RD Sharma Solutions Class 12 Maths Chapter 33 Ex 33.2

Binomial Distribution Ex 33.2 Q1

Let X be a binomial variate with parameters n and p.

$$\text{Mean} = np$$
$$\text{Varience} = npq$$

$$\begin{aligned}\text{Mean} - \text{Variance} &= np - npq \\ &= np(1-q) \\ &= np.p \\ &= np^2\end{aligned}$$

$$\text{Mean} - \text{Variance} > 0$$
$$\text{Mean} > \text{Variance}$$

So, mean can never be less than varience.

Binomial Distribution Ex 33.2 Q2

Let X denote the variance with parameters n and p

$$p + q = 1$$
$$q = 1 - p$$

Goven,Mean $= np = 9$ ---(i)

Variance $= npq = \frac{9}{4}$ ---(ii)

$$\frac{npq}{np} = \frac{\frac{9}{4}}{9}$$ [By dividing (i) by (ii)]

$$q = \frac{1}{4}$$

So, $p = 1 - q$

$$= 1 - \frac{1}{4}$$

$$p = \frac{3}{4}$$

Put p in equation (i),

$$n\left(\frac{3}{4}\right) = 9$$

$\Rightarrow$ $n = \frac{36}{3}$

So, $n = 12$

The distribution is given by

$$= {}^nC_r p^r (q)^{n-r}$$

$$P(X = r) = {}^{12}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r}$$

for $r = 0, 1, 2, ..12$

Binomial Distribution Ex 33.2 Q3

Let n and p be parameters of binomial distribution. Here

$$\text{Mean} = np = 9 \qquad ---(i)$$

$$\text{Variance} = npq = 6 \qquad ---(ii)$$

$$\frac{npq}{np} = \frac{6}{9}$$

$$q = \frac{2}{3}$$

So, $$p = 1 - \frac{2}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$p = \frac{1}{3}$$

Using equation (i), $np = 9$

$$n\left(\frac{1}{3}\right) = 9$$

$$n = 27$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$$

$$r = 0, 1, 2, \ldots 27$$

Binomial Distribution Ex 33.2 Q4

Given that,

$n = 5$

Also, Mean + Varience = 4.8

$np + npq = 4.8$

$np(1+q) = 4.8$

$5p(1+q) = 4.8$

$5(1-q)(1+q) = 4.8$ [Since $p + q = 1$]

$5(1-q^2) = 4.8$

$1 - q^2 = \frac{4.8}{5}$

$q^2 = 1 - \frac{4.8}{5}$

$= \frac{0.2}{5}$

$q^2 = \frac{1}{25}$

$q = \frac{1}{5}$

$\Rightarrow$ $p = 1 - q$

$= 1 - \frac{1}{5}$

$p = \frac{4}{5}$

So, $n = 5,\ p = \frac{4}{5},\ q = \frac{1}{5}$

Here binomial distribution is

$P(X = r) = {}^nC_r p^r q^{n-r}$

$P(X = r) = 5C_r \left(\frac{4}{5}\right)^r \left(\frac{1}{5}\right)^{5-r}$

$r = 0, 1, 2, 3, \ldots 5$

Binomial Distribution Ex 33.2 Q5

Given that,

$$\text{Mean} = np = 20 \quad ---(i)$$

$$\text{Variance} = npq = 16 \quad ---(ii)$$

Let n and p be the parameters of distribution dividing equation (ii) by (i)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put p in equation (i),

$$np = 20$$

$$n\left(\frac{1}{5}\right) = 20$$

$$n = 20 \times 5$$

$$n = 100$$

So, binomial distribution is given by

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{100}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{100-r}$$

$$r = 0, 1, 2, 3, \ldots 100$$

Binomial Distribution Ex 33.2 Q6

Let n and p be the parameters of distribution binomial distribution. So

$q = 1 - p \qquad \text{as } p + q = 1$

$$\text{Mean} + \text{Variance} = \frac{25}{3}$$

$$np + npq = \frac{25}{3}$$

$$np(1+q) = \frac{25}{3}$$

$$np = \frac{25}{3(1+q)} \qquad ---(1)$$

$$\text{Mean} \times \text{Variance} = \frac{50}{3}$$

$$np \times npq = \frac{50}{3}$$

$$n^2p^2q = \frac{50}{3}$$

$$\left[\frac{25}{3(1+q)}\right]^2 . q = \frac{50}{3} \qquad [\text{Using } (1)]$$

$$625q = \frac{50}{3}\left[9(1+q)^2\right]$$

$$625q = 150(1+q)^2$$

$$25q = 6(1+q)^2$$

$$6 + 6q^2 + 12q - 25q = 0$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$3q(2q-3) - 2(2q-3) = 0$$

$$(2q-3)(3q-2) = 0$$

$\Rightarrow \quad 2q - 3 = 0 \quad \text{or} \quad 3q - 2 = 0$

$\Rightarrow \quad q = \frac{3}{2} \quad \text{or} \quad q = \frac{2}{3}$

Since $q \le 1$, so

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Binomial Distribution Ex 33.2 Q7

Let n and p be the parameters of binomial distribution.

Given that,

Mean $= np - 20$ $\quad ---(\text{i})$

Standard deviation $= \sqrt{npq} = 4$

Squaring both the sides,

$npq = 16$ $\quad ---(\text{ii})$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ $\quad$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put value of p in equation (i),

$$np = 20$$

$$\frac{n}{5} = 20$$

$$n = 100$$

$$p = \frac{1}{5}$$

Binomial Distribution Ex 33.2 Q8

Let p denotes the probability of selecting a defective bolt, so

$$p = 0.1$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{9}{10}$$

Given, $n = 400$

(i)

Mean $= np$

$$= 400 \times \frac{1}{10}$$

Mean $= 40$

(ii)

Standard deviation $= \sqrt{npq}$

$$= \sqrt{400 \times \frac{1}{10} \times \frac{9}{10}}$$

$$= \sqrt{36}$$

Standard deviation $= 6$

Binomial Distribution Ex 33.2 Q9

Let n and p be the parameters of binomial distribution.

Given, Mean $= np = 5$ --- (i)

Variance $= npq = \frac{10}{3}$ --- (ii)

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{\frac{10}{3}}{5}$$

$$q = \frac{2}{3}$$

So, $p = 1 - q$ [Since $p + q = 1$]

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = 5$$

$$n = 5 \times 3$$

$$n = 15$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{15}C_{r} \left(\frac{1}{3}\right)^{r} \left(\frac{2}{3}\right)^{15-r}$$

$$r = 0, 1, 2, \ldots 15$$

Binomial Distribution Ex 33.2 Q10

Let p be the probability of a ship returning safely to parts, so

$$p = \frac{9}{10}$$

$$q = 1 - \frac{9}{10} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{10}$$

Given, $n = 500$

$$\text{Mean} = np$$

$$= 500 \times \frac{9}{10}$$

$$\text{Mean} = 450$$

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{500 \times \frac{9}{10} \times \frac{1}{10}}$$

$$= \sqrt{45}$$

$$= 6.71$$

Mean $= 450$

Standard deviation $= 6.71$

Binomial Distribution Ex 33.2 Q11

Given that, parameters for binomial distribution are n and p.

Also, Mean $= np = 16$ --- (i)

Variance $= npq = 8$ --- (ii)

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{8}{16}$$

$$q = \frac{1}{2}$$

So, $p = 1 - \frac{1}{2}$ [as $p + q = 1$]

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{32}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{32-r} \quad \text{--- (iii)}$$

$P(X = 0)$

$$= {}^{32}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0} \quad [\text{Using (iii)}]$$

$$= \left(\frac{1}{2}\right)^{32}$$

$P(X = 1)$

$$= {}^{32}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1}$$

$$= 32.\frac{1}{2}\left(\frac{1}{2}\right)^{31}$$

$$= \left(\frac{1}{2}\right)^{27}$$

$$P(X \geq 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \left[\left(\frac{1}{2}\right)^{32} + \left(\frac{1}{2}\right)^{27}\right]$$
$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{1}{32} + 1\right)$$
$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{33}{32}\right)$$
$$= 1 - \frac{33}{2^{32}}$$

Hence

$$P(X = 0) = \left(\frac{1}{2}\right)^{32}, \; P(X = 1) = \left(\frac{1}{2}\right)^{27}, \; P(X \geq 2) = 1 - \frac{33}{2^{32}}$$

Binomial Distribution Ex 33.2 Q12

Let p be the probability of success in a single throw of die

$$p = \frac{2}{6} \qquad \text{[Since success is occurance of 5 or 6]}$$
$$p = \frac{1}{3}$$
$$q = 1 - \frac{1}{3} \qquad \text{[Since } p + q = 1\text{]}$$
$$q = \frac{2}{3}$$

Given, $n = 8$

$$\text{Mean} = np = \frac{8}{3} = 2.66$$

$$\text{Standard deviation} = \sqrt{npq} = \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} = \frac{4}{3} = 1.33$$

Mean $= 2.66$, Standard deviation $= 1.33$

Binomial Distribution Ex 33.2 Q13

Let n and p be the parameters of binomial distribution.
Let p = probability of having a boy in the family

Given, $p = q$

Since, $p + q = 1$

$$p + p = 1$$
$$2p = 1$$
$$p = \frac{1}{2}$$
$$q = \frac{1}{2}$$
$$n = 8$$

The expected number of boys $= np$

$$= 8 \times \frac{1}{2}$$
$$= 4$$

The expected number of boys $= 4$

Binomial Distribution Ex 33.2 Q14

Let p denot the probability of a defective item produced in the factory, so

$$p = 0.02$$
$$= \frac{2}{100}$$
$$p = \frac{1}{50}$$
$$q = 1 - \frac{1}{50} \qquad [\text{Since } p + q = 1]$$
$$= \frac{49}{50}$$

Given $n = 10,000$

Expected number of defective item $= np$

$$= 10000 \times \frac{1}{50}$$
$$= 200$$

Standard deviation $= \sqrt{npq}$

$$= \sqrt{10000 \times \frac{1}{50} \times \frac{49}{50}}$$
$$= 14$$

Expected No. of defective items $= 200$
Standard deviation $= 14$

Binomial Distribution Ex 33.2 Q15

Let p be the probability of success, so

$$p = \frac{2}{6} \qquad \left[\text{Since success in occurance of 1 or 6 on the die}\right]$$

$$p = \frac{1}{3}$$

Given, $n = 3$

$$q = 1 - p \qquad \left[\text{Since } p + q = 1\right]$$

$$= 1 - \frac{1}{3}$$

$$q = \frac{2}{3}$$

$$\text{Mean} = np$$

$$= 3\left(\frac{1}{3}\right)$$

$$= 1$$

$$\text{Variance} = npq$$

$$= 3 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$

$$= \frac{2}{3}$$

Mean = 1

$$\text{Variance} = \frac{2}{3}$$

Binomial Distribution Ex 33.2 Q16

Let n and p be the parameters of binomial distribution
Given,

$$\text{Mean} = np = 3 \quad \text{---(i)}$$

$$\text{Variance} = npq = \frac{3}{2} \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{3}{2}}{3}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i)

$$np = 3$$

$$n\left(\frac{1}{2}\right) = 3$$

$$n = 6$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^6C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} \quad \text{---(iii)}$$

$$P(X \le 5)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{6-6}, \quad [\text{Using (iii)}]$$

$$= 1 - \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$P(X \le 5) = \frac{63}{64}$$

Binomial Distribution Ex 33.2 Q17

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean} = np = 4 \quad ---(i)$$

$$\text{Variance} = npq = 2 \quad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{2}\right) = 4$$

$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \quad ---(iii)$$

$$P(X \geq 5)$$

$$= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$= {}^8C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8$$

[Using equation (iii)]

$$= \frac{8 \times 7 \times 6}{3 \times 2}\left(\frac{1}{2}\right)^8 + \frac{8 \times 7}{2}\left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 [56 + 28 + 8 + 1]$$

$$= \frac{93}{256}$$

$$P(X \geq 5) = \frac{93}{256}$$

Binomial Distribution Ex 33.2 Q18

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{16}{81}$$

$$= \frac{65}{81}$$

$$P(X \geq 1) = \frac{65}{81}$$

Binomial Distribution Ex 33.2 Q19

Let n and p be the parameters of binomial distribution,

Given, $n = 6$

$$\text{Mean} + \text{Variance} = \frac{10}{3}$$

$$np + npq = \frac{10}{3}$$

$$6p + 6pq = \frac{10}{3}$$

$$6p(1+q) = \frac{10}{3}$$

$$6(1-q)(1+q) = \frac{10}{3} \qquad [\text{Since } p+q=1]$$

$$6(1-q^2) = \frac{10}{3}$$

$$1-q^2 = \frac{10}{18}$$

$$-q^2 = \frac{5}{9} - 1$$

$$-q^2 = -\frac{4}{9}$$

$$q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Hence, the binomial distribution is given by,

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

as $r = 0, 1, 2, \ldots 6$

Binomial Distribution Ex 33.2 Q20

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $=36$

Let p be the probability of success therefore

$p=6/36=1/6$

Let q be the probability of failure therefore $q=1-p=1-1/6=5/6$

Since the dice is thrown 4 times, $n=4$

Let X be the random variable for getting doublet, therefore X can take at max 4 values.

$$P(X=0)={}^4C_0p^0q^4=\left(\frac{5}{6}\right)^4=\frac{625}{1296}$$

$$P(X=1)={}^4C_1p^1q^3=4\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^3=\frac{500}{1296}$$

$$P(X=2)={}^4C_2p^2q^2=\frac{4\cdot 3}{2}\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)^2=\frac{150}{1296}$$

$$P(X=3)={}^4C_3p^3q^1=4\cdot\left(\frac{1}{6}\right)^3\cdot\frac{5}{6}=\frac{20}{1296}$$

$$P(X=4)={}^4C_4p^4q^0=1\cdot\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0=\frac{1}{1296}$$

Mean

$$\mu=\sum_{i=1}^4X_iP(X_i)=0\cdot\frac{625}{1296}+1\cdot\frac{500}{1296}+2\cdot\frac{150}{1296}+3\cdot\frac{20}{1296}+4\cdot\frac{1}{1296}$$
$$=\frac{500+300+60+4}{1296}=\frac{54}{81}=\frac{2}{3}$$

Hence the mean is $=\frac{2}{3}$

Binomial Distribution Ex 33.2 Q21

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

Total number of outcomes $=36$

Let p be the probability of success therefore

$p=6/36=1/6$

Let q be the probability of failure therefore $q=1-p=1-1/6=5/6$

Since there is three rows of dice so $n=3$

Let X be the random variable for getting doublet, therefore X can take at max 3 values.

$$P(X=0)={}^3C_0p^0q^3=\left(\frac{5}{6}\right)^3=\frac{125}{216}$$

$$P(X=1)={}^3C_1p^1q^2=3\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^2=\frac{75}{216}$$

$$P(X=2)={}^3C_2p^2q^1=3\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)=\frac{15}{216}$$

$$P(X=3)={}^3C_3p^3q^0=\left(\frac{1}{6}\right)^3=\frac{1}{216}$$

Mean

$$\mu=\sum_{i=1}^3X_iP(X_i)=0\cdot\frac{125}{216}+1\cdot\frac{75}{216}+2\cdot\frac{15}{216}+3\cdot\frac{1}{216}$$
$$=\frac{75+30+3}{216}=\frac{108}{216}=\frac{1}{2}$$

Hence the mean is $=\frac{1}{2}$

Binomial Distribution Ex 33.2 Q22

Out of 15 bulbs 5 are defective.

Hence, the probability that the drawn bulb is defective is

$$P(\text{Defective}) = \frac{5}{15} = \frac{1}{3}$$

$$P(\text{Not defective}) = \frac{10}{15} = \frac{2}{3}$$

Let X denote the number of defective bulbs out of 4.
Then, X follows binomial distribution with

$n = 4$, $p = \frac{1}{3}$ and $q = \frac{2}{3}$ such that

$$P(X = r) = {}^4C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{4-r}; r = 0,1,2,3,4$$

$$\text{Mean} = \sum_{r=0}^{4} rP(r) = 1 \times {}^4C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + 2 \times {}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$$

$$+3 \times {}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right) + 4 \times {}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$$

$$= \frac{32}{81} + \frac{48}{81} + \frac{24}{81} + \frac{4}{81} = \frac{108}{81} = \frac{4}{3}$$

Binomial Distribution Ex 33.2 Q23

Let p be the probablity of getting 2 when a dice is thrown.

Then $p = \frac{1}{6}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{6}$.

$\therefore$ Expectation $= E(X) = np = 3 \times \frac{1}{6} = \frac{1}{2}$

Binomial Distribution Ex 33.2 Q24

Let p be the probablity of getting an even number on the toss when a dice is thrown.
Let q be the probablity of not getting an even number on the toss when a dice is thrown.

Then $p = \frac{3}{6} = \frac{1}{2}$ and $q = \frac{1}{2}$

Clearly, X follows binomial distribution with $n = 2$, $p = \frac{1}{2}$.

$\therefore$ Variance $= npq = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

Binomial Distribution Ex 33.2 Q25

Let p be the probablity of getting a spade card.
Let q be the probablity of getting a spade card.

Then $p = \frac{13}{52} = \frac{1}{4}$ and $q = \frac{3}{4}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$.

Probablity distribution is given by,

$$P(X = r) = {}^3C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{3-r}; r = 0,1,2$$

$\therefore$ Mean $= np = 3 \times \frac{1}{4} = \frac{3}{4}$