# RD Sharma Solutions Class 12 Maths Chapter 31 Ex 31.7

## Probability Ex 31.7 Q1

Urn I contains 1 white, 2 balck and 3 red balls

Urn II contains 2 white, 1 black and 1 red balls

Urn III contains 4 white, 5 black and 3 red balls.

Consider $E_1, E_2, E_2$ and $A$ be events as:-

$E_1$ = Selecting unr I

$E_2$ = Selecting urn II

$E_3$ = Selecting urn III

$A$ = Drawing 1 white and 1 red balls

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are 3 urns]

$P(A \mid E_1) = P$ [Drawing 1 red and 1 white from urn I]

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$P\left(\frac{A}{E_2}\right) = P$ [Drawing 1 red and 1 white from urn II]

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$P\left(\frac{A}{E_3}\right) = P$ [Drawing 1 red and 1 white from urn III]

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

We have to find,

$$P\,(\text{They come from urn I}) = P\left(\frac{E_1}{A}\right)$$

$$P\,(\text{They come from urn II}) = P\left(\frac{E_2}{A}\right)$$

$$P\,(\text{They come from urn III}) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$p\left(\frac{E_1}{A}\right) = \frac{P\,(E_1)\,P\left(\frac{A}{E_1}\right)}{P\,(E_1)\,P\left(\frac{A}{E_1}\right) + P\,(E_2)\,P\left(\frac{A}{E_2}\right) + P\,(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{36+55+30}{165}}$$

$$= \frac{1}{5}\times\frac{165}{118}$$

$$= \frac{33}{118}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{33+55+30}{165}}$$

$$= \frac{1}{3}\times\frac{165}{118}$$

$$= \frac{55}{118}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{2}{11}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{2}{11}\times\frac{165}{118}$$

$$= \frac{30}{118}$$

Therefore, required probability = $\frac{33}{118}, \frac{55}{118}, \frac{30}{118}$.

## Probability Ex 31.7 Q2

Bag $A$ contains 2 white and 3 red balls

Bag $B$ contains 4 white and 5 red balls.

Consider $E_1, E_2$ and $A$ events as:-

$E_1$ = Selecting bag $A$

$E_2$ = Selecting bag $B$

$A$ = Drawing one red ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing one red ball from bag } A]$$

$$= \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing one red ball from bag } B]$$

$$= \frac{5}{9}$$

To find,

$$P(\text{Drawn, one red ball is from bag } B) = P\left(\frac{E_2}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{3}{5} + \frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{27+25}{45}}$$

$$= \frac{5}{9} \times \frac{45}{52} = \frac{25}{52}$$

Required probability $= \frac{25}{52}$.

## Probability Ex 31.7 Q3

Urn I contains 2 white and 3 black balls
Urn II contains 3 white and 2 black balls
Urn III contains 4 white and 1 black balls
Let $E_1, E_2, E_3$ and $A$ be events as:-

$E_1$ = Selecting urn I
$E_2$ = Selecting urn II
$E_3$ = Selecting urn III
$A$ = A white balls is drawn

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

$$P(E_3) = \frac{1}{2} \qquad \text{[Since there are 3 urns]}$$

$$P(A \mid E_1) = P[\text{Drawing 1 white ball from urn I}]$$
$$= \frac{2}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 white ball from urn II}]$$
$$= \frac{3}{5}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing one whtie ball from urn III}]$$
$$= \frac{4}{5}$$

To find,

$$P(\text{Drawn one white ball from urn I}) = P\left(\frac{E_1}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{10}}{\frac{2+3+4}{10}}$$

$$= \frac{2}{9}$$

Required probability = $\frac{2}{9}$.

## Probability Ex 31.7 Q4

Urn I contains 7 white and 3 black balls

Urn II contains 4 white and 6 black balls

Urn III contains 2 white and 8 black balls

Let $E_1, E_2, E_3$ and $A$ be events as:-

$E_1$ = Selecting urn I

$E_2$ = Selecting urn II

$E_3$ = Selecting urn III

$A$ = Drawing 2 white balls without replacement.

Given,

$$P\left(E_1\right) = 0.20$$

$$P\left(E_2\right) = 0.60$$

$$P\left(E_3\right) = 0.20$$

$P\left(A \mid E_1\right) = P$[Drawing 2 white ball from urn I]

$$= \frac{{}^{7}C_2}{{}^{10}C_2}$$

$$= \frac{\frac{7\times 6}{2}}{\frac{10\times 9}{2}}$$

$$= \frac{7}{15}$$

$P\left(\frac{A}{E_2}\right) = P$[Drawing 2 white ball from urn II]

$$= \frac{{}^{4}C_2}{{}^{10}C_2}$$

$$= \frac{\frac{4\times 3}{2}}{\frac{10\times 9}{2}}$$

$$= \frac{12}{90}$$

$$= \frac{2}{15}$$

$$P\left(\frac{A}{E_3}\right) = P\left[\text{Drawing 2 whtie ball from urn III}\right]$$

$$= \frac{{}^{2}C_2}{{}^{10}C_2}$$

$$= \frac{1}{\frac{10 \times 9}{2}}$$

$$= \frac{1}{45}$$

To find,

$$P\,(\text{2 white balls drawn are from urn III}) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_3}{A}\right) = \frac{P\,(E_3)\,P\left(\frac{A}{E_3}\right)}{P\,(E_1)\,P\left(\frac{A}{E_1}\right) + P\,(E_2)\,P\left(\frac{A}{E_2}\right) + P\,(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.2 \times \frac{1}{45}}{0.2 \times \frac{7}{15} + 0.6 \times \frac{2}{15} + 0.2 \times \frac{1}{45}}$$

$$= \frac{\frac{2}{450}}{\frac{14}{150} + \frac{12}{150} + \frac{2}{450}}$$

$$= \frac{\frac{2}{450}}{\frac{42 + 36 + 2}{450}}$$

$$= \frac{2}{80}$$

$$= \frac{1}{40}$$

Required probability $= \frac{1}{40}$.

## Probability Ex 31.7 Q5

Consider the following events:

$E_1$ = Getting 1 or 2 in a throw of die,

$E_2$ = Getting 3, 4, 5 or 6 in a throw of die,

A = Getting exactly one tail

Clearly,

$$P(E_1) = \frac{2}{6} = \frac{1}{3}, P(E_2) = \frac{4}{6} = \frac{2}{3}, P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$$

Required probablity $= P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{8}{11}$$

## Probability Ex 31.7 Q6

Consider the following events:

$E_1$ = First group wins, $E_2$ = Second group wins, $A$ = New product is introduced.

It is given that

$$P(E_1)=0.6, P(E_2)=0.4, P(A/E_1)=0.7, P(A/E_2)=0.3$$

Required probability $=P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

$$=\frac{0.4\times 0.3}{0.6\times 0.7+0.4\times 0.3}=\frac{12}{54}=\frac{2}{9}$$

Hence required probability is $\frac{2}{9}$

## Probability Ex 31.7 Q7

Given, 5 man out of 100 and 25 women out of 1000 are good orators.

Consider $E_1, E_2$ and $A$ events as:-

$E_1$ = Selected person is male

$E_2$ = Selected perosn is female

$E_3$ = Selected person is an orator

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since number of males and females are equal]

$P(A \mid E_1) = P$ (Selecting a male orator)

$$= \frac{5}{100}$$

$$= \frac{1}{20}$$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a frmale orator)

$$= \frac{25}{1000}$$

$$= \frac{1}{40}$$

To find, $P$ (Orator selected is a male) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{20}}{\frac{1}{2}\times\frac{1}{20}+\frac{1}{2}\times\frac{1}{40}}$$

$$= \frac{\frac{1}{40}}{\frac{1}{40}+\frac{1}{80}}$$

$$= \frac{1}{40}\times\frac{80}{3}$$

$$= \frac{2}{3}$$

Required probability $= \frac{2}{3}$.

## Probability Ex 31.7 Q8

Consider events $E_1, E_2$ and $A$ events as:-

$E_1$ = Letters come from LONDON

$E_2$ = Letters come from CLIFTON

$E_3$ = Two consecutive letters visible on the envelope are ON

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

[Since letters came either from LONDON or CLIFTON]

$$P(A \mid E_1) = P(\text{Two consecutive letters ON from LONDON})$$
$$= \frac{2}{5}$$

[Since LONDON has 2-ON and 5 letters we consider one 'ON' as one letter]

$$P\left(\frac{A}{E_2}\right) = P(\text{Two consecutive letters ON from CLIFTON})$$
$$= \frac{1}{6}$$

[Since CLIFTON has one 'ON' nad 6 letters considering ON as one letter]

(i) To find, $P(\text{ON visible are from LONDON}) = P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{1}{6}}$$

$$= \frac{\frac{2}{10}}{\frac{2}{10} + \frac{1}{12}}$$

$$= \frac{2}{10} \times \frac{60}{17}$$

$$= \frac{12}{17}$$

$$P\left(\frac{E_1}{A}\right) = \frac{12}{17}$$

Required probability $= \frac{12}{17}$

(ii) $$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{6}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{1}{6}}$$

$$= \frac{\frac{1}{12}}{\frac{2}{10} + \frac{1}{12}}$$

$$= \frac{1}{12} \times \frac{60}{17}$$

$$= \frac{5}{17}$$

Required probability $= \frac{5}{17}$.

## Probability Ex 31.7 Q9

Consider $E_1, E_2$ and $A$ events as:-

$E_1$ = Selected student is boy

$E_2$ = Selected student is girl

$E_3$ = A student with IQ more that 150 is selected

$$P(E_1) = \frac{60}{100}$$

$$P(E_2) = \frac{40}{100}$$

$P(A \mid E_1) = P$ (Selected boy has IQ more than 150)

$$= \frac{5}{100}$$

$P\left(\frac{A}{E_2}\right) = P$ (Selected girl has IQ more than 150)

$$= \frac{10}{100}$$

To find, $P$ (Selected student with IQ more than 150 is a boy) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{5}{100} + \frac{40}{100} \times \frac{10}{100}}$$

$$= \frac{300}{300 + 400}$$

$$= \frac{300}{700}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

## Probability Ex 31.7 Q10

Consider $E_1, E_2, E_3$ and $A$ as:-

$E_1$ = Bolt produced by machine $X$

$E_2$ = Bolt produced by machine $Y$

$E_3$ = Bolt produced by machine $Z$

$A$ = A bolt drawn is defective.

$$P\left(E_1\right) = \frac{1000}{6000} = \frac{1}{6}$$

$$P\left(E_2\right) = \frac{2000}{6000} = \frac{1}{3}$$

$$P\left(E_3\right) = \frac{3000}{6000} = \frac{1}{2}$$

$$P\left(A \mid E_1\right) = P\left(\text{Drawing defective bolt from machine } X\right)$$

$$= \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P\left(\text{Drawing defective bolt from machine } Y\right)$$

$$= \frac{1.5}{100}$$

$$= \frac{3}{200}$$

$$P\left(\frac{A}{E_3}\right) = P\left(\text{Drawing defective bolt from machine } Z\right)$$

$$= \frac{2}{100}$$

To find, $P\left(\text{Defective bolt drawn is produced by machine } X\right) = P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right)P\left(\frac{A}{E_1}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right) + P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{600}}{\frac{1}{600} + \frac{3}{600} + \frac{1}{100}}$$

$$= \frac{1}{10}$$

Required probability $= \frac{1}{10}$.

## Probability Ex 31.7 Q11

Let $E_1, E_2, E_3$ and $A$ be the events defined as follows

$E_1$ = scooters

$E_2$ = cars

$E_3$ = trucks

$A$ = vehicle meet with an accident

Since there are 12000 vehicles, therefore

$$P(E_1)=\frac{3000}{12000}=\frac{1}{4}, P(E_2)=\frac{4000}{12000}=\frac{1}{3}, P(E_3)=\frac{5000}{12000}=\frac{5}{12}$$

It is given that $P(A/E_1)$ =Probability that the accident involves a scooter $=0.02$

Similarly $P(A/E_2)=0.03$ and $P(A/E_3)=0.04$

(i)

We are required to find $P(E_1/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{4}\times 0.02}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{3}{19}$$

(ii)

We are required to find $P(E_2/A)$ i.e. given that the vehicle meet with an accident is a car

By Baye's rule

$$P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{3}\times 0.03}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{6}{19}$$

(iii)

We are required to find $P(E_3/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$P(E_3/A)=\frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{5}{12}\times 0.04}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04}$$

$$=\frac{10}{19}$$

## Probability Ex 31.7 Q12

We need to find

$$P\left(\frac{A}{\text{Red}}\right), P\left(\frac{B}{\text{Red}}\right), P\left(\frac{C}{\text{Red}}\right)$$

Now,

$$P\left(\frac{A}{\text{Red}}\right) = \frac{P\left(\frac{\text{Red}}{A}\right)P(A)}{P\left(\frac{\text{Red}}{A}\right)P(A)+P\left(\frac{\text{Red}}{B}\right)P(B)+P\left(\frac{\text{Red}}{C}\right)P(C)+P\left(\frac{\text{Red}}{D}\right)P(D)}$$

$$= \frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0}$$

$$= \frac{1}{1+6+8} = \frac{1}{15}$$

Similarly

$$P\left(\frac{B}{\text{Red}}\right) = \frac{6}{15}$$

$$P\left(\frac{C}{\text{Red}}\right) = \frac{8}{15}$$

## Probability Ex 31.7 Q13

Let $E_1$, $E_2$, and $E_3$ be the respective events of the time consumed by machines A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$

$$P(X|E_2) = 5\% = \frac{5}{100}$$

$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by P ($E_1|A$).

By using Bayes' theorem, we obtain

$$P(E_1|X)=\frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1)+P(E_2)\cdot P(X|E_2)+P(E_3)\cdot P(X|E_3)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100}+\frac{3}{10}\cdot\frac{5}{100}+\frac{1}{5}\cdot\frac{7}{100}}$$

$$=\frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2}+\frac{3}{2}+\frac{7}{5}\right)}$$

$$=\frac{\frac{1}{2}}{\frac{17}{5}}$$

$$=\frac{5}{34}$$

## Probability Ex 31.7 Q14

Consider the following events:

$E_1$ = Item is produced by machine A,

$E_2$ = Item is produced by machine B,

$E_3$ = Item is produced by machine C,

A = Item is defective

Clearly,

$$P(E_1)=\frac{50}{100}=\frac{1}{2}, P(E_2)=\frac{30}{100}=\frac{3}{10}, P(E_2)=\frac{20}{100}=\frac{1}{5}$$

$$P(A/E_1)=\frac{2}{100}, P(A/E_2)=\frac{2}{100}, P(A/E_2)=\frac{3}{100}$$

Re quired probablity $= P(E_1/A)$

$$=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{2}\times\frac{2}{100}}{\frac{1}{2}\times\frac{2}{100}+\frac{3}{10}\times\frac{2}{100}+\frac{1}{5}\times\frac{3}{100}}$$

$$=\frac{5}{11}$$

## Probability Ex 31.7 Q15

Let $E_1, E_2, E_3$ be the events that we choose the first coin, second coin, and third coin respectively in a random toss.

$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$

Let A denote the event when the toss shows heads.

It is given that

$P(A/E_1) = 1, P(A/E_2) = 0.75, P(A/E_3) = .60$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{1}{3}(1)}{\frac{1}{3}(1) + \frac{1}{3}(0.75) + \frac{1}{3}(0.60)} = \frac{1/3}{(1/3) + (1/4) + (1/5)}$$

$$= \frac{1/3}{47/60} = \frac{20}{47}$$

## Probability Ex 31.7 Q16

Consider events $E_1, E_2, E_3$ and $A$ as:-

$E_1$ = Selecting product from machine $A$

$E_2$ = Selecting product from machine $B$

$E_3$ = Selecting product from machine $C$

$A$ = Selecting a standard quality product

$$P(E_1) = \frac{30}{100}$$

$$P(E_2) = \frac{25}{100}$$

$$P(E_3) = \frac{45}{100}$$

$$P(A \mid E_1) = P\,(\text{Selecting defective product from machine } A)$$

$$= \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P\,(\text{Selecting defective product from machine } B)$$

$$= \frac{1.2}{100}$$

$$P\left(\frac{A}{E_3}\right) = P\,(\text{Selecting defective product from machine } C)$$

$$= \frac{2}{100}$$

To find, $P$ (Selecting defective product is produced by machine $B$)

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{12}{1000}}{\frac{30}{100} \times \frac{1}{100} + \frac{25}{100} \times \frac{12}{1000} + \frac{45}{100} \times \frac{2}{100}}$$

$$= \frac{300}{300 + 300 + 900}$$

$$= \frac{300}{1500}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

**Probability Ex 31.7 Q17**

Let $E_1, E_2$ and $A$ be events as:-

$E_1$ = Selecting bicycle from first plant

$E_2$ = Selecting bicycle from second plant

$A$ = Selecting a standard quality bicycle

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A \mid E_1) = P$ (Selecting standard quality bicycle from first plant)

$= \frac{80}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting standard quality bicycle from second plant)

$= \frac{90}{100}$

To find, $P$ (Selected standard quality bicycle is from second plant) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{40}{100} \times \frac{90}{100}}{\frac{60}{100} \times \frac{80}{100} + \frac{40}{100} \times \frac{90}{100}}$$

$$= \frac{3600}{4800 + 3600}$$

$$= \frac{3600}{8400}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

## Probability Ex 31.7 Q18

Urn $A$ contains 6 red and 4 white balls
Urn $B$ contains 2 red and 6 white balls
Urn $C$ contains 1 red and 5 white balls
Consider $E_1, E_2, E_3$ and $A$ events as:-

$E_1$ = Selecting urn $A$
$E_2$ = Selecting urn $B$
$E_3$ = Selecting urn $C$
$A$ = Selecting a red ball

$$P(E_1) = \frac{1}{3}$$

$$P(E_2) = \frac{1}{3}$$

$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are three urns]}$$

$$P(A \mid E_1) = P(\text{Selecting a red ball from urn } A) = \frac{6}{10} = \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Selecting a red ball from urn } B) = \frac{2}{8} = \frac{1}{4}$$

$$P\left(\frac{A}{E_3}\right) = P(\text{Selecting a red ball from urn } C) = \frac{1}{6}$$

To find, $P(\text{Selected red ball is from urn } A) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{6}}$$

**Probability Ex 31.7 Q19**

Let $E_1, E_2, E_3$ be the events that the people are smokers and non-vegetarian, smokers and vegetarian, and non-smokers and vegetarian respectively.

$$P(E_1) = \frac{2}{5}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{7}{20}$$

Let A denote the event that the person has the special chest disease.

It is given that

$$P(A/E_1) = 0.35, P(A/E_2) = 0.20, P(A/E_3) = 0.10$$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{2}{5}(0.35)}{\frac{2}{5}(0.35) + \frac{1}{4}(0.20) + \frac{7}{20}(0.10)} = \frac{7/50}{(7/50) + (1/20) + (7/200)}$$

$$= \frac{7/50}{9/40} = \frac{28}{45}$$

## Probability Ex 31.7 Q20

Let $E_1, E_2, E_3$ and $A$ be events as:-

$E_1$ = Selecting product from machine $A$

$E_2$ = Selecting product from machine $B$

$E_3$ = Selecting product from machine $C$

$A$ = Selecting a defective product

$$P(E_1) = \frac{100}{600} = \frac{1}{6}$$

$$P(E_2) = \frac{200}{600} = \frac{1}{3}$$

$$P(E_3) = \frac{300}{600} = \frac{1}{2}$$

$$P(A \mid E_1) = P(\text{Selecting a defective item from machine } A) = \frac{2}{100}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Selecting a defective item from machine } B) = \frac{3}{100}$$

$$P\left(\frac{A}{E_3}\right) = P(\text{Selecting a defective item machine } C) = \frac{5}{100}$$

To find, $P(\text{Selected defective item is produced by machine } A) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{100}}{\frac{1}{6} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{5}{100}}$$

$$= \frac{\frac{2}{600}}{\frac{2}{600} + \frac{3}{300} + \frac{5}{200}}$$

$$= \frac{2}{600} \times \frac{600}{23}$$

$$= \frac{2}{23}$$

Required probability $= \frac{2}{23}$.

## Probability Ex 31.7 Q21

Bag I contains 1 white and 6 red balls
Bag II contains 4 white and 3 red balls
Let $E_1, E_2$ and $A$ events be:-

$E_1$ = Selecting bag I
$E_2$ = Selecting bag II
$A$ = Selecting a white ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are two bags]}$$

$$P(A \mid E_1) = P\,(\text{Selecting 1 white ball from bag I}) = \frac{1}{7}$$

$$P\left(\frac{A}{E_2}\right) = P\,(\text{Selecting 1 white ball from bag II}) = \frac{4}{7}$$

To find, $P\,(\text{Drawn white ball is from bag I}) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7}+\frac{1}{2}\times\frac{4}{7}}$$

$$= \frac{\frac{1}{14}}{\frac{1}{14}+\frac{4}{14}}$$

$$= \frac{1}{5}$$

Required probability = $\frac{1}{5}$.

## Probability Ex 31.7 Q22

Consider the following events

$E_1 =$ The selected student is a girl

$E_2 =$ The selected student is not a girl

$A =$ The student is taller than 1.75 meters

We have,

$$P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$$

$P(A/E_1) =$ Probability that the student is taller than 1.75 meters given that the student is a girl

$$P(A/E_1) = \frac{1}{100} = 0.01$$

And

$P(A/E_2) =$ Probability that the student is taller than 1.75 meters given that the student is not a girl

$$P(A/E_2) = \frac{4}{100} = 0.04$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.6 \times 0.01}{0.6 \times 0.01 + 0.4 \times 0.04}$$

$$= \frac{\frac{6}{1000}}{\frac{22}{1000}}$$

$$= \frac{3}{11}$$

## Probability Ex 31.7 Q23

Let $E_1, E_2, E_3$ and $A$ be events as:-

$E_1$ = $A$ is appointed

$E_2$ = $B$ is appointed

$E_3$ = $C$ is appointed

$A$ = A change does take place

$P(E_1) = \frac{4}{7}$

$P(E_2) = \frac{1}{7}$

$P(E_3) = \frac{2}{7}$

$P(A \mid E_1) = P$ (Changes take place by $A$)

$= 0.3$

$P\left(\frac{A}{E_2}\right) = P$ (Changes take place by $B$)

$= 0.8$

$P\left(\frac{A}{E_3}\right) = P$ (Changes take place by $C$)

$= 0.5$

To find, $P$ (Changes were taken place by $B$ or $C$) $= P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{7} \times \frac{8}{10} + \frac{2}{7} \times \frac{5}{10}}{\frac{4}{7} \times \frac{3}{10} + \frac{1}{7} \times \frac{8}{10} + \frac{2}{7} \times \frac{5}{10}}$$

$$= \frac{\frac{18}{70}}{\frac{30}{70}}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$.

## Probability Ex 31.7 Q24

Let $E_1, E_2$ and $A$ be events as:-

$E_1$ = Vehicle is scooter

$E_2$ = Vehicle is motorcycle

$A$ = An insured met with accident

$$P(E_1) = \frac{2000}{5000} = \frac{2}{5}$$

$$P(E_2) = \frac{3000}{5000} = \frac{3}{5}$$

$$P(A \mid E_1) = P(\text{Accident of scooter}) = 0.01$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Accident of motorcycle}) = 0.02$$

To find, $P(\text{Accident vehicle was motorcycle}) = P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5} \times \frac{2}{100}}{\frac{2}{5} \times \frac{1}{100} + \frac{3}{5} \times \frac{2}{100}}$$

$$= \frac{\frac{6}{500}}{\frac{2}{500} + \frac{6}{500}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$.

## Probability Ex 31.7 Q25

Consider the following events

$E_1$ = The selected student is a hosteller

$E_2$ = The selected student is not a hosteller.

$A$ = The student has an A grade.

We have,

$$P(E_1) = 30\% = \frac{30}{100} = 0.3$$

$$P(E_2) = 20\% = \frac{20}{100} = 0.2$$

$P(A/E_1)$ = Probability that the student has an A grade given that the student is a hosteller

$$P(A/E_1) = \frac{60}{100} = 0.6$$

And

$P(A/E_2)$ = Probability that the student has an A grade given that the student is not a hosteller

$$P(A/E_2) = \frac{40}{100} = 0.4$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.3 \times 0.6}{0.3 \times 0.6 + 0.2 \times 0.4}$$

$$= \frac{\frac{18}{100}}{\frac{26}{100}}$$

$$= \frac{9}{13}$$

## Probability Ex 31.7 Q26

Let $E_1$, $E_2$, and $E_3$ be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$$\therefore P(A|E_1) = P(\text{coin showing heads, given that it is a two-headed coin}) = 1$$

Probability of heads coming up, given that it is a biased coin= 75%

$$\therefore P(A|E_2) = P(\text{coin showing heads, given that it is a biased coin}) = \frac{75}{100} = \frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$\therefore P(A|E_3) = P(\text{coin showing heads, given that it is an unbiased coin}) = \frac{1}{2}$$

The probability that the coin is two-headed, given that it shows heads, is given by

P ($E_1$|A).

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)}$$

$$=\frac{1}{\frac{9}{4}}$$

$$=\frac{4}{9}$$

## Probability Ex 31.7 Q27

Let A, $E_1$, and $E_2$ respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A)=0.40$$

$$P(E_1)=P(E_2)=\frac{1}{2}$$

$$P(A|E_1)=0.40\times 0.70=0.28$$

$$P(A|E_2)=0.40\times 0.75=0.30$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P $(E_1|A)$.

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$=\frac{14}{29}$$

## Probability Ex 31.7 Q28

We need to find

$$P\left(\frac{\text{Box III}}{\text{Black}}\right)$$

$$= \frac{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III})}{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III}) + P\left(\frac{\text{Black}}{\text{Box II}}\right)P(\text{Box II}) + P\left(\frac{\text{Black}}{\text{Box I}}\right).P(\text{Box I}) + P\left(\frac{\text{Black}}{\text{Box Iv}}\right).P(\text{Box Iv})}$$

$$= \frac{\frac{1}{7} \times \frac{1}{4}}{\frac{1}{7} \times \frac{1}{4} + \frac{2}{8} \times \frac{1}{4} + \frac{3}{18} \times \frac{1}{4} + \frac{4}{13} \times \frac{1}{4}}$$

$$= \frac{\frac{1}{7}}{\frac{1}{7} + \frac{1}{4} + \frac{1}{6} + \frac{4}{13}}$$

$$= \frac{1}{7} \times \frac{7 \times 4 \times 6 \times 13}{4 \times 6 \times 13 + 7 \times 6 \times 13 + 7 \times 4 \times 13 + 7 \times 4 \times 6}$$

$$= \frac{4 \times 6 \times 13}{4 \times 6 \times 13 + 7 \times 6 \times 13 + 7 \times 4 \times 13 + 7 \times 4 \times 6}$$

$= 0.165$

## Probability Ex 31.7 Q29

Let A be the event that the machine produces 2 acceptable items.

Also let $B_1$ be the event of correct set up and $B_2$ represent the event of incorrect set up.

Now, $P(B_1) = 0.8$, $P(B_2) = 0.2$

$P(A/B_1) = 0.9 \times 0.9$ and $P(A/B_2) = 0.4 \times 0.4$

Therefore, $$P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{0.8 \times 0.9 \times 0.9}{0.8 \times 0.9 \times 0.9 + 0.2 \times 0.4 \times 0.4} = \frac{648}{680} = 0.95$$

## Probability Ex 31.7 Q30

Consider events $E_1, E_2$ and $A$ as

$E_1$ = The person selected is actually having T.B.

$E_2$ = The person selected is not having T.B.

$E_3$ = The person diagnosed to have T.B.

Given,

$$P(E_1) = \frac{1}{1000}$$

$$P(E_2) = \frac{999}{1000}$$

$P(A \mid E_1) = P$ (Person diagnosed to have T.B. and he is actually having T.B.)

$= 0.99$

$P\left(\frac{A}{E_2}\right) = P$ (Person diagnosed to have T.B. and he is not a actually having T.B.)

$= 0.001$

To find, $P$ (Person diagnosed to have T.B. is actually having T.B.) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{1000} \times 0.99}{\frac{1}{1000} \times 0.99 + \frac{999}{1000} \times 0.001}$$

$$= \frac{990}{990 + 999}$$

$$= \frac{990}{1989}$$

$$= \frac{110}{221}$$

Required probability $= \frac{110}{221}$.

## Probability Ex 31.7 Q31

Consider events $E_1, E_2$ and $A$ as:-

$E_1$ = The selected person actually has disease

$E_2$ = The selected person has no disease

$A$ = Selected person has disease

$$P(E_1) = \frac{0.2}{100}$$

$$= \frac{2}{1000}$$

$$P(E_2) = \frac{998}{1000}$$

$$P(A \mid E_1) = \frac{90}{100}$$

$$P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

To find, $P$ (Person has disease is actually diseased) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) P\left(\frac{A}{E_1}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{1000} \times \frac{90}{100}}{\frac{2}{1000} \times \frac{90}{100} + \frac{998}{1000} \times \frac{1}{100}}$$

$$= \frac{180}{180 + 998}$$

$$= \frac{180}{1178}$$

$$= \frac{90}{589}$$

Required probability $= \frac{90}{589}$.

**Probability Ex 31.7 Q32**

Let $E_1, E_2, E_3$ and $A$ be events as:-

$E_1$ = Patient has disease $d_1$

$E_2$ = Patient has disease $d_2$

$E_3$ = Patient has disease $D_3$

$A$ = Selected patient has symptom $S$.

$P(E_1) = \frac{1800}{5000} = \frac{18}{50}$

$P(E_2) = \frac{2100}{5000} = \frac{21}{50}$

$P(E_3) = \frac{1100}{5000} = \frac{11}{50}$

$P(A \mid E_1) = P$ (Patient with disease $d_1$ and shows symptom $S$)

$= \frac{1500}{1800}$

$= \frac{5}{6}$

$P\left(\frac{A}{E_2}\right) = P$ (Patient with disease $d_2$ and symptom $S$)

$= \frac{1200}{2100}$

$= \frac{4}{7}$

$P\left(\frac{A}{E_3}\right) = P$ (Patient with disease $d_3$ and symptom $S$)

$= \frac{900}{1100}$

$= \frac{9}{11}$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{5}{6} \times \frac{18}{50}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{3}{10} \times \frac{50}{36}$$

$$= \frac{5}{12}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{21}{50} \times \frac{4}{7}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{6}{25}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{6}{25} \times \frac{50}{36}$$

$$= \frac{1}{3}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{11}{50} \times \frac{9}{11}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{9}{50}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{9}{50} \times \frac{50}{36}$$

$$= \frac{1}{4}$$

So, probabilities of $d_1, d_2, d_3$ diseases are $\frac{5}{12}, \frac{1}{3}, \frac{1}{4}$ respectively.

Hence, the patient is most likely to have $d_1$ diseased.

## Probability Ex 31.7 Q33

Let $E_1, E_2$ and $A$ be events as:-

$E_1$ = 1 occurs on die

$E_2$ = 1 does not occur on die

$A$ = The man reports that it is one

$$P(E_1) = \frac{1}{6}$$

$$P(E_2) = \frac{5}{6}$$

$$P\left(\frac{A}{E_1}\right) = P\,(\text{He reports one when occurs on die})$$

$$= P\,(\text{He speaks truth})$$

$$= \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P\,(\text{He reports one when 1 has not occured})$$

$$= P\,(\text{He does not speak truth})$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

To find, $P\,(\text{It is actually 1 when he reported that it is one on die}) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{5}}{\frac{1}{6} \times \frac{3}{5} + \frac{5}{6} \times \frac{2}{5}}$$

$$= \frac{\frac{3}{30}}{\frac{3}{30} + \frac{10}{30}}$$

$$= \frac{3}{13}$$

Required probability $= \frac{3}{13}$.

## Probability Ex 31.7 Q34

Let $E_1, E_2$ and $A$ events be as:-

$E_1$ = 5 occurs on die

$E_2$ = 5 does not occur on die

$A$ = He reports that it was 5

$$P(E_1) = \frac{1}{6}$$

$$P(E_2) = \frac{5}{6}$$

$$P(A \mid E_1) = P(\text{He reports 5 when 5 occurs on die})$$
$$= P(\text{He speaks truth})$$
$$= \frac{8}{10}$$
$$= \frac{4}{5}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{He reports 5 when 5 does not occur on die})$$
$$= P(\text{He does not speak truth})$$
$$= \frac{1}{5}$$

To find, $P(\text{It was actually 5 when he reports that it is five}) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}$$
$$= \frac{\frac{4}{30}}{\frac{4}{30}+\frac{5}{30}}$$
$$= \frac{4}{9}$$

Required probability $= \frac{4}{9}$.

## Probability Ex 31.7 Q35

$P(\text{Knows}) = \frac{3}{4}$

$P(\text{Guesses}) = \frac{1}{4}$

$P\left(\frac{\text{Correct}}{\text{Guesses}}\right) = \frac{1}{4}$

We need to find

$$P\left(\frac{\text{Knows}}{\text{Correctly}}\right) = \frac{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows})}{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows}) + P\left(\frac{\text{Correctly}}{\text{Guesses}}\right)P(\text{Guesses})}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{12}{13}$$

## Probability Ex 31.7 Q36

Let $E_1$ and $E_2$ be the respective events that a person has a disease and a person has no disease.

Since $E_1$ and $E_2$ are events complimentary to each other,

$P(E_1) + P(E_2) = 1$

$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$$P(A|E_1) = P(\text{result is positive given the person has disease}) = 99\% = 0.99$$

$$P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{0.001\times 0.99}{0.001\times 0.99+0.999\times 0.005}$$

$$=\frac{0.00099}{0.00099+0.004995}$$

$$=\frac{0.00099}{0.005985}$$

$$=\frac{990}{5985}$$

$$=\frac{110}{665}$$

$$=\frac{22}{133}$$