RD Sharma Solutions Class 11 Maths Chapter 3 Ex 3.1

Functions Ex 3.1 Q1

Function = Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B, if

(i) for each $a \in A$ there exists $b \in B$ such that $(a,b) \in f$

(ii) $(a,b) \in f$ and $(a,c) \in f \Rightarrow b = c$

If $(a,b) \in f$, then 'b' is called the image of 'a' under f

If a function f is expresed as the set of ordered pairs, the domain f is the set of all first components of members of f and the range of f is the set of second components of members of f.

Functions Ex 3.1 Q2

Function = Let A and B be two non-empty sets. Then a function 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set b such that:

(i) all elements of set A are associated to element in set B.

(ii) an element of se A is associated to a unique element in set B.

In other words, a function 'f' from a set A to set B associates each element of set A to a unique element of set b.

Functions Ex 3.1 Q3

Function is a type of relation. But in a function no two ordered pairs have the same first element. For eg: R_1 and R_2 are two relations.

Clearly, R_1 is a function, but R_2 is not a function because two ordered pairs $(1,2)$ and $(1,4)$ have the same first element.
This means every function is a relation but every relation is not a function.

Functions Ex 3.1 Q4

We have,

$$f(x) = x^2 - 2x - 3$$

Now,

$$\begin{aligned} f(-2) &= (-2)^2 - 2(-2) - 3 \\ &= 4 + 4 - 3 \\ &= 5 \\ f(-1) &= (-1)^2 - 2(-1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \\ f(-0) &= (-0)^2 - 2 \times 0 - 3 \\ &= -3 \\ f(1) &= (1)^2 - 2 \times 1 - 3 \\ &= 1 - 2 - 3 \\ &= -4 \\ f(2) &= (2)^2 - 2 \times 2 - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

(a) Rang$(f) = \{-4, -3, 0, 5\}$

(b) Clearly, pre-images of $6, -3$ and 5 is ϕ, $\{0,2\}, -2$ respectively.

Functions Ex 3.1 Q5

We have,

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

Now,

$$f(1) = 4 \times 1 + 1 = 5,$$
$$f(-1) = 3 \times (-1) - 2 = -3 - 2 = -5,$$
$$f(0) = 1,$$
and, $f(2) = 4 \times 2 + 1 = 9$

$\therefore \quad f(1) = 5, \quad f(-1) = -5,$
$f(0) = 1, \quad f(2) = 9,$

Functions Ex 3.1 Q6

We have,

$f(x) = x^2$ ---(i)

(a) clearly range of $f = R^+$ (set of all real numbers greater than or equal to zero)

(b) we have,

$\{x : f(x) = 4\}$

$\Rightarrow f(x) = 4$ ---(ii)

Using equation (i) and equation (ii), we get

$x^2 = 4$

$\Rightarrow x = \pm 2$

$\therefore \{x : f(x) = 4\} = \{-2, 2\}$

(c) $\{y : f(y) = -1\}$

$\Rightarrow f(y) = -1$ ---(iii)

Clearly, $x^2 \neq -1$ or $x^2 \geq 0$

$\Rightarrow f(y) \neq -1$

$\therefore \{y : f(y) = -1\} = \phi$

Functions Ex 3.1 Q7

We have,

$f = R^+ \to R$

and $f(x) = \log_e x$ ---(i)

(a) Now,

$f = R^+ \to R$

$\therefore$ the image set of the domain of $f = R$

(b) Now,

$\{x : f(x) = -2\}$

$\Rightarrow f(x) = -2$ ---(ii)

Using equation (i) and equation (ii), we get

$\log_e x = -2$

$\Rightarrow x = e^{-2}$ $[\because \log_a b = c \Rightarrow b = a^c]$

$\therefore \{x : f(x) = -2\} = \{e^{-2}\}$

(c) Now,

$f(xy) = \log_e(xy)$ $[f(x) = \log_e x]$

$= \log_e x + \log_e y$ $[\because \log mn = \log m + \log n]$

$f(x) + f(y)$

$\therefore f(xy) = f(x) + f(y)$

Yes, $f(xy) = f(x) + f(y)$.

Functions Ex 3.1 Q8

(a) we have,

$\{(x, y) = y = 3x, x \in \{1,2,3\}, y \in \{3,6,9,12\}\}$

Putting $x = 1,2,3$ in $y = 3x$, we get

$y = 3,6,9$ respectively

$\therefore R = \{(1,3), (2,6), (3,9)\}$

Yes, it is a function.

(b) we have,

$\{(x, y) : y > x + 1, x = 1,2 \text{ and } y = 2,4,6\}$

Putting $x = 1,2$ in $y > x + 1$, we get

$y > 2$, $y > 3$ respectively.

$\therefore R = \{(1,4), (1,6), (2,4), (2,6)\}$

It is not a function from A to B because two ordered pairs in R have the same first element.

(c) we have,

$$\{(x,y) = x + y = 3,\ x, y \in \{0,1,2,3\}\}$$

Now,

$$y = 3 - x$$

Putting $x = 0,1,2,3,$ we get

$$y = 3,2,1,0 \text{ respectively}$$

$$\therefore \quad R = \{(0,3),\ (1,2),\ (2,1),\ (3,0)\}$$

Yes, this relation is a function.

Functions Ex 3.1 Q9

We have,

$$f : R \to R \text{ and } g : c \to c$$

$\therefore$ Domain $(f) = R$ and Domain $(g) = c$

$\therefore$ Domain $(f) \neq$ Domain $(g) = c$

$\therefore$ $f(x)$ and $g(x)$ are not equal functions.

Functions Ex 3.1 Q10

(i) We have,

$$f(x) = x^2$$

Range of $f(x) = R^+$ (set of all real numbers greater than or equal to zero)

$$= \{x \in R \mid x \geq 0\}$$

(ii) We have,

$$g(x) = \sin x$$

Range of $g(x) = \{x \in R : -1 \leq x \leq 1\}$

(iii) We have,

$$h(x) = x^2 + 1$$

Range of $h(x) = \{x \in R : x \geq 1\}$

Functions Ex 3.1 Q11

(a) We have,

$$f_1 = \{(1,1),\ (2,11),\ (3,1),\ (4,15)\}$$

f_1 is a function from X to Y.

(b) We have,

$$f_2 = \{(1,1),\ (2,7),\ (3,5)\}$$

f_2 is not a function from X to Y because there is an element $4 \in x$ which is not associated to any element of Y.

(c) We have,

$$f_3 = \{(1,5),\ (2,9),\ (3,1),\ (4,5),\ (2,11)\}$$

f_3 is not a function from X to Y because an element $2 \in x$ is associated to two elements 9 and 11 in Y.

Functions Ex 3.1 Q12

We have,

$f(x)$ = highest prime factor of x.

$$\therefore \quad 12 = 3 \times 4,$$
$$13 = 13 \times 1,$$
$$14 = 7 \times 2,$$
$$15 = 5 \times 3,$$
$$16 = 2 \times 8,$$
$$17 = 17 \times 1$$

$$\therefore \quad f = \{(12,3),\ (13,13),\ (14,7),\ (15,5),\ (16,2),\ (17,17)\}$$

$\therefore$ Range$(f) = \{3,13,7,5,2,17\}$

Functions Ex 3.1 Q13

We know that,

if $f : A \to 13$

such that $y \in 3$. Then,

$f^{-1}(y) = \{x \in A : f(x) = y\}$. In other words, $f^{-1}(y)$ is the set of pre-images of y.

Let $f^{-1}\{17\} = x$. Then, $f(x) = 17$

$\Rightarrow \quad x^2 + 1 = 17$

$\Rightarrow \quad x^2 = 17 - 1 = 16$

$\Rightarrow \quad x = \pm 4$

Let $f^{-1}\{-3\} = x$. Then, $f(x) = -3$

$\Rightarrow \quad x^2 + 1 = -3$

$\Rightarrow \quad x^2 = -3 - 1 = -4$

$\Rightarrow \quad x = \sqrt{-4}$

$\therefore \quad f^{-1}\{-3\} = \theta$

Functions Ex 3.1 Q14

We have,

$A = \{p, q, r, s\}$ and $B = \{1, 2, 3\}$

(a) Now,

$R_1 = \{(p,1),\ (q,2),\ (r,1),\ (s,2)\}$

R_1 is a function

(b) Now,

$R_2 = \{(p,1),\ (q,1),\ (r,1),\ (s,1)\}$

R_2 is a function

(c) Now,

$R_3 = \{(p,1),\ (q,2),\ (p,2),\ (s,3)\}$

R_3 is not a function because an element $p \in A$ is associated to two elements 1 and 2 in B.

(d) Now,

$R_4 = \{(p,2),\ (q,3),\ (r,2),\ (s,2)\}$

R_4 is a function.

Functions Ex 3.1 Q15

We have,

$f(n)$ = the highest prime factor of n.

Now,

$9 = 3 \times 3,$

$10 = 5 \times 2,$

$11 = 11 \times 1,$

$12 = 3 \times 4,$

$13 = 13 \times 1$

$\therefore \quad f = \{(9,3),\ (10,5),\ (11,11),\ (12,3),\ (13,13)\}$

Clearly, range $(f) = \{3, 5, 11, 13\}$

Functions Ex 3.1 Q16

We have,

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$$

and, $$g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

Now, $f(3) = (3)^2 = 9$ and $f(3) = 3 \times 3 = 9$

and, $g(2) = (2)^2 = 4$ and $g(2) = 3 \times 2 = 6$

We observe that $f(x)$ takes unique value at each point in its domain $[0, 10]$. However $g(x)$ does not takes unique value at each point in its domain $[0, 10]$.

Hence, $g(x)$ is not a function.

Functions Ex 3.1 Q17

Given $f(x) = x^2$

$$f(1.1) = 1.21$$
$$f(1) = 1$$
$$\frac{f(1.1) - f(1)}{(1.1) - 1} = \frac{1.21 - 1}{1.1 - 1}$$
$$= \frac{0.21}{0.1}$$
$$= 2.1$$

Functions Ex 3.1 Q18

$f : X \to R$ given by $f(x) = x^3 + 1$

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

$f(0) = (0)^3 + 1 = 0 + 1 = 1$

$f(3) = (3)^3 + 1 = 27 + 1 = 28$

$f(9) = (9)^3 + 1 = 81 + 1 = 82$

$f(7) = (7)^3 + 1 = 343 + 1 = 344$

Set of ordered pairs are $\{(-1,0), (0,1), (3,28), (9,82), (7,344)\}$