RD Sharma Solutions Class 11 Maths Chapter 1 Ex 1.7

Sets Ex 1.7 Q1

To show $A' - B' = B - A$

We show that $A' - B' \subseteq B - A$ and vice versa

Let, $x \in A' - B'$

$\Rightarrow \quad x \in A'$ and $x \notin B'$

$\Rightarrow \quad x \notin A$ and $x \in B$ $\quad [\because A \cap A' = \phi$ and $B \cap B' = \phi]$

$\Rightarrow \quad x \in B$ and $x \notin A$

$\Rightarrow \quad x \in B - A$

This is true for all $x \in A' - B$

Hence $A' - B' \subseteq B - A$

Conversely,

Let, $x \in B - A$

$\Rightarrow \quad x \in B$ and $x \notin A$

$\Rightarrow \quad x \notin B'$ and $x \in A'$ $\quad [\because B \cap B' = \phi$ and $A \cap A' = \phi]$

$\Rightarrow \quad x \in A'$ and $x \notin B'$

$\Rightarrow \quad x \in A' - B'$

This is true for all $x \in B - A$

Hence $B - A \subseteq A' - B'$

$\therefore A' - B' = B - A$ Proved.

Sets Ex 1.7 Q2(i)

LHS $= A \cap (A' \cup B)$

$= (A \cap A') \cup (A \cap B)$ $\quad [\because \cap$ distributes over (i)$]$

$= \phi \cup (A \cap B)$ $\quad [\because A \cap A' = \phi]$

$= A \cap B$ $\quad [\because \phi \cup x = x$ for any set $x]$

$=$ RHS

$\therefore$ LHS = RHS Proved.

Sets Ex 1.7 Q2(ii)

For any sets A and B we have by De-morgan's laws

$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

Also,

LHS $= A - (A - B)$

$= A \cap (A - B)'$

$= A \cap (A \cap B')'$

$= A \cap (A' \cup (B')')$ $\quad$ [By De-morgan's law]

$= A \cap (A' \cup B)$ $\quad [\because (B')' = B]$

$= (A \cap A') \cup (A \cap B)$

$= \phi \cup (A \cap B)$ $\quad [\because A \cap A' = \phi]$

$= A \cap B$ $\quad [\because \phi \cup x = x$, for any set $x]$

$=$ RHS

$\therefore$ LHS = RHS Proved.

Sets Ex 1.7 Q2(iii)

LHS $= A \cap (A \cup B')$

$= A \cap (A' \cap B')$ $\quad$ [By De-morgan's law]

$= (A \cap A') \cap B'$ $\quad$ [By associative law]

$= \phi \cap B'$ $\quad [\because A \cap A' = \phi]$

$= \phi$

$=$ RHS

$\therefore$ LHS = RHS Proved.

Sets Ex 1.7 Q2(iv)

RHS $= A \Delta (A \cap B)$

$= (A - (A \cap B)) \cup (A \cap B - A)$ $\quad [\because E \Delta F = (E - F) \cup (F - E)]$

$= (A \cap (A \cap B)') \cup (A \cap B \cap A')$ $\quad [\because E - F = E \cap F']$

$= (A \cap (A' \cup B')) \cup (A \cap A' \cap B)$ $\quad$ [By De-morgan's law & associative law]

$= (A \cap A') \cup (A \cap B') \cup (\phi \cap B)$ $\quad [\because \cap$ distributes over $\cup$ and $A \cap A' = \phi]$

$= \phi \cup (A \cap B') \cup \phi$ $\quad [\because \phi \cap B = \phi]$

$= A \cap B'$ $\quad [\because \phi \cup x = x$ for any set $x]$

$= A - B$ $\quad [\because A \cap B' = A - B]$

$=$ LHS

$\therefore$ LHS = RHS Proved.

Sets Ex 1.7 Q3

We have, $A C B$

To show: $C - B \subset C - A$

Let, $x \in C - B$

$\Rightarrow \quad x \in C$ and $x \notin B$

$\Rightarrow \quad x \in C$ and $x \notin A \quad [\because A \subset B]$

$\Rightarrow \quad x \in C - A$

Thus, $x \in C - B \Rightarrow x \in C - A$

This is true for all $x \in C - B$

$\therefore \quad C - B \subset C - A$

Sets Ex 1.7 Q4(i)

i. $(A \cup B) - B = (A - B) \cup (B - B)$

$= (A - B) \cup \phi$

$= A - B$

Sets Ex 1.7 Q4(ii)

ii. $A - (A \cap B) = (A - A) \cap (A - B)$

$= \phi \cap (A - B)$

$= A - B$

Sets Ex 1.7 Q4(iii)

iii.Let $x \in A - (A - B) \Leftrightarrow x \in A$ and $x \notin (A - B)$

$\Leftrightarrow x \in A$ and $x \in (A \cap B)$

$\Leftrightarrow x \in A \cap (A \cap B)$

$\Leftrightarrow x \in (A \cap B)$

$\therefore A - (A - B) = (A \cap B)$

Sets Ex 1.7 Q4(iv)

iv.Let $x \in A \cup (B - A) \Rightarrow x \in A$ or $x \in (B - A)$

$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$

$\Rightarrow x \in B$

$\Rightarrow x \in (A \cup B) \quad [\because B \subset (A \cup B)]$

This is true for all $x \in A \cup (B - A)$

$\therefore A \cup (B - A) \subset (A \cup B)$.........(1)

Conversely,

Let, $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ or $x \in (B - A) \quad [\because B \subset (B - A)]$

$\Rightarrow x \in A \cup (B - A)$

$\therefore (A \cup B) \subset A \cup (B - A)$..........(2)

From (1) and (2), we get

$A \cup (B - A) = (A \cup B)$

Sets Ex 1.7 Q4(v)

v.Let $x \in A$.

Then either $x \in (A - B)$ or $x \in (A \cap B)$

$\Rightarrow x \in (A - B) \cup (A \cap B)$

$\therefore A \subset (A - B) \cup (A \cap B)$..........(1)

Conversely,

Let $x \in (A - B) \cup (A \cap B)$

$\Rightarrow x \in (A - B)$ or $x \in (A \cap B)$

$\Rightarrow x \in A$ and $x \notin B$ or $x \in A$ and $x \in B$

$\Rightarrow x \in A$

$\therefore (A - B) \cup (A \cap B) \subset A$..............(2)

$\therefore$ From (1) and (2), we get

$(A - B) \cup (A \cap B) = A$