + 5000y$$

$$\text{Subject to } 25000x + 40000y \leq 70,00,000$$

$$x + y \leq 250$$

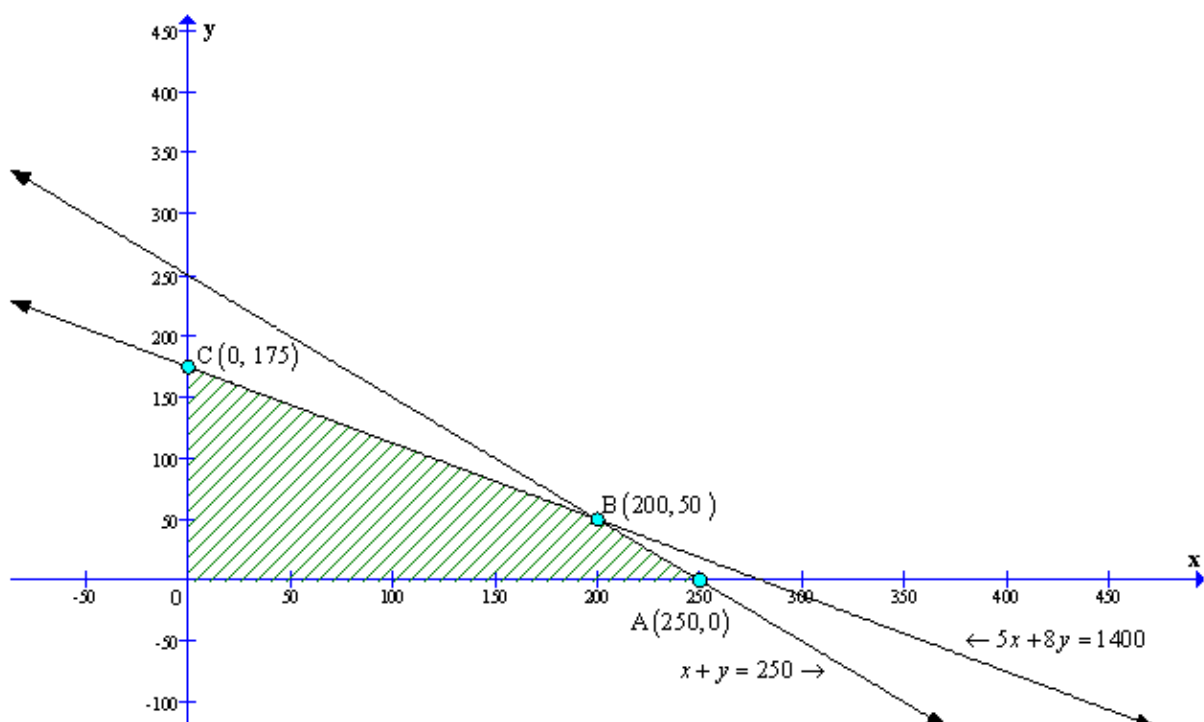
$$\text{and } x \geq 0, y \geq 0$$

To solve the LPP we draw the lines,

$$5x + 8y = 1,400$$

$$x + y = 250$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(250, 0), B(200,50) and C(0, 175).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 4500x + 5000y$
A(250, 0)	$Z = 11,25,000$
B(200, 50)	$Z = 11,50,000$
C(0, 175)	$Z = 8,75,000$

The merchant should stock 200 personal computer and 50 portable computers to earn maximum profit. The maximum profit that can be eared is Rs. 11,50,000.

### Linear Programming Ex 30.4 Q46

Let  $x$  be the number of dolls of type A and  $y$  be the number of dolls of type B should be produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 12x + 16y$$

$$\text{Subject to } x + y \leq 1200$$

$$\frac{1}{2}x - y \geq 0$$

$$x - 3y \leq 600$$

$$\text{and } x \geq 0, y \geq 0$$

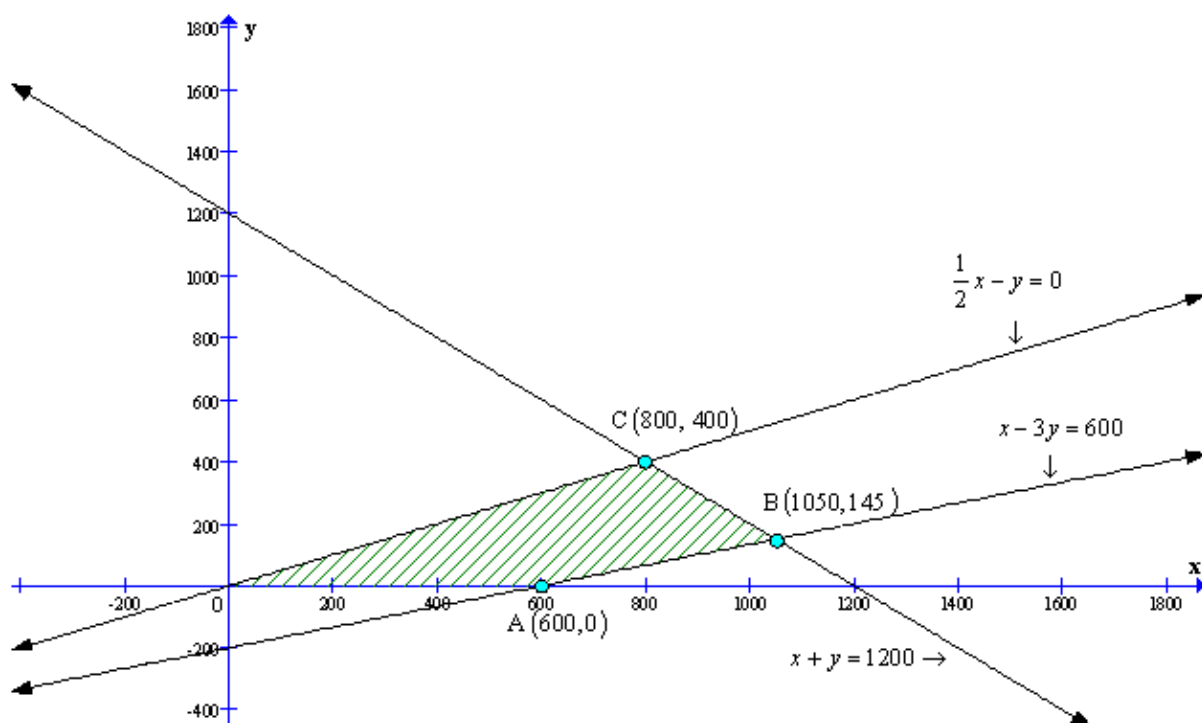
To solve the LPP we draw the lines,

$$x + y = 1200$$

$$\frac{1}{2}x - y = 0$$

$$x - 3y = 600$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(600, 0), B(1050, 145) and C(800, 400).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 12x + 16y$
A(600, 0)	$Z = 7200$
B(1050, 145)	$Z = 14920$
C(800, 400)	$Z = 16000$

The toy company should manufacture 800 dolls of type A and 400 dolls of type B to earn maximum profit. The maximum profit that can be earned is Rs. 16,000.

### Linear Programming Ex 30.4 Q47

Let  $x$  kg of fertiliser  $F_1$  and  $y$  kg of fertiliser  $F_2$  should be used to minimise the cost.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 6x + 5y$$

$$\text{Subject to } 10x + 5y \geq 1400$$

$$6x + 10y \geq 1400$$

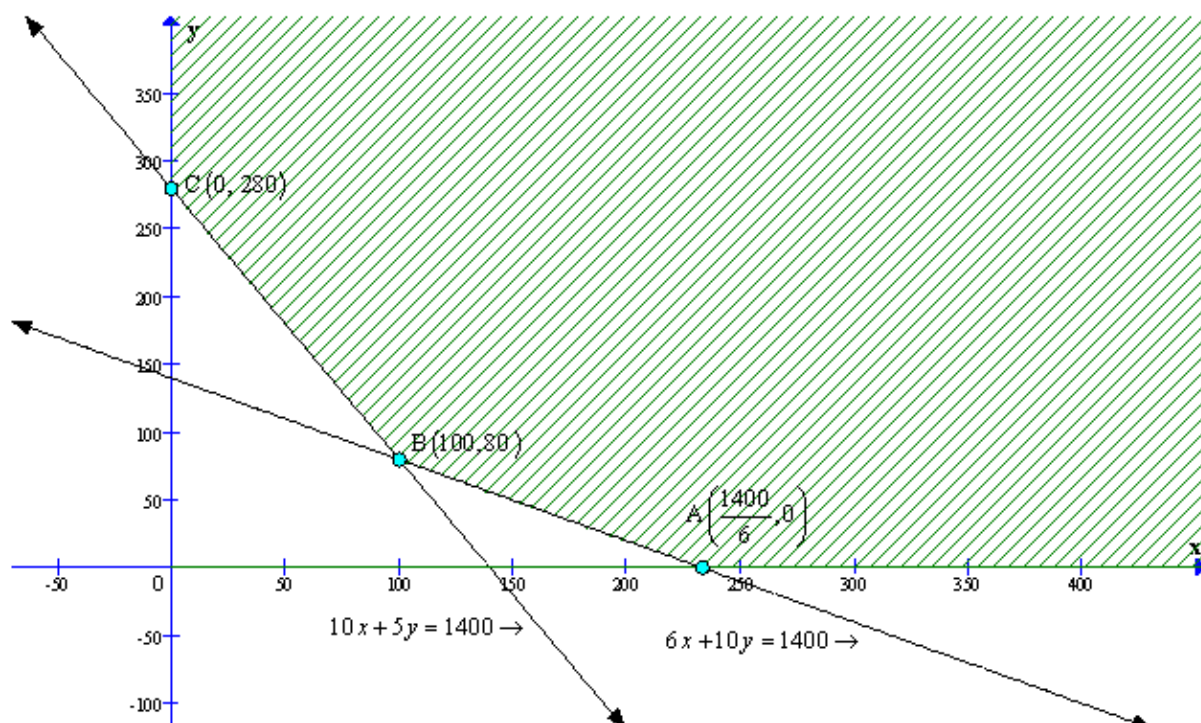
$$\text{and } x \geq 0, y \geq 0$$

To solve the LPP we draw the lines,

$$10x + 5y = 1400$$

$$6x + 10y = 1400$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are

$$A\left(\frac{1400}{6}, 0\right), B(100, 80) \text{ and } C(0, 280).$$

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 6x + 5y$
$A\left(\frac{1400}{6}, 0\right)$	$Z = 1400$
$B(100, 80)$	$Z = 1000$
$C(0, 280)$	$Z = 1400$

100 kg of fertiliser  $F_1$  and 80 kg of fertiliser  $F_2$  to earn minimise the cost.

The maximum cost Rs. 1,000.

### Linear Programming Ex 30.4 Q48

Let  $x$  units of item M and  $y$  units of item N should be produced to maximise the cost.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 600x + 400y$$

$$\text{Subject to } x + 2y \leq 12$$

$$2x + y \leq 12$$

$$x + 1.25y \geq 5$$

$$\text{and } x \geq 0, y \geq 0$$

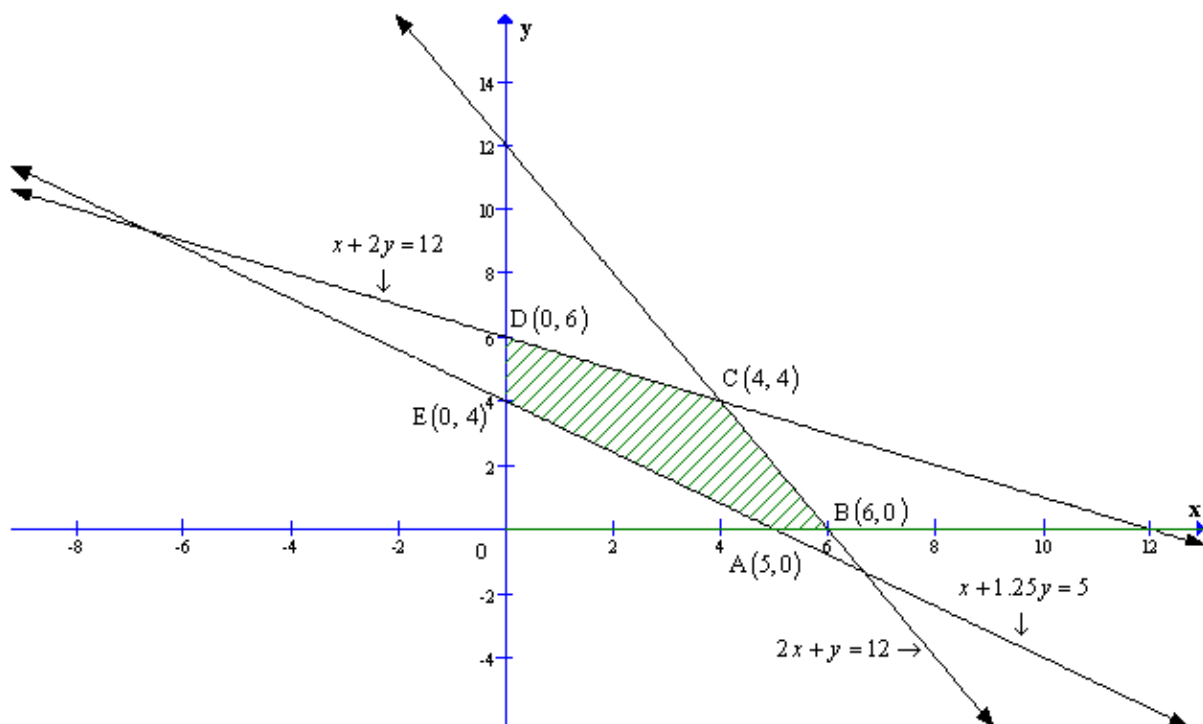
To solve the LPP we draw the lines,

$$x + 2y = 12$$

$$2x + y = 12$$

$$x + 1.25y = 5$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDE are A(5, 0), B(6, 0), C(4, 4), D(0,6) and E(0, 4).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 600x + 400y$
A(5, 0)	$Z = 3000$
B(6, 0)	$Z = 3600$
C(4, 4)	$Z = 4000$
D(0, 6)	$Z = 2400$
E(0, 4)	$Z = 1600$

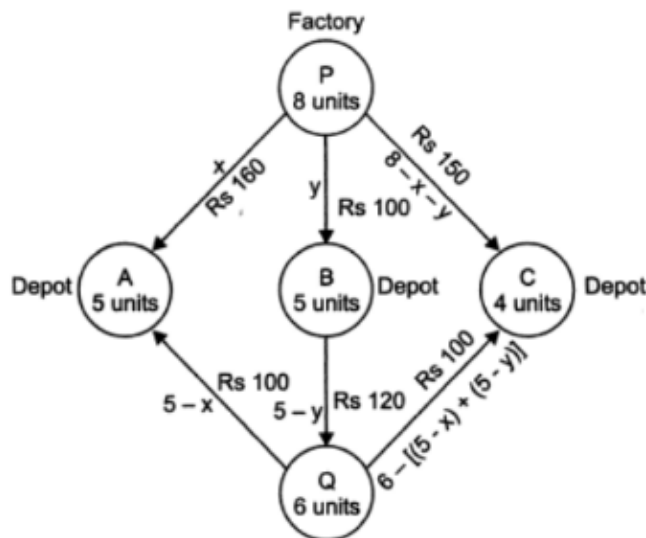
4 units of item M and 4 units of item N should be produced to maximise the profit.  
The maximum profit is Rs. 4,000.

### Linear Programming Ex 30.4 Q49

Let  $x$  and  $y$  units of commodity be transported from factory P to the depots at A and B respectively.

Then  $(8 - x - y)$  units will be transported to depot at C.

The flow is shown below.



Hence we have,  $x \geq 0$ ,  $y \geq 0$  and  $8 - x - y \geq 0$

i.e.  $x \geq 0$ ,  $y \geq 0$  and  $x + y \geq 8$

Now, the weekly requirement of the depot at A is 5 units of the commodity.

Since  $x$  units are transported from the factory at P, remaining  $(5 - x)$  units need to be transported from the factory at Q.

$$\therefore 5 - x \geq 0 \Rightarrow x \leq 5$$

Similarly,  $(5 - y)$  and  $6 - (5 - x + 5 - y) = x + y - 4$  units are to be transported from the factory at Q to the depots at B and C respectively.

$$\therefore 5 - y \geq 0 \text{ and } x + y - 4 \geq 0$$

$$\Rightarrow y \leq 5 \text{ and } x + y \geq 4$$

Total transportation cost  $Z$  is given by

$$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$$

$$Z = 10(x - 7y + 190)$$

So the mathematical model of given LPP is as follows.

$$\text{Minimize } Z = 10(x - 7y + 190)$$

Subject to  $x + y \leq 8$

$$x \leq 5, y \leq 5$$

$$x + y \geq 4$$

$$x \geq 0, y \geq 0$$

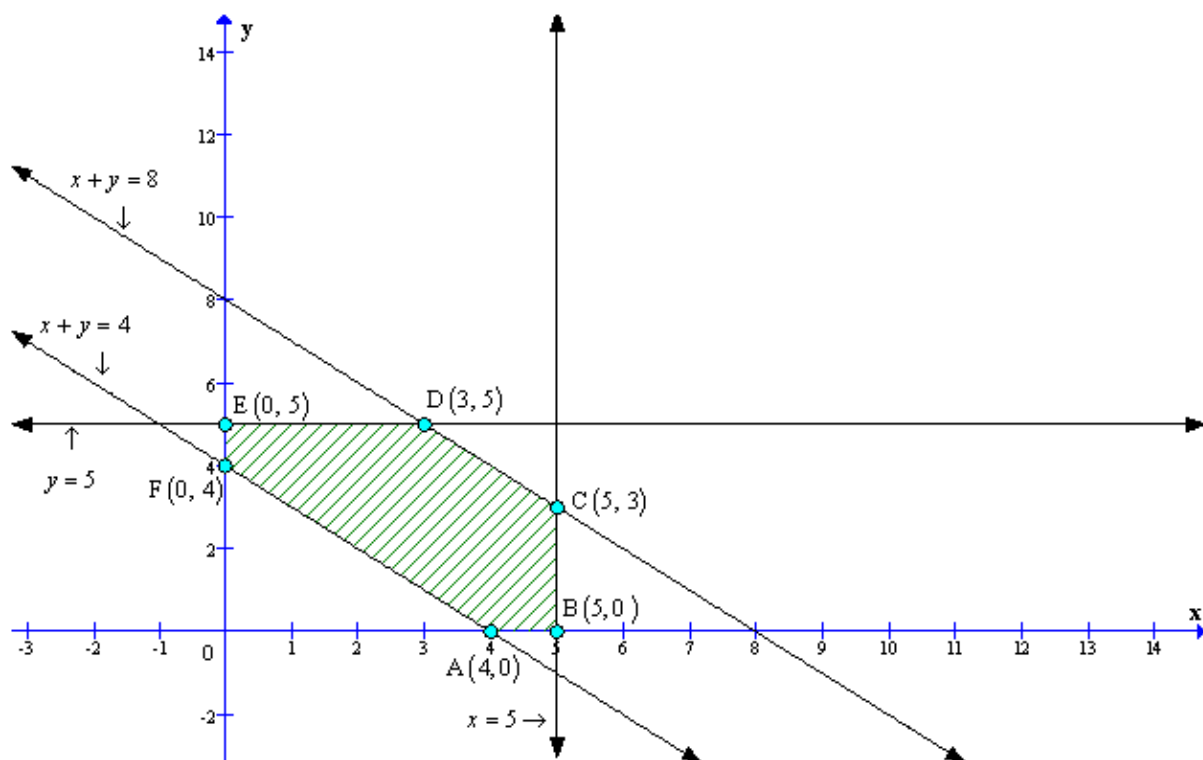
To solve the LPP we draw the lines,

$$x + y = 8$$

$$x = 5, y = 5$$

$$x + y = 4$$

The feasible region of the LPP is shaded in graph.





The coordinates of the vertices (Corner - points) of shaded feasible region ABCDEF are A(4, 0), B(5, 0), C(5, 3), D(3, 5), E(0, 5) and F(0, 4).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 10(x - 7y + 190)$
A(4, 0)	$Z = 1940$
B(5, 0)	$Z = 1950$
C(5, 3)	$Z = 1740$
D(3, 5)	$Z = 1580$
E(0, 5)	$Z = 1550$
F(0, 4)	$Z = 1620$

Deliver 0, 5, 3 units from factory at P and 5, 0, 1 from the factory at Q to the depots at A, B and C respectively. The minimum transportation cost is Rs. 1550.

### Linear Programming Ex 30.4 Q50

Let the mixture contains  $x$  toys of type A and  $y$  toys of type B.

Type of toys	No. of toys	Machine I (in min)	Machine II (in min)	Machine III (in min)	Profit Rs.
A	$x$	$12x$	$18x$	$6x$	$7.5x$
B	$y$	$6y$	$0$	$9y$	$5y$
Total	$x+y$	$12x + 6y$	$18x$	$6x + 9y$	$7.5x + 5y$
Requirement		$360$	$360$	$360$	

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 7.5x + 5y$$

$$\text{Subject to } 12x + 6y \leq 360 \Rightarrow 2x + y \leq 60$$

$$18x \leq 360 \Rightarrow x \leq 20$$

$$6x + 9y \leq 360 \Rightarrow 2x + 3y \leq 120$$

$$\text{and } x \geq 0, y \geq 0$$

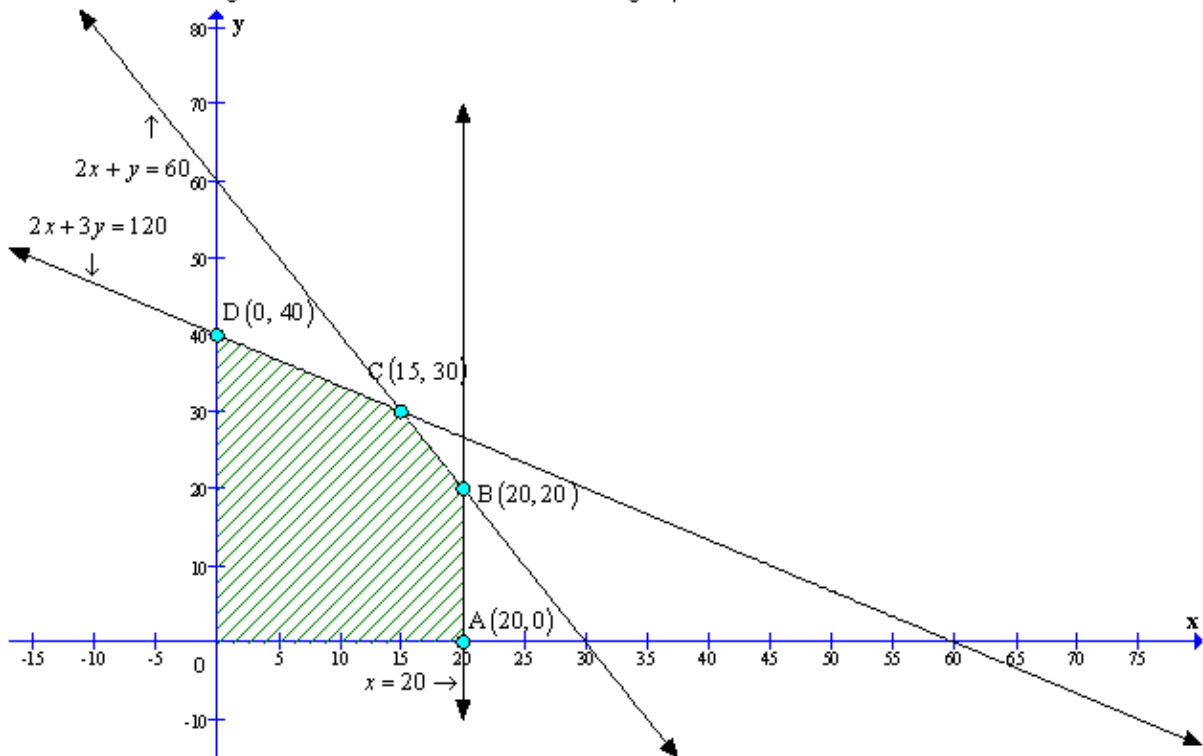
To solve the LPP we draw the lines,

$$2x + y = 60$$

$$x = 20$$

$$2x + 3y = 120$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(20, 0), B(20, 20), C(15, 30) and D(0, 40).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 7.5x + 5y$
A(20, 0)	$Z = 150$
B(20, 20)	$Z = 250$
C(15, 30)	$Z = 262.5$
D(0, 40)	$Z = 200$

Manufacturer should make 15 toys of type A and 30 toys of type B to maximize the profit.

The maximum profit that can be earned is Rs. 262.5

Let  $x$  be the number of executive class tickets and  $y$  be the number of economic class tickets.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 1000x + 600y$$

$$\text{Subject to } x + y \leq 200$$

$$x \geq 20$$

$$y \geq 4x \Rightarrow -4x + y \geq 0$$

$$\text{and } x \geq 0, y \geq 0$$

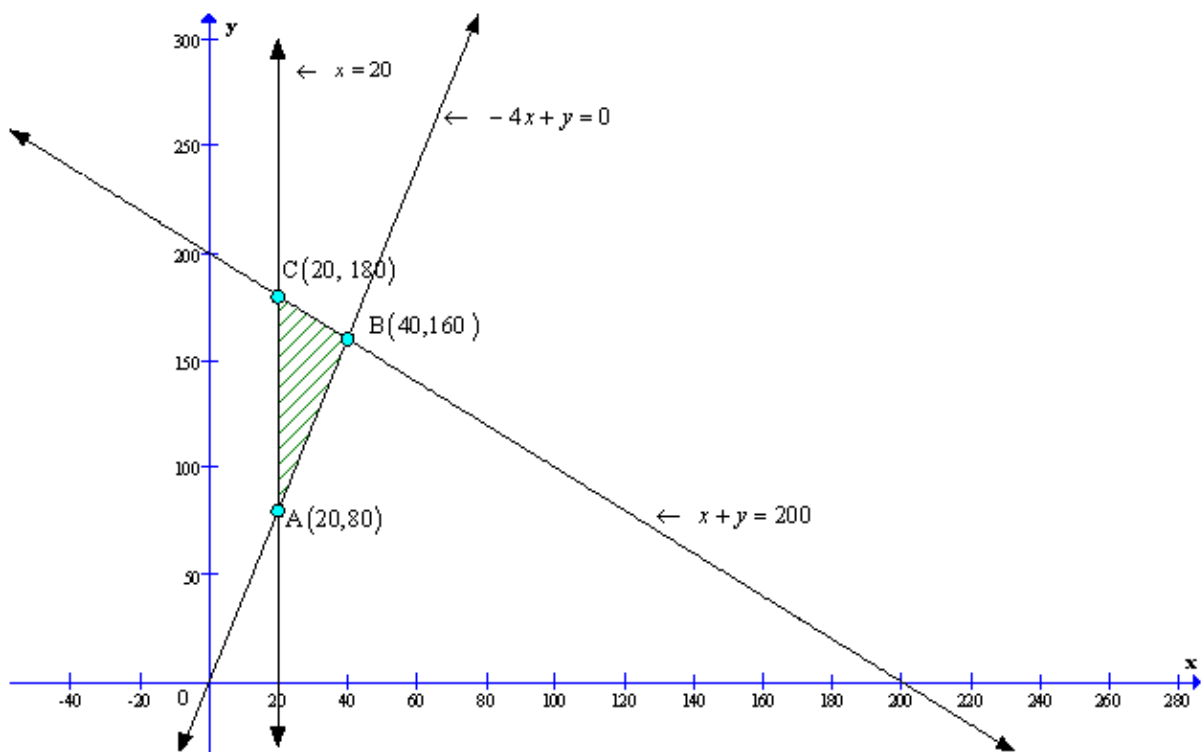
To solve the LPP we draw the lines,

$$x + y = 200$$

$$x = 20$$

$$-4x + y = 0$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 80), B(40, 160) and C(20, 180).

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 1000x + 600y$
A(20, 80)	$Z = 68,000$
B(40, 160)	$Z = 1,36,000$
C(20, 180)	$Z = 1,28,000$

40 tickets of executive class and 160 tickets of economic class must be sold to maximize the profit.

The maximum profit that can be earned is Rs. 1,36,000.

Then the mathematical model of the LPP is as follows:

$$\text{Maximize } Z = 100x + 120y$$

$$\text{Subject to } 2x + 3y \leq 30$$

$$3x + y \leq 17$$

$$\text{and } x \geq 0, y \geq 0$$

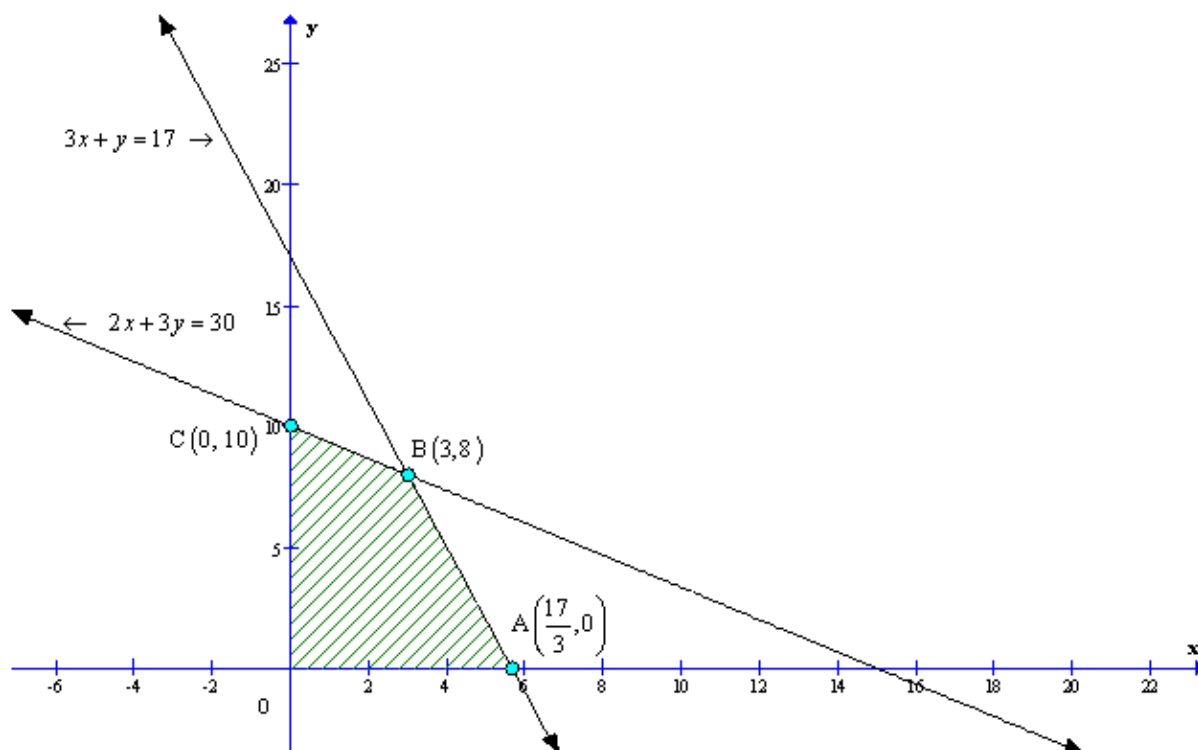
To solve the LPP we draw the lines,

$$2x + 3y = 30$$

$$3x + y = 17$$

The feasible region of the LPP is shaded in graph.

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are

$$A\left(\frac{17}{3}, 0\right), B(3, 8) \text{ and } C(0, 10).$$

The values of the objective of function at these points are given in the following table:

Point $(x_1, x_2)$	Value of objective function $Z = 100x + 120y$
$A\left(\frac{17}{3}, 0\right)$	$Z = 566.67$
$B(3, 8)$	$Z = 1260$
$C(0, 10)$	$Z = 1200$

3 units of workers and 8 units of capital must be used to maximize the profit.

The maximum profit that can be earned is Rs. 1260.

Yes, because efficiency of a person does not depend on sex (male or female).