RD Sharma Solutions Class 11 Maths Chapter 1 Ex 1.8

Sets Ex 1.8 Q1

$n(A \cup B) = 50,\ n(A) = 28,\ n(B) = 32,$ where $n(x)$ doesnotes the cardinal number of the set x.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 50 = 28 + 32 - n(A \cap B)$

$\Rightarrow \quad 50 = 60 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 60 - 50$

$= 10$

$\therefore \quad n(A \cap B) = 10$

Sets Ex 1.8 Q2

We have,

$n(P) = 40,\ n(P \cup Q) = 60,\ n(P \cap Q) = 10,$ to find $n(Q)$.

We know $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$\Rightarrow \quad 60 = 40 + n(Q) - 10$

$\Rightarrow \quad 60 = 30 + n(Q)$

$\Rightarrow \quad n(Q) = 60 - 30$

$= 30$

Hence, Q has 30 elements.

Sets Ex 1.8 Q3

Let $n(P)$ denote the number of teachers who teach Physics and $n(Q)$ denote the number of teachers who teach Mathematics.

We have,

$n(P \text{ or } M) = 20$

i.e $n(P \cup M) = 20$

$n(M) = 12$

and $\quad n(P \cap M) = 4$

To find: $n(P)$

We know $n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$\Rightarrow \quad 20 = n(P) + 12 - 4$

$\Rightarrow \quad 20 = n(P) + 8$

$\Rightarrow \quad n(P) = 20 - 8$

$= 12$

$\therefore$ There are 12 Physics teachers.

Sets Ex 1.8 Q4

Let,

$n(P)$ denote the total number of people

$n(C)$ denote the number of people who like coffee and

$n(T)$ denote the number of people who like tea.

Then, $n(P) = 70$

$n(C) = 37$

$n(T) = 52$

We are given that each person likes at least one of the two drinks, i.e., $P = C \cup T$

To find: $n(C \cap T)$

We know $n(P) = n(C) + n(T) - n(C \cap T)$

$\Rightarrow \quad 70 = 37 + 52 - n(C \cap T)$

$\Rightarrow \quad 70 = 89 - n(C \cap T)$

$\Rightarrow \quad n(C \cap T) = 89 - 70$

$= 19$

Hence, 19 people like both coffee and tea.

Sets Ex 1.8 Q5(i)

$n(A) = 20,\ n(A \cup B) = 42$ and $n(A \cap B) = 4,$ to find: $n(B)$

We know $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 42 = 20 + n(B) - 4$

$\Rightarrow \quad 42 = 16 + n(B)$

$\Rightarrow \quad n(B) = 42 - 16$
$= 26$

$\therefore \ n(B) = 26$

Sets Ex 1.8 Q5(ii)

To find: $n(A - B)$

We know that if A and B are disjoint sets, then

$A \cap B = \phi$

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= n(A) + n(B) - n(\phi)$

$\Rightarrow \quad n(A \cup B) = n(A) + n(B) \qquad [\because n(\phi) = 0]$

Now,

$A = (A - B) \cup (A \cap B)$

i.e A is the disjoint union of $A - B$ and $A \cap B$

$\therefore \quad n(A) = n(A - B) \cup (A \cap B)$
$= n(A - B) + n(A \cap B) \qquad [\because A - B \text{ and } A \cap B \text{ are disjoint}]$

$\Rightarrow \quad 20 = n(A - B) + 4$

$\Rightarrow \quad n(A - B) = 20 - 4$
$= 16$

$\therefore \ n(A - B) = 16$

Sets Ex 1.8 Q5(iii)

To find: $B - A$

On a similar lines we have B is the disjoint union of $B - A$ and $A \cap B$

i.e $B = (B - A) \cup (A \cap B)$

$\therefore \quad n(B) = n(B - A) + n(A \cap B)$

$\Rightarrow \quad 26 = n(B - A) + 4 \qquad$ [using (i)]

$\Rightarrow \quad n(B - A) = 26 - 4$
$= 22$

$\therefore \ n(B - A) = 22$

Sets Ex 1.8 Q6

Let $n(P)$ denote the total percentage of Indians $n(O)$ denotes the percentage of Indians who like oranges, and $n(B)$ denotes the percentage of Indians who like bananas.

Then, $n(P) = 100$, $n(O) = 76$ and $n(B) = 62$

To find: $n(O \cap B)$

Now,

$n(P) = n(O) + n(B) - n(O \cap B)$

$\Rightarrow \quad 100 = 76 + 62 - n(O \cap B)$

$\Rightarrow \quad 100 = 138 - n(O \cap B)$

$\Rightarrow \quad n(O \cap B) = 138 - 100$
$= 38$

$\therefore$ 38% of Indians like both oranges and bananas.

Sets Ex 1.8 Q7

(i)

Let,

$n(P)$ denote the total number of persons,

$n(H)$ denote the number of persons who speak Hindi and

$n(E)$ denote the number of persons who speak English.

Then,

$n(P) = 950, \ n(H) = 750, \ n(E) = 460$

To find: $n(H \cap E)$

$n(P) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow \quad 950 = 750 + 460 - n(H \cap E)$

$\Rightarrow \quad 950 = 2110 - n(H \cap E)$

$\Rightarrow \quad n(H \cap E) = 2110 - 950$
$= 260$

Hence, 260 persons can speak both Hindi and English.

(ii)
Clearly H is the disjoint union of $H-E$ & $H\cap$
i.e $H=(H-E)\cup(H\cap E)$

$\therefore \quad n(H)=n(H-E)+n(H\cap E)$ $\quad \left[\begin{array}{l}\because \text{if } A \,\&\, B \text{ are disjoint then} \\ n(A\cup B)=n(A)+n(B)\end{array}\right]$

$\Rightarrow \quad 750=n(H-E)+260$

$\Rightarrow \quad n(H-E)=750-260$

$=490$

Hence, 490 persons can speak Hindi only.

(iii)
On a similar lines we have

$E=(E-H)\cup(H\cap E)$

i.e E is the disjoint union of $E-H$ & $H\cap E$

$\therefore \quad n(E)=n(E-H)+n(H\cap E)$

$\Rightarrow \quad 460=n(E-H)+260$

$\Rightarrow \quad n(E-H)=460-260$

$=200$

Hence, 200 persons can speak English only.

Sets Ex 1.8 Q8

(i)
Let,
$n(P)$ denote the total number of persons,
$n(T)$ denote number of persons who drink tea and
$n(C)$ denote number of persons who drink coffee.

Then, $n(P)=50,\ n(T-C)=14,\ n(\mathrm{T})=30$
To find: $n(T\cap C)$

Clearly T is the disjoint union of $T-C$ and $T\cap C$

$\therefore \quad T=(T-C)\cup(T\cap C)$

$\therefore \quad n(T)=n(T-C)+n(T\cap C)$

$\Rightarrow \quad 30=14+n(T\cap C)$

$\Rightarrow \quad n(T\cap C)=30-14$

$=16$

Hence, 16 persons drink tea and coffee both.

(ii)
To find: $C-T$
We know $n(P)=n(C)+n(T)-n(T\cap C)$

$\Rightarrow \quad 50=n(C)+30-16$

$\Rightarrow \quad 50=n(C)+14$

$\Rightarrow \quad n(C)=50-14$

$=36$

New C is the disjoint union of $C-T$ and $T\cap C$

$\therefore \quad C=(C-T)\cup(C\cap T)$

$\Rightarrow \quad n(C)=n(C-T)+n(C\cap T)$

$\Rightarrow \quad 36=n(C-T)+16$ $\quad [\because n(T\cap C)=n(C\cap T)=16]$

$\Rightarrow \quad n(C-T)=36-16$

$=20$

Hence, 20 persons drink coffee but not tea.

Sets Ex 1.8 Q9

(i)
Let $n(P)$ denote total number of people $n(H)$ denote number of people who read newspaper H $n(T)$ denote number of people who read newspaper T and $n(I)$ denote number of people who read newspaper I

Then, $n(P) = 60,\ n(H) = 25,\ n(T) = 26,\ n(I) = 26$

$n(H \cap I) = 9,\ n(H \cap T) = 11,\ n(T \cap I) = 8,\ n(H \cap T \cap I) = 3$

We need to find the number of people who read at least one of the newspaper, i.e., $n(H \text{ or } T \text{ or } I)$, i.e., $n(H \cup T \cup I)$ we know that if A, B, C are 3 sets, then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\therefore\ n(H \cup T \cup I) = n(H) + n(T) + n(I) - n(H \cap T) - n(T \cap I) - n(H \cap I) + n(H \cap T \cap I)$$

$$= 25 + 26 + 26 - 9 - 11 - 8 + 3$$
$$= 25 + 52 - 28 + 3$$
$$= 25 + 52 - 25$$
$$= 52$$

Hence, 52 people read at least one of the newspaper.

(ii)

The venn diagram representing people reading newspapers H, T and I is shown above.

The shaded region shows the number of people who read newspaper H only, newspaper T only and newspaer I only respectively.

The number of people who read newspaper H only equals

$$25 - (8 + 3 + 6)$$
$$= 25 - 17$$
$$= 8$$

The number of people who read newspaper T only

$$= 26 - (8 + 3 + 5)$$
$$= 26 - 16$$
$$= 10$$

And, the number of people who read newspaper I only

$$= 26 - (6 + 3 + 5)$$
$$= 26 - 14$$
$$= 12$$

Hence, the number of people, who read exactly one newspaper $= 8 + 10 + 12 = 30$.

Sets Ex 1.8 Q10

Let,

$n(P)$ denote total number of members,

$n(B)$ denote number of members in the basket ball team

$n(H)$ denote number of members in the hockey team and

$n(F)$ denote number of members in the football team.

Then, $n(B) = 21,\ n(H) = 26,$ and $n(F) = 29$

Also, $n(H \cap B) = 14,\ n(H \cap F) = 15,\ n(F \cap B) = 12,\ n(H \cap B \cap F) = 8$

Now,

$$P = B \cup H \cup F$$

$$\therefore\quad n(P) = n(B \cup H \cup F)$$
$$= n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$$

$$\Rightarrow\quad n(P) = 21 + 26 + 29 - 14 - 15 - 12 + 8$$
$$= 76 - 41 + 8$$
$$= 43$$

Hence, there are 43 members in all.

Sets Ex 1.8 Q11

Let,

$n(P)$ denote the total number of people,

$n(H)$ the number of people who speak Hindi and

$n(B)$ the number of people who speak Bengali.

Then, $n(P) = 1000,\ n(H) = 750,\ n(B) = 400$

We have $P = (H \cup B)$

$$\therefore\quad n(P) = n(H \cup B)$$
$$= n(H) + n(B) - n(H \cap B)$$

$$\Rightarrow\quad 1000 = 750 + 400 - n(H \cap B)$$

$\Rightarrow \quad 1000 = 1150 - n(H \cap B)$

$\Rightarrow \quad n(H \cap B) = 1150 - 1000$

$= 150$

Hence, 150 people can speak both Hindi and Bengali now $H = (H - B) \cup (H \cap B)$, the union being disjoint

$\therefore \quad n(H) = n(H - B) + n(H \cap B)$

$\Rightarrow \quad 750 = n(H - B) + 150$

$\Rightarrow \quad n(H - B) = 750 - 150$

$= 600$

Hence, 600 people can speak Hindi only

On a similar lines we have $B = (B - H) \cup (H \cap B)$

$\Rightarrow \quad n(B) = n(B - H) + n(H \cap B)$

$\Rightarrow \quad 400 = n(B - H) + 150$

$\Rightarrow \quad n(B - H) = 400 - 150$

$= 250$

Hence, 250 people can speak Bengali only.

Sets Ex 1.8 Q12

Let,

$n(P)$ denote the total number of television viewers,

$n(F)$ be the number of people who watch football,

$n(H)$ be the number of people who watch hockey and

$n(B)$ be the number of people who watch basket ball.

Then, $n(P) = 500$, $n(F) = 285$, $n(H) = 195$, $n(B) = 115$, $n(F \cap B) = 45$, $n(F \cap H) = 70$,
$n(H \cap B) = 50$ and $n(F \cup H \cup B) = 50$

Now,

$n((F \cup H \cup B')) = n(P) - n(F \cup H \cup B)$

$\Rightarrow \quad 50 = 500 - (n(F) + n(H) + n(B) - n(F \cap H) - n(H \cap B) - n(F \cap B) + n(F \cap H \cap B))$

$\Rightarrow \quad 50 = 500 - (285 + 195 + 115 - 70 - 50 - 45 + n(F \cap H \cap B))$

$\Rightarrow \quad 50 = 500 - 430 - n(F \cap H \cap B)$

$\Rightarrow \quad 50 = 70 - n(F \cap H \cap B)$

$\Rightarrow \quad n(F \cap H \cap B) = 70 - 50$

$= 20$

Hence, 20 people watch all the 3 games

Number of people who watch only football

$= 285 - (50 + 20 + 25)$

$= 285 - 95$

$= 190$

Number of people who watch only hockey

$= 195 - (50 + 20 + 30)$

$= 195 - 100$

$= 95$

And, number of people who watch only basket ball

$= 115 - (25 + 20 + 30)$

$= 115 - 75$

$= 40$

Number of people who watch exactly one of the three games

= number of people who watch either football only or hockey only or basket ball only

$= 190 + 95 + 40$ $\quad [\because$ they are pairwise disjoint$]$

$= 325$

Hence, 325 people watch exactly one of the three games.

Sets Ex 1.8 Q13

(i)

Let $n(P)$ denote total number of persons

$n(A)$ denote number of people who read magazine A

$n(B)$ denote number of people who read magazine B

and $n(C)$ denote number of people who read magazine C

and $n(C)$ denote number of people who read magazine C

Then, $n(P) = 100,\ n(A) = 28,\ n(B) = 30,\ n(C) = 42,\ n(A \cap B) = 8,$
$n(A \cap C) = 10, n(B \cap C) = 5,\ n(A \cap B \cap C) = 3$

Now,
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 28 + 30 + 42 - 8 - 10 - 5 + 3$$
$$= 100 - 23 + 3$$
$$= 100 - 20$$
$$= 80$$

$\therefore$ Number of people who read none of the three magazines
$$= n(A \cup B \cup C)'$$
$$= n(P) - n(A \cup B \cup C)$$
$$= 100 - 80$$
$$= 20$$

Hence, 20 people read none of the three magazines.

(ii)
$$n(C\ only) = 42 - (7 + 3 + 2)$$
$$= 42 - 12$$
$$= 30$$

Sets Ex 1.8 Q14

(i)
Let $n(P)$ denote total number of students
$n(E)$ denote number of students studying English language
$n(H)$ denote number of students studying Hindi language and
$n(S)$ denote number of students studying Sanskrit language

Then, $n(P) = 100,\ n(E - H) = 23,\ n(E \cap S) = 8,\ n(E) = 26,\ n(S) = 48,$
$n(S \cap H) = 8,\ n((E \cup H \cup S)') = 24$

Number of students studying English only = 18

We have,
$$n((E \cup H \cup S)') = 24$$
$$\Rightarrow \quad n(P) - n(E \cup H \cup S) = 24$$
$$\Rightarrow \quad 100 - 24 = n(E \cup H \cup S)$$
$$\Rightarrow \quad n(E \cup H \cup S) = 76$$

We have $n(E \cup H \cup S) = n(E) + n(H) + n(S) - n(E \cap H) - n(H \cap S) - n(E \cap S)$
$+ n(E \cap H \cap S)$
$$\Rightarrow \quad 76 = 26 + n(H) + 48 - 3 - 8 - 8 + 3$$
$$\Rightarrow \quad 76 = 26 + n(H) + 48 - 16$$
$$\Rightarrow \quad 76 = 26 + 32 + n(H)$$
$$\Rightarrow \quad n(H) = 76 - 58$$
$$= 18$$

$\therefore$ 18 students were studying Hindi.

(ii)
From (i) we have $n(E \cap H) = 3$

$\therefore$ 3 students were studying both English and Hindi.

Sets Ex 1.8 Q15

Let $n(P_1)$ be the number of persons liking product P_1
$n(P_2)$ be the number of persons liking product P_2
and $n(P_3)$ be the number of persons liking product P_3

Then, $n(P_1) = 21,\ n(P_2) = 26,\ n(P_3) = 29,\ n(P_1 \cap P_2) = 14,$
$n(P_1 \cap P_3) = 12,\ n(P_2 \cap P_3) = 14,\ n(P_1 \cap P_2 \cap P_3) = 8$

$= 29 - 18$

$= 11$

Hence, 11 persons liked product P_3 only.