RD Sharma Solutions Class 11 Maths Chapter Ex 1.6

Sets Ex 1.6 Q1

The smallest set A such that

$A \cup \{1,2\} = \{1,2,3,5,9\}$ is $\{3,5,9\}$

$\because \quad \{3,5,9\} \cup \{1,2\} = \{1,2,3,5,9\}$

Any other set B such that $B \cup \{1,2\} = \{1,2,3,5,9\}$ will contain A. For example we contake B to be $\{1,3,5,9\}$ or $\{1,2,3,5,9\}$. Clearly B contains $A = \{3,5,9\}$.

Sets Ex 1.6 Q2(i)

i. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B \cap C = \{5,6\}$

$A \cup (B \cap C) = \{1,2,4,5,6\} \ldots\ldots\ldots (1)$

$(A \cup B) = \{1,2,3,4,5,6\}$

$(A \cup C) = \{1,2,4,5,6,7\}$

$(A \cup B) \cap (A \cup C) = \{1,2,4,5,6\} \ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sets Ex 1.6 Q2(ii)

ii. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B \cup C = \{2,3,4,5,6,7\}$

$A \cap (B \cup C) = \{2,4,5\} \ldots\ldots\ldots (1)$

$(A \cap B) = \{2,5\}$

$(A \cap C) = \{4,5\}$

$(A \cap B) \cup (A \cap C) = \{2,4,5\} \ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sets Ex 1.6 Q2(iii)

iii. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B - C = \{2,3\}$

$A \cap (B - C) = \{2\} \ldots\ldots\ldots (1)$

$(A \cap B) = \{2,5\}$

$(A \cap C) = \{4,5\}$

$(A \cap B) - (A \cap C) = \{2\} \ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Sets Ex 1.6 Q2(iv)

iv. $A = \{1,2,4,5\}, B = \{2,3,5,6\}, C = \{4,5,6,7\}$

$B \cup C = \{2,3,4,5,6,7\}$

$A - (B \cup C) = \{1\} \ldots\ldots\ldots (1)$

$(A - B) = \{1,4\}$

$(A - C) = \{1,2\}$

$(A - B) \cap (A - C) = \{1\} \ldots\ldots\ldots\ldots (2)$

From eq^n (1) and eq^n (2), we get

$A-(B\cup C)=(A-B)\cap(A-C)$

Sets Ex 1.6 Q2(v)

v. $A=\{1,2,4,5\}, B=\{2,3,5,6\}$, C [illegible]

$B\cap C=\{5,6\}$

$A-(B\cap C)=\{1,2,4\}$........(1)

$(A-B)=\{1,4\}$

$(A-C)=\{1,2\}$

$(A-B)\cup(A-C)=\{1,2,4\}$............(2)

From eq^n(1) and eq^n(2), we get

$A-(B\cap C)=(A-B)\cup(A-C)$

Sets Ex 1.6 Q2(vi)

vi. $A=\{1,2,4,5\}, B=\{2,3,5,6\}, C=\{4,5,6,7\}$

$B\Delta C=(B-C)\cup(C-B)=\{2,3\}\cup\{4,7\}=\{2,3,4,7\}$

$A\cap(B\Delta C)=\{2,4\}$........(1)

$(A\cap B)=\{2,5\}$

$(A\cap C)=\{4,5\}$

$(A\cap B)\Delta(A\cap C)=[(A\cap B)-(A\cap C)]\cup[(A\cap C)-(A\cap B)]$

$(A\cap B)\Delta(A\cap C)=\{2\}\cup\{4\}=\{2,4\}$............(2)

From eq^n(1) and eq^n(2), we get

$A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$

Sets Ex 1.6 Q3(i)

$U=\{2,3,5,7,9\}$ is the universal set

$A=\{3,7\}$, $B=\{2,5,7,9\}$

$A\cup B=\{x: x\in A \text{ or } x\in B\}$

$=\{2,3,5,7,9\}$

LHS $=(A\cup B)'$

$=\{2,3,5,7,9\}'$

$=U-A\cup B$

$=\phi$

RHS $=A'\cap B'$

$A'=\{x\in U: x\notin A\}$

$=\{2,5,9\}$

$B'=\{x\in U: x\notin B\}$

$=\{3\}$

$\therefore A'\cap B'=\{2,5,9\}\cap\{3\}$

$=\phi$ [$\because$ the two sets are disjoint]

$\therefore$ LHS = RHS Proved

Sets Ex 1.6 Q3(ii)

LHS $=(A\cap B)'$

Now,

$A\cap B=\{x \mid x\in A \text{ and } x\in B\}$

$=\{7\}$

$\therefore (A\cap B)'=\{7\}'$

$=\{x\in U: x\notin 7\}$

$=\{2,3,5,9\}$

RHS $=A'\cup B'$

Now, $A'=\{2,5,9\}$ [form (i)]

and $B'=\{3\}$ [from (i)]

$\therefore A'\cup B'=\{2,3,5,9\}$

Hence, LHS = RHS Prove

Sets Ex 1.6 Q4(i)

i. Let $x \in B$. Then

$\Rightarrow x \in B \cup A$

$\Rightarrow x \in A \cup B$

$\therefore B \subset (A \cup B)$

Sets Ex 1.6 Q4(ii)

ii. Let $x \in A \cap B$. Then

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$

$\therefore (A \cap B) \subset B$

Sets Ex 1.6 Q4(iii)

iii. Let $x \in A \subset B$. Then

$\Rightarrow x \in B$

Let and $x \in A \cap B$

$\Leftrightarrow x \in A$ and $x \in B$

$\Leftrightarrow x \in A$ and $x \in A$ $\quad (\because A \subset B)$

$\therefore (A \cap B) = A$

Sets Ex 1.6 Q5

(i)

In order to show that the following four statements are equivalent, we need to show that $(1) \Rightarrow (2)$, $(2) \Rightarrow (3)$, $(3) \Rightarrow (4)$ and $(4) \Rightarrow (1)$

We first show that $(1) \Rightarrow (2)$

We assume that $A \subset B$, and use this to show that $A - B = \phi$

Now $A - B = \{x \in A : x \notin B\}$. As $A \subset B$,

$\therefore$ Each element of A is an element of B,

$\therefore$ $A - B = \phi$

Hence, we have proved that $(1) \Rightarrow (2)$.

(ii)

We new show that $(2) \Rightarrow (3)$

So assume that $A - B = \phi$

To show: $A \cup B = B$

$\because$ $A - B = \phi$

$\therefore$ Every element of A is an element of B

[$\because$ $A - B = \phi$ only when ther is some element in A which is not in B]

So $A \subset B$ and therefore $A \cup B = B$

So $(2) \Rightarrow (3)$ is true.

(iii)

We new show that $(3) \Rightarrow (4)$

Assume that $A \cup B = B$

To show: $A \cap B = A$

$\because$ $A \cup B = B$

$\therefore$ $A \subset B$ and so $A \cap B = A$

So $(3) \Rightarrow (4)$ is true.

(iv)

Finally we show that $(4) \Rightarrow (1)$, which will prove the equivalence of the four statements.

So, assume that $A \cap B = A$

To show: $A \subset B$

$A \cap B = A$ therefore $A \subset B$ and so $(4) \Rightarrow (1)$ is true

$\because A \cap B = A$, therefore $A \subset B$, and so $(4) \Rightarrow (1)$ is true.

Hence, $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4)$.

Sets Ex 1.6 Q6(i)

Let $A = \{1,2,3\}$, $B = \{2,4,6\}$ and $C = \{2,5,7\}$

Then,

$A \cap B = \{2\}$

and $A \cap C = \{2\}$

Hence, $A \cap B = A \cap C$, but clearly $B \neq C$.

Sets Ex 1.6 Q6(ii)

Given $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$ [by definition of $C - B$]

$\Rightarrow x \in C$ and $x \notin A$ [$\because A \subset B$]

This can be seen by the venn diagram above

$\Rightarrow x \in C - A$ [by definition of $C - A$]

Thus $x \in C - B \Rightarrow x \in C - A$. This is true for all $x \in C - B$

$\therefore C - B \subset C - A$

Sets Ex 1.6 Q7

(i)

$A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$ [$\because$ union $\cup$ is distributive over intersection $\cap$]

$= A \cap (A \cup B)$ [$\because A \cup A = A$]

$= A$ [$\because A \subset (A \cup B)$, as union of two sets is bigger than each of the individual sets]

Hence, $A \cup (A \cap B) = A$ Proved.

(ii)

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ [$\because \cap$ distributes over $\cup$]

$= A \cup (A \cap B)$ [$\because A \cap A = A$]

$= A$ [using (i)]

Sets Ex 1.6 Q8

To find sets A, B and C such that $A \cap B \neq \phi$, $A \cap C = \phi$ and $B \cap C = \phi$ and $A \cap B \cap C = \phi$

Take $A = \{1,2,3\}$

$B = \{2,4,6\}$

and $C = \{3,4,7\}$

Then,

$A \cap B = \{2\}$

$\therefore A \cap B \neq \phi$

$A \cap C = \{3\}$

$\therefore A \cap C \neq \phi$

$B \cap C = \{4\}$

$\therefore B \cap C \neq \phi$

However A, B and C have no elements in common,

$\therefore A \cap B \cap C = \phi$

Sets Ex 1.6 Q9

Given $A \cap B = \phi$, i.e., A and B are disjoint sets this can represented by venn diagram as follows

To show: $A \subseteq B'$

This is clear from the venn diagram itself

$\because A$ is lying in the complement of B, but we give a proof of it.

So let $x \in A$

$\because A \cap B = \phi$,

$\therefore x \notin B$

and so $x \in B'$

Thus $x \in A \Rightarrow$ ›

Hence, $A \subseteq B'$

Sets Ex 1.6 Q10

We need to show that $(A - B) \cap (A \cap B) = \phi$, $(A \cap B) \cap (B - A) = \phi$ and $(A - B) \cap (B - A) = \phi$

The 3 sets $A - B$, $A \cap B$ and $B - A$ may be represented by a venn diagram as follows

It is clear from the diagram that the 3 sets are pairwise disjoint, but we shall give a proff of it.

We first show that $(A - B) \cap (A \cap B) = \phi$

Let $x \in (A - B)$

$\Rightarrow \quad x \in A$ and $x \notin B$ [by definition of $A - B$]

$\Rightarrow \quad x \notin A \cap B$. This is true for all $x \in (A - B)$

Hence $(A - B) \cap (A \cap B) = \phi$

On a similar lines, it can be seen that $(A \cap B) \cap (B - A) = \phi$

Finally, we show that $(A - B) \cap (B - A) = \phi$

We have,

$$A - B = \{x \in A : x \notin B\}$$

and $\quad B - A = \{x \in B : x \notin A\}$

Hence, $(A - B) \cap (B - A) = \phi$.

Sets Ex 1.6 Q11

We need to show $(A \cup B) \cap (A \cap B') = A$

Now,

$(A \cup B) \cap (A \cap B') = ((A \cup B) \cap A) \cap B'$ [Using associative property]

$= ((A \cap A) \cup (B \cap A)) \cap B'$ $\left[\because A \cap A = A \text{ and } B \cap A = A \cap B, \text{ by commutative law}\right]$

$= A \cap B'$ $[\because A \cup (A \cap B) = A]$

$= A$

Sets Ex 1.6 Q12(i)

We have $A' \cup B = \cup$, the universal set

To show: $A \subset B$

Let, $x \in A$

$\Rightarrow \quad x \notin A'$ $[\because A \cap A' = \phi]$

$\because \quad x \in A$ and $A \subset \cup$

$\Rightarrow \quad x \in \cup$

$\Rightarrow \quad x \in (A' \cup B)$ $[\because \cup = A' \cup B]$

$\Rightarrow \quad x \in A'$ or $x \in B$

But, $x \notin A'$,

$\therefore \quad x \in B$

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore \quad A \subset B$

Sets Ex 1.6 Q12(ii)

We have $B' \subset A'$

To show: $A \subset B$

Let, $x \in A$

$\Rightarrow \quad x \notin A'$ $[\because A \cap A' = \phi]$

$\Rightarrow \quad x \notin B'$ $[\because B' \subset A']$

$\Rightarrow \quad x \in B$ $[\because B \cap B' = \phi]$

Thus, $x \in A \Rightarrow x \in B$

This is true for all $x \in A$

$\therefore \quad A \subset B$

Sets Ex 1.6 Q13

This is a false statement

Let, $A = \{1\}$ and $B = \{2\}$

Then,

$P(A) = \{\phi, \{1\}\}$

and $P(B) = \{\phi, \{2\}\}$

$\therefore$ $P(A) \cup P(B) = \{\phi, \{1\}, \{2\}\}$

Now,

$A \cup B = \{1, 2\}$

and $P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Hence, $P(A) \cup P(B) \neq P(A \cup B)$

Sets Ex 1.6 Q14(i)

i. We know that $(A \cap B) \subset A$ and $(A - B) \subset A$

$\Rightarrow (A \cap B) \cap (A - B) \subset A \ldots\ldots\ldots(1)$

Let and $x \in (A \cap B) \cap (A - B)$

$\Rightarrow x \in (A \cap B)$ and $x \in (A - B)$

$\Rightarrow x \in A$ and $x \in B$ and $x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in A$ $[\because x \in B$ and $x \notin B$ are not possible simultaneously$]$

$\Rightarrow x \in A$

$\therefore (A \cap B) \cap (A - B) \subset A \ldots\ldots(2)$

From (1) and (2), we get

$A = (A \cap B) \cap (A - B)$

Sets Ex 1.6 Q14(ii)

ii. Let $x \in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$

$\therefore A \cup (B - A) \subset (A \cup B) \ldots\ldots\ldots(1)$

Let and $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A \cup (B - A)$

$\therefore (A \cup B) \subset A \cup (B - A) \ldots\ldots(2)$

From (1) and (2), we get

$A \cup (B - A) = A \cup B$

Sets Ex 1.6 Q15

Since each X_r has 5 elements and each element of S belongs to exactly 10 of X_r's.

$\therefore S = \bigcup_{r=1}^{20} X_r \Rightarrow \frac{1}{10}\sum_{r=1}^{20} n(X_r) = \frac{1}{10}(5 \times 20) = 10 \ldots\ldots(i)$

Since each Y_r has 2 elements and each element of S belongs to exactly 4 of X_r's.

$\therefore S = \bigcup_{r=1}^{n} X_r \Rightarrow \frac{1}{4}\sum_{r=1}^{n} n(Y_r) = \frac{1}{4}(2n) = \frac{n}{2} \ldots(ii)$

From (i) and (ii), we get

$10 = \frac{n}{2} \Rightarrow n = 20$