RD Sharma Solutions Class 12 Maths Chapter 31 Ex 31.5

Probability Ex 31.5 Q1

There are two bags.

One bag (1) Contain 6 black and 3 white balls

other bag (2) Contain 5 black and 4 white balls

One ball is drawn from each bag

$$P(\text{One black from bag 1}) = \frac{6}{9}$$

$$P(B_1) = \frac{2}{3}$$

$$P(\text{One black from bag 2}) = \frac{5}{9}$$

$$P(B_2) = \frac{5}{9}$$

$$P(\text{One white from bag 1}) = \frac{3}{9}$$

$$P(W_1) = \frac{1}{3}$$

$$P(\text{One white from bag 2}) = \frac{4}{9}$$

$$P(W_2) = \frac{4}{9}$$

$$P(\text{Two balls of same colour})$$
$$= P\left[(W_1 \cap W_2) \cup (B_1 \cap B_2)\right]$$
$$= P(W_1 \cap W_2) + P(B_1 \cap B_2)$$
$$= P(W_1)P(W_2) + P(B_1)P(B_2)$$
$$= \frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{5}{9}$$
$$= \frac{4}{27} + \frac{10}{27}$$
$$= \frac{14}{27}$$

$$\text{Required probability} = \frac{14}{27}$$

Probability Ex 31.5 Q2

There are two bags.

Bag (1) contain 3 red and 5 black balls

Bag (2) contain 6 red and 4 black balls

$$P(\text{One red ball from bag 1}) = \frac{3}{8}$$
$$P(R_1) = \frac{3}{8}$$

$$P(\text{One black ball from bag 1}) = \frac{5}{8}$$
$$P(B_1) = \frac{5}{8}$$

$$P(\text{One red ball from bag 1}) = \frac{6}{10}$$
$$P(R_2) = \frac{3}{5}$$

$$P(\text{One black ball from bag 2}) = \frac{4}{10}$$
$$P(B_2) = \frac{2}{5}$$

One ball is drawn from each bag.

$$P(\text{One ball is red and the other is black})$$
$$= P\left[(R_1 \cap B_2) \cup (B_1 \cap R_2)\right]$$
$$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$$
$$= P(R_1)P(B_2) + P(B_1)P(R_2)$$
$$= \frac{3}{8} \times \frac{2}{5} + \frac{5}{8} \times \frac{3}{5}$$
$$= \frac{6}{40} + \frac{15}{40}$$
$$= \frac{21}{40}$$

$$\text{Required probability} = \frac{21}{40}$$

Probability Ex 31.5 Q3

Given, box contains 10 black and 8 red balls.
Two balls are drawn with replacement.

(i)

$$P(\text{Both the balls are red})$$
$$= P(R_1 \cap R_2)$$
$$= P(R_1)P(R_2)$$
$$= \frac{8}{18} \times \frac{8}{18}$$
$$= \frac{16}{81}$$

Required probability $= \frac{16}{81}$

(ii)

$$P(\text{first ball is black and second is red})$$
$$= P(B \cap R)$$
$$= P(B)P(R)$$
$$= \frac{10}{18} \times \frac{8}{18}$$
$$= \frac{20}{81}$$

Required probability $= \frac{20}{81}$

(iii)

$$P(\text{one of them red and other black})$$
$$= P((B \cap R) \cup (R \cap B))$$
$$= P(B \cap R) + P(R \cap B)$$
$$= P(B)P(R) + P(R)P(B)$$
$$= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18}$$
$$= \frac{20+20}{81}$$
$$= \frac{40}{81}$$

Required probability $= \frac{40}{81}$

Probability Ex 31.5 Q4

Two cards are drawn without replacement.

There are total 4 ace.

A = Getting Ace

$$P(\text{Exactly one ace out of 2 cards})$$
$$= P\left(\left(A \cap \overline{A}\right) \cup \left(\overline{A} \cap A\right)\right)$$
$$= P(A)P\left(\frac{\overline{A}}{A}\right) + P\left(\overline{A}\right)P\left(\frac{A}{\overline{A}}\right)$$
$$= \frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{4}{51}$$
$$= \frac{96}{663}$$
$$= \frac{32}{221}$$

Required probability $= \dfrac{32}{221}$

Probability Ex 31.5 Q5

Given,

A speaks truth in 75% cases.

B speaks truth in 80% cases.

$$P(A) = \frac{75}{100} \quad \Rightarrow P\left(\overline{A}\right) = \frac{25}{100}$$
$$P(B) = \frac{80}{100} \quad \Rightarrow P\left(\overline{B}\right) = \frac{20}{100}$$

$$P(A \text{ and } B \text{ contradict each other})$$
$$= P\left[\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right]$$
$$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$$
$$= P(A)P\left(\overline{B}\right) + P\left(\overline{A}\right)P(B)$$
$$= \frac{75}{100} \cdot \frac{20}{100} + \frac{25}{100} \cdot \frac{80}{100}$$
$$= \frac{1500}{1000} + \frac{2000}{10000}$$
$$= \frac{3500}{10000}$$
$$= 35\%$$

Required probability $= 35\%$

Probability Ex 31.5 Q6

Given,

Probability of selection of Kamal $(K) = \frac{1}{3}$

$$P(K) = \frac{1}{3}$$

Probability of selection of Monika $(M) = \frac{1}{5}$

$$P(M) = \frac{1}{5}$$

(i)

$P(\text{Both of them selected})$

$= P(K \cap M)$

$= P(K)\,P(M)$

$= \frac{1}{3} \cdot \frac{1}{5}$

$= \frac{1}{15}$

Required probability $= \frac{1}{15}$

(ii)

$P(\text{None of them will be selected})$

$= P(\overline{K} \cap \overline{M})$

$= P(\overline{K})\,P(\overline{M})$

$= [1 - P(K)][1 - P(M)]$

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Required probability $= \frac{8}{15}$

(iii)

$$P(\text{At least one of them selected})$$
$$= 1 - P(\text{None of them selected})$$
$$= 1 - P\left(\overline{M} \cap \overline{K}\right)$$
$$= 1 - P\left(\overline{M}\right).P\left(\overline{K}\right)$$
$$= 1 - [1 - P(M)][1 - P(K)]$$
$$= 1 - \left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{3}\right)$$
$$= 1 - \frac{4}{5}.\frac{2}{3}$$
$$= 1 - \frac{8}{15}$$
$$= \frac{7}{15}$$

Required probability $= \frac{7}{15}$

(iv)

$$P(\text{Only one of them will be selected})$$
$$= P\left[\left(K \cap \overline{M}\right) \cup \left(\overline{K} \cap M\right)\right]$$
$$= P\left(K \cap \overline{M}\right) + P\left(\overline{K} \cap M\right)$$
$$= P(K)P\left(\overline{M}\right) + P\left(\overline{K}\right)P(M)$$
$$= \frac{1}{3}[1 - P(M)] + [1 - P(K)]\frac{1}{5}$$
$$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{3}\right].\frac{1}{5}$$
$$= \frac{1}{3}.\frac{4}{5} + \frac{2}{3}.\frac{1}{5}$$
$$= \frac{4}{15} + \frac{2}{15}$$
$$= \frac{6}{15}$$
$$= \frac{2}{5}$$

Required probability $= \frac{2}{5}$

Probability Ex 31.5 Q7

Bag contain 3 white, 4 red, 5 black balls.
Two balls are drawn without replacement.

$$P(\text{One ball is white and other black})$$
$$= P\left[(W \cap B) \cup (B \cap W)\right]$$
$$= P(W \cap B) + P(B \cap W)$$
$$= P(W)P\left(\frac{B}{W}\right) + P(B)P\left(\frac{W}{B}\right)$$
$$= \frac{3}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{3}{11}$$
$$= \frac{15}{132} + \frac{15}{132}$$
$$= \frac{30}{132}$$
$$= \frac{5}{22}$$

$$\text{Required probability } = \frac{5}{22}$$

Probability Ex 31.5 Q8

A bag contains 8 red and 6 green balls.
Three balls are drawn without replacement

$$P(\text{at least 2 balls are green})$$
$$= P\left[(G_1 \cap G_2 \cap R_1) \cup (G_1 \cap R_1 \cap G_2) \cup (R_1 \cap G_1 \cap G_2) \cup (G_1 \cap G_2 \cap G_3)\right]$$
$$= P(G_1 \cap G_2 \cap R_1) + P(G_1 \cap R_1 \cap G_2) + P(R_1 \cap G_1 \cap G_2) + P(G_1 \cap G_2 \cap G_3)$$
$$= P(G_1)P\left(\frac{G_2}{G_1}\right)P\left(\frac{R_1}{G_1 \cap G_2}\right) + P(G_1)P\left(\frac{R_1}{G_1}\right)P\left(\frac{G_2}{R_1 \cap G_1}\right) +$$
$$P(R_1)P\left(\frac{G_1}{R_1}\right)P\left(\frac{G_2}{G_1 \cap R_1}\right) + P(G_1)P\left(\frac{G_2}{G_1}\right)P\left(\frac{G_3}{G_1 \cap G_2}\right)$$
$$= \frac{6}{14} \times \frac{5}{13} \times \frac{8}{12} + \frac{6}{14} \times \frac{8}{13} \times \frac{5}{12} + \frac{8}{14} \times \frac{6}{13} \times \frac{5}{12} + \frac{6}{14} \times \frac{5}{13} \times \frac{4}{12}$$
$$= \frac{1}{14} \times \frac{1}{13} \times \frac{1}{12} \times (240 + 240 + 240 + 120)$$
$$= \frac{840}{14 \times 13 \times 12}$$
$$= \frac{5}{13}$$

$$\text{Required probability } = \frac{5}{13}$$

Probability Ex 31.5 Q9

Given, Probability of Arun's (A) selection $= \frac{1}{4}$

$$P(A) = \frac{1}{4}$$

Probability of Tarun's (T) rejection $= \frac{2}{3}$

$$P(\overline{T}) = \frac{2}{3}$$

$$P(\overline{A}) = 1 - P(A)$$

$$\Rightarrow \quad P(\overline{A}) = 1 - \frac{1}{4}$$

$$\Rightarrow \quad P(\overline{A}) = \frac{3}{4}$$

$$P(T) = 1 - P(\overline{T})$$

$$\Rightarrow \quad P(T) = 1 - \frac{2}{3}$$

$$\Rightarrow \quad P(T) = \frac{1}{3}$$

P(At least one of them will be selected)

$= 1 - P$(None of them selected)

$$= 1 - P(\overline{A} \cap \overline{T})$$

$$= 1 - P(\overline{A})P(\overline{T})$$

$$= 1 - \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Probability Ex 31.5 Q10

Let E be event of occuring head in a toss of fair coin.

$$P(E) = \frac{1}{2}$$

$$P(\overline{E}) = \frac{1}{2}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw

$$= P(E) = \frac{1}{2}$$

Probability that A wins in 3rd throw

$$= P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^3$$

Probability that A wins in 5th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^5$$

Hence,

Probability of winning A

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\text{Probability that } B \text{ wins} = 1 - P(A \text{ wins})$$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Required probability = $\frac{1}{3}$

Probability Ex 31.5 Q11

Two cards are drawn without replacement from a pack of 52 cards.

There are 26 black and 26 red cards

$$P(\text{one red and other black card})$$

$$= P\left[(R \cap B) \cup (B \cap R)\right]$$

$$= P(R \cap B) + P(B \cap R)$$

$$= P(R)P\left(\frac{B}{R}\right) + P(B)P\left(\frac{R}{B}\right)$$

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$

$$= \frac{13}{51} + \frac{13}{51}$$

$$= \frac{26}{51}$$

Required probability $= \frac{26}{51}$

Probability Ex 31.5 Q12

Tickets are numbered from 1 to 10.
Two tickets are drawn.

Consider, A = Multiple of 5
B = Multiple of 4

$P(A) = \frac{2}{10}$ [Since 5, 10 are multiple of 5]

$P(A) = \frac{1}{5}$

$P(B) = \frac{2}{10}$

$P(B) = \frac{1}{5}$ [Since 4, 8 are multiple of 4]

P (One number multiple of 5 and other multiple of 4)

$= P\left[(A \cap B) \cup (B \cap A)\right]$

$= P(A \cap B) + P(B \cap A)$

$= P(A) P\left(\frac{B}{A}\right) + P(B) P\left(\frac{A}{B}\right)$

$= \frac{1}{5} \times \frac{2}{9} + \frac{1}{5} \times \frac{2}{9}$

$= \frac{4}{45}$

Required probability $= \frac{4}{45}$

Probability Ex 31.5 Q13

Given, In a family Husband (H) tells a lie in 30% cases and Wife (W) tells a lie in 35%

$P(H) = 30\%$, $P(\overline{H}) = 70\%$

$P(W) = 35\%$, $P(\overline{W}) = 65\%$

P (Both contradict each other)

$= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right]$

$= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right)$

$= P(H) P\left(\overline{W}\right) + P\left(\overline{H}\right) P(W)$

$= \frac{30}{100} \times \frac{65}{100} + \frac{70}{100} \times \frac{35}{100}$

$= \frac{1950 + 2450}{10000}$

$= \frac{4400}{10000}$

$= 0.44$

Required probability = 0.44

Probability Ex 31.5 Q14

Given, Probability of Husband's (H) selection = $\frac{1}{7}$

$$P(H) = \frac{1}{7}$$

Probability of Wife's (W) selection = $\frac{1}{5}$

$$P(W) = \frac{1}{5}$$

(a)

$$\begin{aligned}
&\text{P(Both of them will be selected)} \\
&= (H \cap W) \\
&= P(H)P(W) \\
&= \frac{1}{7} \times \frac{1}{5} \\
&= \frac{1}{35}
\end{aligned}$$

Required probability = $\frac{1}{35}$

(b)

$$\begin{aligned}
&\text{P(Only one of them will be selected)} \\
&= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right] \\
&= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right) \\
&= P(H)P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W) \\
&= P(H)\left[1 - P(W)\right] + \left[1 - P(H)\right]P(W) \\
&= \frac{1}{7}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{7}\right]\frac{1}{5} \\
&= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5} \\
&= \frac{10}{35} \\
&= \frac{2}{7}
\end{aligned}$$

Required probability = $\frac{2}{7}$

(c)

P (None of them selected)

$= \left(\overline{H} \cap \overline{W}\right)$

$= P\left(\overline{H}\right) P\left(\overline{W}\right)$

$= \left(1 - P(H)\right)\left(1 - P(W)\right)$

$= \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{6}{7} \times \frac{4}{5}$

$= \frac{24}{35}$

Required probability $= \frac{24}{35}$

Probability Ex 31.5 Q15

A bag contains 7 white, 5 black and 4 red balls.
Four balls are drawn without replacement

P (At least three balls are black)

$= P$ (3 black balls and one not black or 4 black balls)

$= P$ (3 black and one not black) $+ P$ (4 black balls)

$$= \frac{{}^5C_3 \times {}^{11}C_1}{{}^{16}C_4} + \frac{{}^5C_4}{{}^{16}C_4}$$

$$= \frac{\frac{5!}{3!2!} \times 11 + \frac{5!}{4!1!}}{\frac{16!}{4!12!}} \qquad \left[\text{Since } {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$= \frac{\frac{5.4}{2} \times 11 + 5}{\frac{16.15.14.13}{4.3.2}}$$

$$= \frac{(110 + 5)}{1820}$$

$$= \frac{115}{1820}$$

$$= \frac{23}{364}$$

Required probability $= \frac{23}{364}$

Probability Ex 31.5 Q16

Given,

A speaks truth 3 out of four times
B speaks truth 4 out of five times
C speads truth 5 out of six times.

$\Rightarrow \quad P(A) = \frac{3}{4}, P(B) = \frac{4}{5}, P(C) = \frac{5}{6}$

P (Reported truth fully by majority of witnesses)

$$= P\left(\left(A \cap B \cap \overline{C}\right) \cup \left(A \cap \overline{B} \cap C\right) \cup \left(\overline{A} \cap B \cap C\right) \cup \left(A \cap B \cap C\right)\right)$$
$$= P\left(A \cap B \cap \overline{C}\right) + P\left(A \cap \overline{B} \cap C\right) + P\left(\overline{A} \cap B \cap C\right) + P\left(A \cap B \cap C\right)$$
$$= P(A)P(B)P\left(\overline{C}\right) + P(A)P\left(\overline{B}\right)P(C) + P\left(\overline{A}\right)P(B)P(C) + P(A)P(B)P(C)$$
$$= P(A)P(B)\left(1 - P(C)\right) + P(A)\left(1 - P(B)\right)P(C) + \left(1 - P(A)\right)P(B)P(C) + P(A)P(B)P(C)$$
$$= \frac{3}{4} \times \frac{4}{5}\left(1 - \frac{5}{6}\right) + \frac{3}{4}\left(1 - \frac{4}{5}\right)\frac{5}{6} + \left(1 - \frac{3}{4}\right)\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$
$$= \frac{3}{4}.\frac{4}{5}.\frac{1}{6} + \frac{3}{4}.\frac{1}{5}.\frac{5}{6} + \frac{1}{4}.\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}$$
$$= \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{2}$$
$$= \frac{12 + 15 + 20 + 60}{120}$$
$$= \frac{107}{120}$$

Required probability $= \frac{107}{120}$

Probability Ex 31.5 Q17

Bag A has 4 white balls and 2 black balls;
Bag B has 3 white balls and 5 black balls.

(i) $P(A_W \text{ and } B_W) = P(A_W)P(B_W) = \frac{4}{6}.\frac{3}{8} = \frac{1}{4}$

(ii) $P(A_B \text{ and } B_B) = P(A_B)P(B_B) = \frac{2}{6}.\frac{5}{8} = \frac{5}{24}$

(iii) $P(A_W \text{ and } B_B \text{ or } A_B \text{ and } B_W) = P(A_W)P(B_B) + P(A_B)P(B_W)$

$$= \frac{4}{6}.\frac{5}{8} + \frac{2}{6}.\frac{3}{8}$$
$$= \frac{20}{48} + \frac{6}{48}$$
$$= \frac{26}{48} = \frac{13}{24}$$

Probability Ex 31.5 Q18

Number of white balls = 4
Number of black balls = 7
Number of red balls = 5
Total balls = 16

Number of ways in which 4 balls can be drawn from 16 balls = ${}^{16}C_4$
Let A = getting at least two white ball = getting 2, 3, 4 white balls

Number of ways of choosing 2 white balls = ${}^4C_2 \times {}^{12}C_2$

Number of ways of choosing 3 white balls = ${}^4C_3 \times {}^{12}C_1$

Number of ways of choosing 4 white balls = ${}^4C_4 \times {}^{12}C_0$

$$\therefore \qquad P(A) = \frac{{}^4C_2 \times {}^{12}C_2 + {}^4C_3 \times {}^{12}C_1 + {}^4C_4 \times {}^{12}C_0}{{}^{16}C_4} = \frac{67}{256}$$

Probability Ex 31.5 Q19

Three cards are drawn with replacement from a pack of cards.
There are 4 Kings, 4 Queens, 5 Jacks.

$$P(1 \text{ King}, 1 \text{ Queen}, 1 \text{ Jack})$$

$$= P\left((K \cap Q \cap J) \cup (K \cap J \cap Q) \cup (J \cap K \cap Q) \cup (J \cap Q \cap K) \cup (Q \cap K \cap L) \cup (Q \cap J \cap K)\right)$$
$$= P(K \cap Q \cap J) + P(K \cap J \cap Q) + P(J \cap K \cap Q) + P(J \cap Q \cap K) + P(Q \cap K \cap J) + P(Q \cap J \cap K)$$
$$= P(K)P(Q)P(J) + P(K)P(J)P(Q) + P(J)P(K)P(Q) + P(J)P(Q)P(K) + P(Q)P(K)P(J)$$
$$+ P(Q)P(J)P(K)$$
$$= \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52} + \frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52}$$
$$= \frac{6}{13.13.13}$$
$$= \frac{6}{2197}$$

Required probability = $\frac{6}{2197}$

Probability Ex 31.5 Q20

Given, Bag (1) contains 4 red and 5 black balls.

Bag (2) contains 3 red and 7 black balls

One ball is drawn at random from each bag.

(i)

P (Balls are of different colours)

$= P\left((R_1 \cap B_2) \cup (B_1 \cap R_2)\right)$

$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$

$= P(R_1)P(B_2) + P(B_1)P(R_2)$

$= \frac{4}{9} \cdot \frac{7}{10} + \frac{5}{9} \cdot \frac{9}{10}$

$= \frac{28}{90} + \frac{15}{90}$

$= \frac{43}{93}$

(ii)

P (Balls are of the same colour)

$= P\left((B_1 \cap B_2) \cup (R_1 \cap R_2)\right)$

$= P(B_1 \cap B_2) + P(R_1 \cap R_2)$

$= P(B_1)P(B_2) + P(R_1)P(R_2)$

$= \frac{5}{9} \cdot \frac{7}{10} + \frac{4}{9} \cdot \frac{3}{10}$

$= \frac{35}{90} + \frac{12}{90}$

$= \frac{47}{90}$

Required probability $= \frac{47}{90}$

Probability Ex 31.5 Q21

Let A be the event that "A hits the target",
B be the event that "B hits the target" and
C be the event that "C hits the target".
Then A, B and C are independent events such that
$P(A) = \frac{3}{6} = \frac{1}{2}$; $P(B) = \frac{2}{6} = \frac{1}{3}$; $P(C) = \frac{4}{4} = 1$

The target is hit by at least 2 shots in the following mutually exclusive ways :
(i) A hits, B hits and C does not hit, i.e., $A \cap B \cap \bar{C}$
(ii) A hits, B does not hit and C hits, i.e., $A \cap \bar{B} \cap C$
(iii) A does not hit, B hits and C hits, i.e., $\bar{A} \cap B \cap C$
(iv) A hits, B hits and C hits, i.e., $A \cap B \cap C$
Hence, by the addition theorem for mutually exclusive events, the probability that at least 2 shots hit.
$= P(i) + P(ii) + P(iii) + P(iv)$
$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) +$
$P(A \cap B \cap C)$
$= P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})\,P(C) + P(\bar{A})\,P(B)\,P(C) +$
$P(A)\,P(B)\,P(C)$
$= P(A)\,P(B)\,[1 - P(C)] + P(A)\,[1 - P(B)]\,P(C) +$
$[1 - P(A)]\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= \frac{1}{2} \times \frac{1}{3} \times (1 - 1) + \frac{1}{2} \times \left(1 - \frac{1}{3}\right) \times 1 + \left(1 - \frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= \frac{1}{2} \times \frac{1}{3} \times 0 + \frac{1}{2} \times \left(\frac{2}{3}\right) \times 1 + \left(\frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= 0 + \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$
$= \frac{2}{3}$

Probability Ex 31.5 Q22

Given,

The probability of A passing exam $= \frac{2}{9}$

The probability of B passing exam $= \frac{5}{9}$

And they are independent.

$$\Rightarrow \quad P(A) = \frac{2}{9}, \; P(B) = \frac{5}{9}$$

(i)

P (Only A passing the exam)

$$= P\left(A \cap \overline{B}\right)$$

$$= P(A).P(B)$$

$$= P(A).\left(1 - P(B)\right)$$

$$= \frac{2}{9}\left(1 - \frac{5}{9}\right)$$

$$= \frac{2}{9}\left(\frac{4}{9}\right)$$

$$= \frac{8}{81}$$

(ii)

P (Only one of them passing exam)

$$= P\left(\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right)$$

$$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$$

$$= P(A)P\left(\overline{B}\right) + P\left(\overline{A}\right)P(B)$$

$$= P(A)\left(1 - P(B)\right) + \left(1 - P(A)\right)P(B)$$

$$= \frac{2}{9}\left(1 - \frac{5}{9}\right) + \left(1 - \frac{2}{9}\right)\frac{5}{9}$$

$$= \frac{2}{9}.\frac{4}{9} + \frac{7}{9}.\frac{5}{9}$$

$$= \frac{8}{81} + \frac{35}{81}$$

$$= \frac{43}{81}$$

Required probability $= \frac{43}{81}$

Probability Ex 31.5 Q23

Given,

Urn A contains 4 red (R_1) and 3 black (B_1) balls

Urn B contains 5 red (R_2) and 4 black (B_2) balls

Urn C contains 4 red (R_3) and 4 black (B_3) balls.

P (3 balls drawn consists of 2 red and a black ball)

$= P\left[(R_1 \cap R_2 \cap R_3) \cup (R_1 \cap B_2 \cap R_3) \cup (B_1 \cap R_2 \cap R_3)\right]$

$= P(R_1 \cap R_2 \cap R_3) + P(R_1 \cap B_2 \cap R_3) + P(B_1 \cap R_2 \cap R_3)$

$= P(R_1) + P(R_2) + P(R_3) + P(R_1) + P(B_2) + P(R_3) + P(B_1) + P(R_2) + P(R_3)$

$= \frac{4}{7} \cdot \frac{5}{9} \cdot \frac{4}{8} + \frac{4}{7} \cdot \frac{4}{9} \cdot \frac{4}{8} + \frac{3}{7} \cdot \frac{5}{9} \cdot \frac{4}{8}$

$= \frac{80 + 64 + 60}{504}$

$= \frac{204}{504}$

$= \frac{17}{42}$

Required probability $= \frac{17}{42}$

Probability Ex 31.5 Q24

Given,

Probability of getting A grade in mathematics $(m) = 0.2$

$\Rightarrow \quad P(m) = 0.2$

Probability of getting A grade in physics $(p) = 0.3$

$\Rightarrow \quad P(p) = 0.3$

Probability of getting A grade in chemistry $(p) = 0.5$

$\Rightarrow \quad P(c) = 0.5$

(i)

$P(\text{Getting } A \text{ grade in all subjects})$
$= P(m \cap p \cap c)$
$= P(m) + P(p) + P(c)$
$= 0.2 \times 0.3 \times 0.5$
$= 0.03$

Required probability $= 0.03$

(ii)

$P(\text{Getting } A \text{ in no subject})$
$= P(\overline{m} \cap \overline{p} \cap \overline{c})$
$= P(\overline{m}) + P(\overline{p}) + P(\overline{c})$
$= (1 - P(m))(1 - P(p))(1 - P(c))$
$= (1 - 0.2)(1 - 0.3)(1 - 0.5)$
$= (0.8)(0.7)(0.5)$
$= 0.28$

Required probability $= 0.28$

(iii)

$P(\text{Getting } A \text{ grade in two subjects})$
$= P((m \cap p \cap \overline{c}) \cup (m \cap \overline{p} \cap c) \cup (\overline{m} \cap p \cap c))$
$= P(m)P(p)P(\overline{c}) + P(m)P(\overline{p})P(c) + P(\overline{m})P(p)P(c)$
$= P(m)P(p)(1 - P(c)) + P(m)(1 - P(p))P(c) + (1 - P(m))P(p)P(c)$
$= (0.2)(0.3)(1 - 0.5) + (0.2)(1 - 0.3)(0.5) + (1 - 0.2)(0.3)(0.5)$

Probability Ex 31.5 Q25

Sum of 9 can be obtained by

$$E = \{(3,6),(4,5),(5,4),(6,3)\}$$

Probability of throwing 9 = $\frac{4}{36}$

$$P(E) = \frac{1}{9},\ P(\overline{E}) = \frac{8}{9}$$

$\Rightarrow \quad P(A) = P(B) = \frac{1}{9}$

$\Rightarrow \quad P(\overline{A}) = P(\overline{B}) = \frac{8}{9}$

A and B take turns in throwing two dice.
Let A starts the game.

$P(A \text{ wins the game})$

$= P(A \cup \overline{A} \cap \overline{B} \cap A \cup \overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A \cup \ldots)$

$= P(A) + P(\overline{A} \cap \overline{B} \cap A) + P(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A) + \ldots$

$= P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A})P(\overline{B})P(\overline{A})P(\overline{B})P(A) + \ldots$

$= \frac{1}{9} + \frac{8}{9}.\frac{8}{9}.\frac{1}{9} + \frac{8}{9}.\frac{8}{9}.\frac{8}{9}.\frac{8}{9}.\frac{1}{9} + \ldots$

$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\right]$

$= \frac{1}{9}\left[\frac{1}{1 - \left(\frac{8}{9}\right)^2}\right]$ $\quad \left[\begin{array}{l}\text{Since for a G.P. with first term 9 and common ratio r,} \\ S_\infty = \frac{a}{1-r}\end{array}\right]$

$= \frac{1}{9}\left[\frac{1}{1 - \frac{64}{81}}\right]$

$= \frac{1}{9}\left[\frac{81}{81 - 64}\right]$

$= \frac{9}{17}$

$P(B \text{ wins the game}) = 1 - P(A \text{ wins the game})$

$= 1 - \frac{9}{17}$

$= \frac{8}{17}$

Chances of winning of $A : B$

$= \frac{9}{17} : \frac{8}{17}$

$= 9 : 8$

Chances of winning $A : B = 9 : 8$

Probability Ex 31.5 Q26

Let E be event of getting a head.

$$P(E) = \frac{1}{2} \qquad \Rightarrow P(\overline{E}) = \frac{1}{2}$$

If A starts the game,

$\Rightarrow$ A wins the game in 1^{st}, 4^{th}, 7^{th},... toss of coin.

$$
\begin{aligned}
&P(A \text{ wins}) \\
&= P\left(E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \ldots\right) \\
&= P\left(\overline{E}\right) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap E\right) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E\right) + \ldots \\
&= P(E) + P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P(E) + P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P(E) + \ldots \\
&= \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots \\
&= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^7 + \ldots \\
&= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\right] \\
&= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right] \\
&= \frac{1}{2}\left[\frac{1}{1 - \frac{1}{8}}\right] \\
&= \frac{1}{2}\left[\frac{8}{7}\right] \\
&= \frac{4}{7}
\end{aligned}
$$

B wins in 2^{nd}, 5^{th}, 8^{th},... toss of coin

$$
\begin{aligned}
&P(B \text{ wins}) \\
&= P\left(\overline{E} \cap E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \ldots\right) \\
&= P\left(\overline{E} \cap E\right) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup\right) + \ldots \\
&= P\left(\overline{E}\right)P(E) + P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P\left(\overline{E}\right)P(E) + \ldots \\
&= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots
\end{aligned}
$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \left(\frac{1}{2}\right)^2 \left[1 + \left(\frac{1}{2}\right)^3 + \ldots\right]$$

$$= \frac{1}{4}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^3}\right] \qquad \left[\text{Since for G.P. } S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{1}{4}\left[\frac{1}{1 - \frac{1}{8}}\right]$$

$$= \frac{1}{4}\left[\frac{8}{7}\right]$$

$$= \frac{2}{7}$$

$$P(C \text{ wins}) = 1 - P(A \text{ wins}) - P(B \text{ wins})$$

$$= 1 - \frac{4}{7} - \frac{2}{7}$$

$$= \frac{1}{7}$$

Probabilities of winning A, B and C are $\frac{4}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively.

Probability Ex 31.5 Q27

Let E be the even of getting a six

$$P(E) = \frac{1}{6}$$

$$P(\overline{E}) = \frac{5}{6}$$

A wins if he gets a six in 1st or 4th or 7th... throw

A wins in first throw $= P(E) = \frac{1}{6}$

A wins in 4th throw if he fails in 1^{st}, B fails in 2^{nd}, C fails in 3^{rd} throw.

Probability of winning A in 4th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6}$$

Similarly, Probability of winning A in 7th thrwo $= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$

$$= \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6}$$

Hence, probability of winning of A

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \ldots$$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Using } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \frac{125}{216}}\right]$$

$$= \frac{1}{6} \times \frac{216}{91}$$

$$= \frac{36}{91}$$

B wins if he gets a six in 2nd or 5th or 8th ...throw.

B wins in 2nd throw $= P(\bar{E})P(E)$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$$

B wins in 5th throw if A fails in first, B fails in 2nd, C fails in 3rd, A fails in 4th.

Probability of winning B in 5th throw

$$= P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E)$$

$$= \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$$

Probability of winning B in 8th thrwo

$$= \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

Hence, probability of winning B

$$= \left(\frac{5}{6}\right)\frac{1}{6} + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

$$= \frac{5}{6}.\frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\frac{125}{216}}\right]$$

$$= \frac{5}{36}\times\left[\frac{216}{91}\right]$$

$$= \frac{30}{91}$$

Probability of winning $C = 1 - P(A \text{ wins}) - P(B \text{ wins})$

$$= 1 - \frac{36}{91} - \frac{30}{91}$$

$$= \frac{25}{91}$$

The respective probabilities of winning of A, B and C are $\frac{36}{91}$, $\frac{30}{91}$ and $\frac{25}{91}$.

Probability Ex 31.5 Q28

Let E be events of throwing 10 on a pair of dice,

$$E = \{(4,6),(5,5),(6,4)\}$$

$$P(E) = \frac{3}{37}$$

$$P(E) = \frac{1}{12}$$

$$P(\overline{E}) = \frac{11}{12}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw $= P(E) = \frac{1}{12}$

Probability that A wins in 3rd throw

$$= P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)$$

Probability that A wins in 5th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

Hence,

Probability of winning A

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) + \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\right]$$

$$= \frac{1}{12}\left[\frac{1}{1 - \left(\frac{11}{12}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

Probability Ex 31.5 Q29

Bag A has 3 red and 5 black balls

Bag B has 2 red and 3 black balls

One ball is drawn from bag A and two from bag B.

P(One red from bag A and 2 black from bag B Or one black from bag A and 1 red and one black from bag B)

$$= P\left(R_1 \cap \left(2B_2\right)\right) + P\left(B_1 \cap R_2 \cap B_2\right)$$

$$= P\left(R_1\right) P\left(2B_2\right) + P\left(B_1\right) P\left(R_2\right) P\left(B_2\right)$$

$$= \frac{3}{8} \cdot \frac{{}^3C_2}{{}^5C_2} + \frac{5}{8} \cdot \frac{2}{5} \cdot \frac{3}{4}$$

$$= \frac{3}{8} \cdot \frac{3}{\left(\frac{5.4}{2}\right)} + \frac{5}{8} \cdot \frac{2}{5} \cdot \frac{3}{4}$$

$$= \frac{18}{160} + \frac{30}{160}$$

$$= \frac{48}{160} = \frac{3}{10}$$

Required probability $= \dfrac{3}{10}$

Probability Ex 31.5 Q30

Given,

Probability of Fatima's (F) selection = $\frac{1}{7}$

$P(F) = \frac{1}{7} \quad \Rightarrow P(\bar{F}) = \frac{6}{7}$

Probability of John's (J) selection = $\frac{1}{5}$

$P(J) = \frac{1}{5} \quad \Rightarrow P(\bar{J}) = \frac{4}{5}$

(i)

$P(\text{Both of them selected})$
$= P(F \cap J)$
$= P(F).P(J)$
$= \frac{1}{7} \times \frac{1}{5}$
$= \frac{1}{35}$

Required probability = $\frac{1}{35}$

(ii)

$P(\text{only one of them selected})$
$= P((F \cap \bar{J}) \cup (\bar{F} \cap J))$
$= P(F)P(\bar{J}) + P(\bar{F})P(J)$
$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$
$= \frac{4+6}{35}$
$= \frac{10}{35}$
$= \frac{2}{7}$

Required probability = $\frac{2}{7}$

(iii)

$P(\text{None of them selected})$
$= P(\bar{F} \cap \bar{J})$
$= P(\bar{F})P(\bar{J})$
$= \frac{6}{7} \times \frac{4}{5}$
$= \frac{24}{35}$

Required probability = $\frac{24}{35}$

Probability Ex 31.5 Q31

Bag contains 3 blue, 5 red marble. One marble is drawn, its colour noted and replaced, then again a marble drawn and its colour is noted.

(i)

$$P\text{(Blue followed by red)}$$
$$= P(B \cap R)$$
$$= P(B)\,P(R)$$
$$= \frac{3}{8} \times \frac{5}{8}$$
$$= \frac{15}{64}$$

Required probability $= \dfrac{15}{64}$

(ii)

$$P\text{(Blue and red in any order)}$$
$$= P((B \cap R) \cup (R \cap B))$$
$$= P(B \cap R) + P(R \cap B)$$
$$= P(B)\,P(R) + P(R)\,P(B)$$
$$= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$$
$$= \frac{30}{64}$$
$$= \frac{15}{32}$$

Required probability $= \dfrac{15}{32}$

(iii)

$$P\text{(of the same colour)}$$
$$= P((R_1 \cap R_2) \cup (B_1 \cap B_2))$$
$$= P(R_1)\,P(R_2) + P(B_1)\,P(B_2)$$
$$= \frac{5}{8} \times \frac{5}{8} + \frac{3}{8} \times \frac{3}{8}$$
$$= \frac{25+9}{64}$$
$$= \frac{34}{64}$$
$$= \frac{17}{32}$$

Probability Ex 31.5 Q32

An urn contains 7 red and 4 blue balls.
Two balls are drawn with replacement.

(i)

P (Getting 2 red balls)

$= P(R_1 \cap R_2)$

$= P(R_1).P(R_2)$

$= \frac{7}{11} \times \frac{7}{11}$

$= \frac{49}{121}$

Required probability = $\frac{49}{121}$

(ii)

P (Getting 2 blue balls)

$= P(B_1 \cap B_2)$

$= P(B_1).P(B_2)$

$= \frac{4}{11} \times \frac{4}{11}$

$= \frac{16}{121}$

Required probability = $\frac{16}{121}$

(iii)

P (Getting one red and one blue ball)

$= P((R \cap B) \cup (B \cap R))$

$= P(R)P(B) + P(B)P(R)$

$= \frac{7}{11} \times \frac{4}{11} + \frac{4}{11} \times \frac{7}{11}$

$= \frac{28+28}{121}$

$= \frac{56}{121}$

Required probability = $\frac{56}{121}$

Probability Ex 31.5 Q33

A card is drawn, out come noted, the card is replaced, pack reshuffled, another card is drawn.

(i)

We know that, there are four suits club(C), spade(S), heart(H) diamand(D), each contains 13 cards.

$$P(\text{Both the cards are of same suit})$$
$$= P\left((C_1 \cap C_2) \cup (S_1 \cap S_2) \cup (H_1 \cap H_2) \cup (D_1 \cap D_2)\right)$$
$$= P(C_1 \cap C_2) + P(S_1 \cap S_2) + P(H_1 \cap H_2) + P(D_1 \cap D_2)$$
$$= P(C_1)P(C_2) + P(S_1)P(S_2) + P(H_1)P(H_2) + P(D_1)P(D_2)$$
$$= \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52} + \frac{13}{52}\cdot\frac{13}{52}$$
$$= \left(\frac{1}{4}\cdot\frac{1}{4}\right)^4$$
$$= \frac{1}{4}$$

Required probability $= \frac{1}{4}$

(ii)

We know that, there are four ace and 2 red queens.

$$P(\text{first card an ace and second card a red queen})$$
$$= P(\text{Getting an ace})\,P(\text{Getting a red queen})$$
$$= \frac{4}{52} \times \frac{2}{52}$$
$$= \frac{1}{338}$$

Required probability $= \frac{1}{338}$

Probability Ex 31.5 Q34

(i)

Out of 100 students two friends can enter the sections in ${}^{100}C_2$ ways.

Let A = event both enter in section A (40 students)

B = event both enter in section B (60 students)

$$P(A) = \frac{{}^{40}C_2}{{}^{100}C_2},\ P(B) = \frac{{}^{60}C_2}{{}^{100}C_2}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{{}^{40}C_2 + {}^{60}C_2}{{}^{100}C_2}$$

$$= \frac{\frac{40 \times 39}{2} + \frac{60 \times 59}{2}}{\frac{100 \times 99}{2}}$$

$$= \frac{780 + 1770}{4950}$$

$$= \frac{2550}{4950}$$

$$= \frac{17}{33}$$

$$P(\text{Both enter same section}) = \frac{17}{33}$$

(ii)

$P(\text{Both enter different section})$

$$= 1 - P(\text{Both enter same section})$$

$$= 1 - \frac{17}{33}$$

$$= \frac{16}{33}$$

$$P(\text{Both enter different section}) = \frac{16}{32}$$

Probability Ex 31.5 Q35

Probability of getting six in any toss of a dice = $\frac{1}{6}$

Probability of not getting six in any toss of a dice = $\frac{5}{6}$

A and B toss the die alternatively.

Hence probability of A's win

$$= P(A) + P(\overline{A}\overline{B}A) + P(\overline{A}\overline{B}\overline{A}\overline{B}A) + P(\overline{A}\overline{B}\overline{A}\overline{B}\overline{A}\overline{B}A) + \ldots\ldots.$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots..$$

$$= \frac{1/6}{1-(5/6)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Similarly, probability of B's win

$$= P(\overline{A}B) + P(\overline{A}\overline{B}\overline{A}B) + P(\overline{A}\overline{B}\overline{A}\overline{B}\overline{A}B) + \ldots\ldots$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{\frac{5}{6} \times \frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

Since the probabilities are not equal,

the decision of the refree was not a fair one.