RD Sharma Solutions Class 12 Maths Chapter 31 Ex 31.4

Probability Ex 31.4 Q1(i)

A coin is tossed thrice

Sample space $= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = The first throw results in head

$A = \{HHT, HTH, HHH, HTT\}$

B = The last throw in tail

$B = \{HHT, HTT, THT, TTT\}$

$A \cap B = \{HHT, HTT\}$

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$P(A).P(B) = \frac{1}{2} \cdot \frac{1}{2}$$

$$P(A).P(B) = \frac{1}{4}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Probability Ex 31.4 Q1(ii)

Sample space for a coin thrown thrice is

$$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$$

A = the number of head is odd

$$A = \{HTT, THT, TTH, HHH\}$$

B = the number of tails is odd

$$B = \{THH, HTH, HHT, TTT\}$$

$$A \cap B = \{\ \} = \varnothing$$

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(A \cap B) = \frac{0}{8} = 0$$

$$P(A).P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(A).P(B) \neq P(A \cap B)$$

So, A and B are not independent events.

Probability Ex 31.4 Q1(iii)

Sample space for throwing a coin thrice

$$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$$

A = the number of heads is two

$$A = \{HHT, THH, HTH\}$$

B = the last throw results in head

$$B = \{HHH, HTH, THH, TTH\}$$

$$A \cap B = \{THH, HTH\}$$

$$P(A) = \frac{3}{8}$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$P(A).P(B) = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$$

$$P(A).P(B) \neq P(A \cap B)$$

So, A and B are not independent events.

Probability Ex 31.4 Q2

A pair of dice are thrown. It has 36 elements in its sample space.

A = Occurence of number 4 on first die

$A = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

B = Occurence of 5 on second die

$B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$A \cap B = \{(4,5)\}$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A).P(B) = \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Probability Ex 31.4 Q3(i)

A card is drawn from 52 cards

It has 4 kings, 4 Queen, 4 Jack

A = the card drawn is a king or a queen

$$P(A) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$P(A) = \frac{2}{13}$$

B = the card drawn is a queen or a jack

$$P(B) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$= \frac{2}{13}$$

$A \cap B$ = The card drawn is a queen

$$P(A \cap B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$$P(A)P(B) = \frac{2}{13} \times \frac{2}{13}$$
$$= \frac{4}{169}$$

$$P(A)P(B) \neq P(A \cap B)$$

Hence, A and B are not independent.

Probability Ex 31.4 Q3(ii)

A card is drawn from pack of 52 cards
There are 26 black and four kings in which 2 kings are black.

A = the card drawn is black

$$P(A) = \frac{26}{52}$$
$$P(A) = \frac{1}{2}$$

B = the card drawn is a king

$$P(B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$A \cap B$ = The card drawn is a black king

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{13}$$
$$= \frac{1}{26}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Probability Ex 31.4 Q3(iii)

A card is drawn from a pack of 52 cards
There are 13 spades and 4 Ace in which one card is ace of spade

A = the card drawn is spade

$$P(A) = \frac{13}{52}$$
$$P(A) = \frac{1}{4}$$

B = the card drawn is an ace

$$P(B) = \frac{4}{52}$$
$$P(B) = \frac{1}{13}$$

$A \cap B$ = The card drawn is an ace of spade

$$P(A \cap B) = \frac{1}{52}$$

$$P(A).P(B) = \frac{1}{4} \times \frac{1}{13}$$
$$= \frac{1}{52}$$

$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q4

A coin is tossed three times,

Sample space $= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

A = first toss is Head

$A = \{HHH, HHT, HTH, HTT\}$

$P(A) = \frac{4}{8}$

$P(A) = \frac{1}{2}$

B = second toss is Head

$= \{HHH, HHT, THH, THT\}$

$P(B) = \frac{4}{8}$

$P(B) = \frac{1}{2}$

C = exactly two Head in a row

$C = \{HHT, THH\}$

$P(C) = \frac{2}{8}$

$P(C) = \frac{1}{4}$

$A \cap B = \{HHH, HHT\}$

$P(A \cap B) = \frac{2}{8}$

$= \frac{1}{4}$

$B \cap C = \{HHT, THH\}$

$P(B \cap C) = \frac{2}{8}$

$P(B \cap C) = \frac{1}{4}$

$A \cap C = \{HHT\}$

$P(A \cap C) = \frac{1}{8}$

(i)

$$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

(ii)

$$P(B)\,P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(B)\,P(C) \neq P(B \cap C)$$

So, B and C are not independent events.

(iii)

$$P(A).P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(A).P(C) = P(A \cap C)$$

Hence, A and C are independent events.

Probability Ex 31.4 Q5

Given,

$$P(A) = \frac{1}{4}, P(B) = \frac{1}{3} \text{ and } P(A \cup B) = \frac{1}{2}$$

We know that,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$= \frac{3 + 4 - 6}{12}$$
$$P(A \cap B) = \frac{1}{12}$$

$$P(A)\,P(B) = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$
$$P(A)\,P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Probability Ex 31.4 Q6

Given that A and B are independent events and $P(A) = 0.3$, $P(B) = 0.6$

(i)

$$P(A \cap B) = P(A)P(B) \qquad \left[\text{Since, } A \text{ and } B \text{ are indenpendent events}\right]$$
$$= 0.3 \times 0.6$$

$$P(A \cap B) = 0.18$$

(ii)

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$
$$= 0.3 - 0.18$$

$$P(A \cap \overline{B}) = 0.12$$

(iii)

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.6 - 0.18$$

$$P(\overline{A} \cap B) = 0.42$$

(iv)

$$P(\overline{A} \cap \overline{B}) = P(A)\,P(B)$$
$$= \left[1 - P(A)\right]\left[1 - P(B)\right]$$
$$= (1 - 0.3)(1 - 0.6)$$
$$= 0.7 \times 0.4$$

$$P(\overline{A} \cap \overline{B}) = 0.28$$

(v)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.6 - 0.18$$

$$P(A \cup B) = 0.72$$

(vi)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.18}{0.6}$$

$$P\left(\frac{A}{B}\right) = 0.3$$

(vii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.18}{0.3}$$

$$P\left(\frac{B}{A}\right) = 0.6$$

Probability Ex 31.4 Q7

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since A, B are independent

$\therefore$ $P(A \cap B) = P(A).P(B)$

Also $P(\text{not } B) = 0.65 \Rightarrow P(B) = 0.35$

Hence, we have

$$0.85 = P(A) + 0.35 - P(A)(0.35)$$

$\Rightarrow$ $0.5 = P(A)[1 - 0.35]$

$\Rightarrow$ $\dfrac{0.5}{.65} = P(A)$

$\Rightarrow$ $P(A) = 0.77$

Probability Ex 31.4 Q8

We are given

$PP\left(\overline{A}\cap B\right)=\frac{2}{15}$

$P\left(A\cap \overline{B}\right)=\frac{1}{6}$

Since A, B are independent,

$\therefore \quad P\left(\overline{A}\right)P\left(B\right)=\frac{2}{15}\Rightarrow \quad \left[1-P\left(A\right)\right]P\left(B\right)=\frac{2}{15} \qquad ---\text{(i)}$

and $\quad P\left(A\right)P\left(\overline{B}\right)=\frac{1}{6}\Rightarrow \quad P\left(A\right)\left[1-P\left(B\right)\right]=\frac{1}{6} \qquad ---\text{(ii)}$

From (i) we get

$$P\left(B\right)=\frac{2}{15}\times\frac{1}{1-P\left(A\right)}$$

Substituting this value in equation (ii) we get,

$$P\left(A\right)\left[1-\frac{2}{15\left(1-P\left(A\right)\right)}\right]=\frac{1}{6}$$

$$\Rightarrow \quad P\left(A\right)\left[\frac{15\left(1-P\left(A\right)\right)-2}{15\left(1-P\left(A\right)\right)}\right]=\frac{1}{6}$$

$$\Rightarrow \quad 6P\left(A\right)\left(13-15P\left(A\right)\right)=15\left(1-P\left(A\right)\right)$$

$$\Rightarrow \quad 2P\left(A\right)\left(13-15P\left(A\right)\right)=5-5P\left(A\right)$$

$$\Rightarrow \quad 26P\left(A\right)-30\left[P\left(A\right)\right]^2+5P\left(A\right)-5=0$$

$$\Rightarrow \quad -30\left[P\left(A\right)\right]^2+31P\left(A\right)-5=0$$

This is a quadratic equation in $x=P\left(A\right)$ given as

$$-30x^2+31x-5=0$$

$$\Rightarrow \quad 30x^2-31x+5=0$$

$$\therefore \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Where $a = +30$, $b = -31$, $c = +5$

$$\Rightarrow \quad x = \frac{31 \pm \sqrt{(-31)^2 - 4(30)(5)}}{60}$$

$$= \frac{31 \pm \sqrt{961 - 600}}{60}$$

$$= \frac{31 \pm 19}{60}$$

$$= \frac{50}{60}, \frac{12}{60}$$

$$= \frac{5}{6}, \frac{1}{5}$$

$$\therefore \quad P(A) = \frac{5}{6} \text{ or } \frac{1}{5}$$

Now

$$P(A)[1 - P(B)] = \frac{1}{6}$$

Putting $P(A) = \frac{5}{6}$

$$\frac{5}{6}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{1}{5}$$

$$P(B) = 1 - \frac{1}{5}$$

$$P(B) = \frac{4}{5}$$

Putting $P(A) = \frac{1}{5}$

$$\frac{1}{5}[1 - P(B)] = \frac{1}{6}$$

$$1 - P(B) = \frac{5}{6}$$

$$P(B) = 1 - \frac{5}{6}$$

$$P(B) = \frac{1}{6}$$

Hence $P(B) = \frac{4}{5}$ or $\frac{1}{6}$

Probability Ex 31.4 Q9

Given,

$$P\left(A \cap B\right) = \frac{1}{6}$$

$$P\left(\overline{A} \cap \overline{B}\right) = \frac{1}{3}$$

We know that,

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = \left(1 - P\left(A\right)\right)\left(1 - P\left(B\right)\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A \cap B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + \frac{1}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6+1-2}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{5}{6}$$

$$P\left(A\right) = \frac{5}{6} - P\left(B\right) \qquad ---\left(i\right)$$

Given, $P\left(A \cap B\right) = \frac{1}{6}$

$$P\left(A\right)P\left(B\right) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P\left(B\right)\right]P\left(B\right) = \frac{1}{6} \qquad \left[\text{Using equation } \left(i\right)\right]$$

$\Rightarrow \quad \frac{5}{6}P\left(B\right) - \left\{P\left(B\right)\right\}^2 = \frac{1}{6}$

$\Rightarrow \quad \left\{P\left(B\right)\right\}^2 - \frac{5}{6}P\left(B\right) + \frac{1}{6} = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 5P\left(B\right) + 1 = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 3P\left(B\right) - 2P\left(B\right) + 1 = 0$

$\Rightarrow \quad 3P\left(B\right)\left[2P\left(B\right) - 1\right] - 1\left[2P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad \left[2P\left(B\right) - 1\right]\left[3P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad 2P\left(B\right) - 1 = 0 \text{ or } 3P\left(B\right) - 1 = 0$

Given,

$$P(A \cap B) = \frac{1}{6}$$

$$P(\overline{A} \cap \overline{B}) = \frac{1}{3}$$

We know that,

$$P(\overline{A} \cap \overline{B}) = P(A)P(B)$$

$$\frac{1}{3} = (1 - P(A))(1 - P(B))$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A)P(B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + P(A \cap B)$$

$$\frac{1}{3} = 1 - P(B) - P(A) + \frac{1}{6}$$

$$P(A) + P(B) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P(A) + P(B) = \frac{5}{6}$$

$$P(A) = \frac{5}{6} - P(B) \qquad \text{---(i)}$$

Given, $P(A \cap B) = \frac{1}{6}$

$$P(A)P(B) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P(B)\right]P(B) = \frac{1}{6} \qquad \text{[Using equation (i)]}$$

$\Rightarrow \quad \frac{5}{6}P(B) - \{P(B)\}^2 = \frac{1}{6}$

$\Rightarrow \quad \{P(B)\}^2 - \frac{5}{6}P(B) + \frac{1}{6} = 0$

$\Rightarrow \quad 6\{P(B)\}^2 - 5P(B) + 1 = 0$

$\Rightarrow \quad 6\{P(B)\}^2 - 3P(B) - 2P(B) + 1 = 0$

$\Rightarrow \quad 3P(B)[2P(B) - 1] - 1[2P(B) - 1] = 0$

$\Rightarrow \quad [2P(B) - 1][3P(B) - 1] = 0$

$\Rightarrow \quad 2P(B) - 1 = 0 \text{ or } 3P(B) - 1 = 0$

Probability Ex 31.4 Q10

Given, A and B are independent events and $P(A \cup B) = 0.60$, $P(A) = 0.2$
A and B are independent events,

So, $P(A \cap B) = P(A)P(B)$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.6 = 0.2 + P(B) - P(A)P(B)$$
$$0.6 - 0.2 = P(B) - 0.2P(B)$$
$$0.4 = 0.8P(B)$$
$$P(B) = \frac{0.4}{0.8}$$

$$P(B) = 0.5$$

Probability Ex 31.4 Q11

A die is tossed twice.

Let A = Getting a number greater than 3 on first toss
B = Getting a number greater than 3 on second toss

$$P(A) = \frac{3}{6}$$ [Since, number greater than 3 on die are 4, 5, 6.]
$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{3}{6}$$
$$P(B) = \frac{1}{2}$$

P(Getting a number greater than 3 on each toss)
$= P(A \cap B)$ [Since, A and B are independent events]
$= P(A)P(B)$
$= \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$

Required Probability $= \frac{1}{4}$

Probability Ex 31.4 Q12

Given,

Probability that A can solve a problem $= \frac{2}{3}$

$\Rightarrow \quad P(A) = \frac{2}{3}$

$\Rightarrow \quad P(\overline{A}) = 1 - \frac{2}{3}$

$P(\overline{A}) = \frac{1}{3}$

Probability that B can solve the same problem $= \frac{3}{5}$

$\Rightarrow \quad P(B) = \frac{3}{5}$

$\Rightarrow \quad P(\overline{B}) = 1 - \frac{3}{5}$

$P(\overline{B}) = \frac{2}{5}$

P (None of them solve the problem)

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}).P(\overline{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Required probability $= \frac{2}{15}$

Probability Ex 31.4 Q13

Given an unbiased die is tossed twice

A = Getting 4,5 or 6 on the first toss

B = 1,2,3 or 4 on second toss

$\Rightarrow \quad P(A) = \frac{3}{6}$

$P(A) = \frac{1}{2}$

and, $\quad P(B) = \frac{4}{6}$

$P(B) = \frac{2}{3}$

P (Getting 4,5 or 6 on the first toss and 1,2,3 or 4 on second toss)

$= P(A \cap B)$

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3}$

Required probability $= \frac{1}{3}$

Probability Ex 31.4 Q14

Given bag contains 3 red and 2 black balls.

A = Getting one red ball

$\Rightarrow \quad P(A) = \frac{3}{5}$

B = Getting one black ball

$\Rightarrow \quad P(B) = \frac{2}{5}$

(i)

P (Getting two red balls)

$= P(A)P(A)$

$= \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

P (Getting two red balls) $= \frac{9}{25}$

(ii)

P (Getting two black balls)

$= P(B).P(B)$

$= \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

P (Getting two black balls) $= \frac{4}{25}$

(iii)

P (Getting first red and second black ball)

$= P(A)P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

P (Getting first red and second black ball) $= \frac{6}{25}$

Probability Ex 31.4 Q15

Three cards are drawn with replacement consider,

A = drawing a king

B = drawing a queen

C = drawing a jack

$\Rightarrow \quad P(A) = \frac{4}{52}$ [Since there are 4 kings]

$P(A) = \frac{1}{13}$

$\Rightarrow \quad P(B) = \frac{4}{52}$ [Since there are 4 queens]

$P(B) = \frac{1}{13}$

$\Rightarrow \quad P(C) = \frac{4}{52}$ [Since there are 4 jacks]

$P(C) = \frac{1}{13}$

P (Cards drawn are king, queen and jack)

$= P(A \cap B \cap C) + P(A \cap C \cap B) + P(B \cap A \cap C)$

$P(B \cap C \cap A) + P(C \cap A \cap B) + P(C \cap B \cap A)$

[Since order of drawing them may be different]

$= P(A)P(B)P(C) + P(A)P(C)P(B) + P(B)P(A)P(C)$

$+P(B)P(C)P(A) + P(C)P(A)P(B) + P(C)P(B)P(A)$

$= \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13} + \frac{1}{13}.\frac{1}{13}.\frac{1}{13}$

$= \left(\frac{1}{13}.\frac{1}{13}.\frac{1}{13}\right) \times 6$

$= \frac{6}{2197}$

Required probability $= \frac{6}{2197}$

Probability Ex 31.4 Q16

Given,

Prat X has 9 out of 100 defective

$\Rightarrow$ Part X has 91 out of 100 non defective

Part Y has 5 out of 100 defective

$\Rightarrow$ Part Y has 95 out of 100 non defective

Consider,

X = A non defective part X

Y = A non defective part Y

$\Rightarrow$ $P(X) = \frac{91}{100}$ abd $P(Y) = \frac{95}{100}$

$= P$ (Assembled product will not be defective)

$= P$ (Niether X defective nor Y defective)

$= P(X \cap Y)$

$= P(X)P(Y)$

$= \frac{91}{100} \times \frac{95}{100}$

$= 0.8645$

Required probability $= 0.8645$

Probability Ex 31.4 Q17

Given,

Probability that A hits a target $= \frac{1}{3}$

$\Rightarrow$ $P(A) = \frac{1}{3}$

Probability that B hits the target $= \frac{2}{5}$

$\Rightarrow$ $P(B) = \frac{2}{5}$

P (Target will be hit)

$= 1 - P$ (target will not be hit)

$= 1 - P$ (Niether A non B hits the target)

$= 1 - P\left(\overline{A} \cap \overline{B}\right)$

$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)$

$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]$

$= 1 - \left[1 - \frac{1}{3}\right]\left[1 - \frac{2}{5}\right]$

$= 1 - \frac{2}{3} \cdot \frac{3}{5}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

Required probability $= \frac{3}{5}$

Probability Ex 31.4 Q18

Given,

An anti aircraft gun can take a maximum 4 shots at an enemy plane

Consider,

A = Hitting the plane at first shot

B = Hetting the plane at second shot

C = Hetting the plane at third shot

D = Hetting the plane at fourth shot

$\Rightarrow \quad P(A) = 0.4,\ P(B) = 0.3,\ P(C) = 0.2,\ P(D) = 0.1$

P (Gun hits the plane)

$= 1 - P$ (Gun does not hit the plane)

$= 1 - P$ (Non of the four shots hit the plane)

$= 1 - P\left(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D}\right)$

$= 1 - P\left(\overline{A}\right) P\left(\overline{B}\right) P\left(\overline{C}\right) P\left(\overline{D}\right)$

$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]\left[1 - P(C)\right]\left[1 - P(D)\right]$

$= 1 - \left[1 - 0.4\right]\left[1 - 0.3\right]\left[1 - 0.2\right]\left[1 - 0.1\right]$

$= 1 - (0.6)(0.7)(0.8)(0.9)$

$= 1 - 0.3024$

$= 0.6976$

Required probability $= 0.6976$

Probability Ex 31.4 Q19

Given,

The odds against a certain event (say, A) are 5 to 2

$$\Rightarrow \quad P(\overline{A}) = \frac{5}{5+2}$$

$$P(\overline{A}) = \frac{5}{7}$$

The odds in favour of another event (say, B) are 6 to 5

$$\Rightarrow \quad P(B) = \frac{6}{5+6}$$

$$P(B) = \frac{6}{11}$$

$$P(\overline{B}) = 1 - \frac{6}{11}$$

$$P(\overline{B}) = \frac{5}{11}$$

(a)

$P(\text{At least one of the events will occur})$

$= 1 - P(\text{None of events occur})$

$= 1 - P(\overline{A} \cap \overline{B})$

$= 1 - P(\overline{A})P(\overline{B})$ [Since events are independent]

$= 1 - \frac{5}{7} \times \frac{5}{11}$

$= 1 - \frac{25}{77}$

$= \frac{52}{77}$

Required probability $= \frac{52}{77}$

(b)

$P(\text{None of the events will occur})$

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A})P(\overline{B})$

$= \frac{5}{7} \times \frac{5}{11}$

$= \frac{25}{11}$

Probability Ex 31.4 Q20

Given, A die is thrown thrice.

Consider,

A = Getting an odd number in a throw of die

$P(A) = \frac{3}{6}$ [Since there are 1,3,5 odd number on die]

$P(A) = \frac{1}{2} \Rightarrow P(\overline{A}) = \frac{1}{2}$

P (Getting an odd number at least once)

$= 1 - P$ (Getting no odd number)

$= 1 - P(\overline{A} \cap \overline{A} \cap \overline{A})$

$= 1 - P(\overline{A})P(\overline{A})P(\overline{A})$

$= 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Required probability $= \frac{7}{8}$

Probability Ex 31.4 Q21

The box contains 10 black balls and 8red balls.

Then $P(\text{black ball}) = \frac{10}{18}$

$P(\text{red ball}) = \frac{8}{18}$

(i) $P(\text{Both balls are red}) = \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$

(ii) P (First ball is black and second is red)

$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$

(iii) P (one of them is black and other is red)

$= \frac{10}{18} \cdot \frac{8}{18} + \frac{8}{18} \cdot \frac{10}{18}$

$= 2\left(\frac{20}{81}\right)$

$= \frac{40}{81}$

Probability Ex 31.4 Q22

Given, Urn contains 4 red and 7 black balls.
Two balls drawn at random with replacement.

Consider,

R = Getting one red ball from urn.

$P(R) = \frac{4}{11}$

B = Getting one blue ball from urn.

$P(B) = \frac{7}{11}$

(i)

P (Getting 2 red balls)

$= P(R).P(R)$

$= \frac{4}{11} \times \frac{4}{11}$

$= \frac{16}{121}$

Required probability $= \frac{16}{121}$

(ii)

P (Getting two blue balls)

$= P(B)P(B)$

$= \frac{7}{11} \times \frac{7}{11}$

$= \frac{49}{121}$

Required probability $= \frac{49}{121}$

(iii)

P (Getting one red and one blue ball)

$= P(R)P(B) + P(B)P(R)$

$= \frac{4}{11} \times \frac{7}{11} + \frac{7}{11} \times \frac{4}{11}$

$= \frac{28}{121} + \frac{28}{121}$

$= \frac{56}{121}$

Probability Ex 31.4 Q23

Given that the events 'A coming in time' and 'B coming in time' are independent.
Let 'A' denote the event of 'A coming in time'.

Then, '$\overline{A}$' denotes the complementary event of A.

Similarly we define B and $\overline{B}$.

P(only one coming in time) $= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A) \times P(\overline{B}) + P(\overline{A}) \times P(B)$...(since A and B are independent events)

$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

The advantage of coming to school in time is that you will not miss any part of the lecture and will be able to learn mo

Probability Ex 31.4 Q24

$$S = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

$n(S) = 36$

E be the event of geting a total of 4.

$E = \{(1,3),(3,1),(2,2)\}$

$n(E) = 3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

F be the event of geting a total of 9 or more.

$F = \{(3,6),(6,3),(4,5),(5,4),(4,6),(6,4),(5,5),(5,6),(6,5),(6,6)\}$

$n(F) = 10$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

G be the event of geting a total divisible by 5.

$G = \{(1,4),(4,1),(2,3),(3,2),(4,6),(6,4),(5,5)\}$

$n(G) = 7$

$$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

No pair is independent.

Probability Ex 31.4 Q25

Events are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence or non-occurrence of the other.

(i) $p_1 p_2 = P(A)P(B)$

$\Rightarrow$ Both A and B occour.

(ii) $(1-p_1)p_2 = (1-P(A))P(B) = P(\overline{A})P(B)$

$\Rightarrow$ Event A does not occour, but event B occours.

(iii) $1-(1-p_1)(1-p_2) = [1-(1-P(A))(1-P(B))] = (1-P(\overline{A})P(\overline{B}))$

$\Rightarrow$ At least one of the events A or B occours.

(iv) $p_1 + p_2 = 2p_1 p_2$

$\Rightarrow P(A) + P(B) = 2P(A)P(B)$

$\Rightarrow P(A) + P(B) - 2P(A)P(B) = 0$

$\Rightarrow P(A) - P(A)P(B) + P(B) - P(A)P(B) = 0$

$\Rightarrow P(A)(1-P(B)) + P(B)(1-P(A)) = 0$

$\Rightarrow P(A)P(\overline{B}) + P(B)P(\overline{A}) = 0$

$\Rightarrow P(A)P(\overline{B}) = P(B)P(\overline{A})$

$\Rightarrow$ Exactly one of A and B occours.