RD Sharma Solutions Class 11 Maths Chapter 1 Ex 1.4

Sets Ex 1.4 Q1

(i) False, $\because$ the two sets A and B need not be comparable.

(ii) False, $\because$ $\{1\}$ is a finite subset of the infinite set N of natural numbers.

(iii) True, $\because$ the order (or cardinal number) of any subset of a set is less than or equal to the order of the set.
(order (or cardinal number) of a set is the number of elements in the set).

(iv) False, $\because$ the empty set ϕ has no proper subset.

(v) False, $\because$ $\{a,b,a,b,...\} = \{a,b\}$ (repetition is not allowed)

$\therefore$ $\{a,b,a,b,...\}$ is a finite set.

(vi) True, $\because$ equivalent sets have the same cardinal number.

(vii) False,
One knows that if the cardinal number of a set A is n, then the power set of A denoted by $P(A)$ which is the set of all subsets of A, has the cardinal number 2^n.

If the cardinal number of A is infinite, then the cardinal number of $P(A)$ is also infinite.
Hence, the above statement is true provided the set is infinite.

Sets Ex 1.4 Q2

(i) True, $\because$ 1 is an element of the set $\{1,2,3\}$.

(ii) False, $\because$ a is an element and not a subset of the set $\{b,c,a\}$.

(iii) False, $\because$ $\{a\}$ is a subset of the set $\{a,b,c\}$ and not an element.

(iv) True, $\because$ repetition is not allowed in a set.

(v) False, $\because$ the set $\{x : x + 8 = 8\}$ is the single ton set $\{0\}$ which is not the null set ϕ.

Sets Ex 1.4 Q3

We have,

$$A = \{x : x \text{ satisfies } x^2 - 8x + 12 = 0\}$$
$$= \{x : x^2 - 6x - 2x + 12 = 0\}$$
$$= \{x : x(x-6) - 2(x-6) = 0\}$$
$$= \{x : (x-6)(x-2) = 0\}$$

$= \{x : x = 6, 2\}$
$= \{6, 2\}$

$B = \{2, 4, 6\}$
$C = \{2, 4, 6, 8, \ldots\}$
$D = \{6\}$

We know that if E and F are two sets, then E is a subset of F. i.e., $E \subseteq F$ if $x \in E \Rightarrow x \in F$. E is called a proper subset of F if E is strictly contained in F and is denoted by $E \subset F$.

Clearly,
$D \subset A \{\because 6 \in D \text{ and } 6 \in A\}$
$A \subset B \{\because 2, 6 \in A \text{ and they also belong to } B\}$
Similarly, $B \subset C$

Hence, $D \subset A \subset B \subset C$.

Sets Ex 1.4 Q4(i)

The given statement is 'True'.

If $m \in z$, then m can be written as $\frac{m}{1}$, which is of the form $\frac{p}{q}$, where p and q are relatively prime integers and $q \neq 0$.

This implies that $m \in Q$, the set of rational numbers.

Thus, $m \in z \Rightarrow m \in Q$

Hence $Z \subseteq Q$

Sets Ex 1.4 Q4(ii)

The given statement is 'True'.
$\because$ Crows are also Birds.

Sets Ex 1.4 Q4(iii)

The given statement is 'False'.
$\because$ A rectangle need not be a square.

Sets Ex 1.4 Q4(iv)

The given statement is 'True'.

If z is a complex number, then it can be written as $z = x + iy$, where x and y are real numbers and are called the real and imaginary parts of the complex number z.

If x is a real number, then
$x = x + i.0 \in C$,
where C is the set of complex numbers.

Thus $x \in R \Rightarrow x \in C$

Hence, the set of all real numbers is contained in the set of all complex numbers.

Sets Ex 1.4 Q4(v)

False, $\because$ $a \in P$ but $a \notin B$
Note that $\{a\}$ is an element of B which is different from the element 'a'.

Sets Ex 1.4 Q4(vi)

$A = \{L, I, T, E\}$ $[\because \text{repetition is not allowed}]$
$B = \{T, I, L, E\}$ $[\because \text{repetition is not allowed}]$
$= \{L, I, T, E\}$ $\left[\because \text{the manner in which the elements are listed does not matter}\right]$

$\because$ Each element of A is an element of B and vice-versa

$\therefore$ $A = B$

Hence, the given statement is true.

Sets Ex 1.4 Q5

(i) False,
The correct statement is $a \in \{a, b, c\}$.

The correct form is $\{a\} \subset \{a,b,c\}$.

(iii) False, $\because$ a is not an element of $\{\{a\},b\}$
The correct form is $\{a\} \in \{\{a\},b\}$

(iv) False, $\because \{a\}$ is not a subset of $\{\{a\},b\}$ hence it cannot be contained in it.
The correct form is $\{a\} \in \{\{a\},b\}$. Another correct form could be $\{\{a\}\} \subset \{\{a\},b\}$.

(v) False, $\because$ $\{b,c\}$ is an element and not a subset of $\{a,\{,bc\}\}$.
The correct form is $\{b,c\} \in \{a,\{b,c\}\}$.

(vi) False, $\because$ $\{a,b\}$ is not a subset of $\{a,\{b,c\}\}$
The correct form is $\{a,b\} \not\subset \{a,\{b,c\}\}$.

(vii) False, $\because$ ϕ is not an element of $\{a,b\}$.
The correct form is $\phi \subset \{a,b\}$.

(viii) True, $\because$ empty set ϕ is a subset of every set.

(ix) False, $\because \{x : x+3=3\} = \{x : x=0\} = \{0\}$
The correct form is $\{x : x+3=3\} \neq \phi$.

Sets Ex 1.4 Q6

(i) False, $\{c,d\}$ is an element of A and not a subset of A.

(ii) True, $\because \{c,d\}$ is indeed an element of A.

(iii) True, $\because \{c,d\}$ is a subset of A.

(iv) True,

(v) False, $\because$ a belongs to A and not a subset of A. An element of a set belongs to it whereas a subset of it is contained in it.

(vi) True, $\because \{a,b,e\}$ is a subset of A.

(vii) False, $\because \{a,b,e\}$ is a subset of A, so it does not belong to A.

(viii) False, $\because \{a,b,c\}$ is not a subset of A.

(ix) False, $\because \phi$ is a subset and not an element of A.

(x) False, $\because \phi$ and not $\{\phi\}$ is a subset of A.

Sets Ex 1.4 Q7

(i) False, $\because$ 1 is not an element of A.

(ii) False, $\because \{1,2,3\}$ is not a subset of A, it is an element of A.

(iii) True, $\because \{6,7,8\}$ is indeed an element of A.

(iv) True, $\because$ $\{\{4,5\}\}$ is indeed a subset of A.

(v) False, $\because$ ϕ is a subset and not an element of A.

(vi) True, $\because$ ϕ is a subset of every set, and hence a subset of A.

Sets Ex 1.4 Q8

(i) True, $\because$ ϕ indeed belongs to A.

(ii) True, $\because$ $\{\phi\}$ is an element of A.

(iii) False, $\because$ $\{1\}$ is not an element of A.

(iv) True, $\because$ $\{2,\phi\}$ is a subset of A.

(v) False, $\because$ 2 is not a subset of A, it

(vi) True, $\because \{2,\{1\}\}$ is not a subset of

(vii) True, $\because$ $\{\{2\},\{1\}\}$ is not a subset of A.

(viii) True, $\because \{\phi,\{\phi\},\{1,\phi\}\}$ is a subset of A.

(ix)True, $\because \{\{\phi\}\}$ is a subset of A.

Sets Ex 1.4 Q9

(i) We know that, if a set has n elements, then its power set has 2^n elements.

Here, n = 1, so there 2^1=2 subsets of the given set.

The possible subsets are $\phi,\{a\}$.

(ii) The set has two elements, so power set has 2^2 = 4 elements, namely $\phi,\{0\},\{1\},\{0,1\}$.

(iii) The set has 3 elemets , so power set has 2^3 = 8 elements,
namely $\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,b,c\}$.

(iv) The set has 2 elements, so power set has 2^2 = 4 elements, namely, $\phi,\{1\},\{\{1\}\},\{1,\{1\}\}$.

(v) The set has 1 element, so power set has ^{1} = 2 elements, namely $\phi,\{\phi\}$.

Sets Ex 1.4 Q10

(i) We know that if A is a set and B a subset of A, then B is called a proper subset of A
if $B \subseteq A$ and $B \neq A$, ϕ and is written as $B \subset A$ or $B \subseteq A$.

Hence, the proper subsets are given by $\{1\},\{2\}$.

(ii) The proper subsets are given by $\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\}$.

(iii) The only subsets of the given set are ϕ & $\{1\}$.
Hence, there are no proper subsets.

Sets Ex 1.4 Q11

We know that, if A is a set having n elements then power set of A,
namely $P(A)$ has 2^n elements. Out of this A is not proper subset.

Hence, the total number of proper subsets of a set consisting of n elements in 2^n - 1

Sets Ex 1.4 Q12

The symbol '$\Leftrightarrow$' stands for if and only if (in short if).
In order to show that two sets A and B are equal we show that $A \subseteq B$ and $B \subseteq A$.

We have $A \subseteq \phi$. $\because$ ϕ is a subset of every set

$\therefore$ $\phi \subseteq A$

Hence $A = \phi$
To show the backward implication, suppose that $A = \phi$
$\because$ every set is a subset of itself
$\therefore$ $\phi = A \subseteq \phi$

Hence, proved.

Sets Ex 1.4 Q13

We have $A \subseteq B$, $B \subseteq C$ and $C \subseteq A$, so $A \subseteq B \subseteq C \subseteq A$
Now, A is a subset of B and B is a subset of C, so

A is a subset of C, i.e., $A \subseteq C$

Also, $C \subseteq A$

Hence, $A = C$

Sets Ex 1.4 Q14

$\because$ an empty set has zero element.

∴ power set of ϕ has $2^0 = 1$ element.

Sets Ex 1.4 Q15

(i)

The set of right triangles is a subset of the set of all triangles in the plane. So, the set of all triangles in the plane forms a universal set for the set of right triangles.

(ii)

The set of isosceles triangles forms a subset of the set of all triangles in the plane.

Hence the set of all triangles in the plane forms a universal set for the set of isosceles triangles.

Sets Ex 1.4 Q16

$$X = \{8^n - 7n - 1 : x \in N\}$$
$$Y = \{4n(n-1) : n \in N\}$$

In order to show that $x \subseteq y$ we show tat every element of X is an element of Y.

So let $x \in X \Rightarrow x = 8^m - 7m - 1$ for some $m \in N$

$$\Rightarrow \quad x = (1+7)^m - 7m - 1$$
$$= \left({}^mC_0 1^m + {}^mC_1 1^{m-1} 7 + \ldots + {}^mC_{m-1} 1^1 7^{m-1} + {}^mC_m 7^m\right) - 7m - 1$$

[using binomial expansion]

$$= 1 + 7m + {}^mC_2 7^2 + {}^mC_3 7^3 + \ldots + {}^mC_m 7^m - 7m - 1$$
$$= {}^mC_2 7^2 + {}^mC_3 7^3 + \ldots + {}^mC_m 7^m$$
$$= 49\left({}^mC_2 + {}^m mC_3 + \ldots + {}^mC_m 7^{m-2}\right),\ m \ge 2$$
$$= 49t_m,\ m \ge 2,\ \text{where } t_m = {}^mC_2 + {}^mC_3 7 + \ldots + {}^mC_m 7^{m-2}$$

Is some positive integer depending on $m \ge 2$

For $m = 1$

$$x = 8^1 - 7 \times 1 - 1$$
$$= 8 - 8$$
$$= 0$$

Hence, X contains all positive integral multiples of 49.

Also, Y consistes of all positive integral multiples of 49, including 0, for $n = 1$.

Thus, we conclude that $X \subseteq Y$.