RD Sharma Solutions Class 11 Maths Chapter 5 Ex 5.1

Trigonometric Functions Ex 5.1 Q1

$$\text{LHS} = \sec^4\theta - \sec^2\theta$$
$$= \sec^2\theta(\sec^2\theta - 1)$$
$$= (1+\tan^2\theta)\tan^2\theta \qquad [\because \sec^2\theta = 1+\tan^2\theta]$$
$$= \tan^2\theta + \tan^4\theta$$
$$= \tan^4\theta + \tan^2\theta$$
$$= \text{RHS}$$

LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q2

$$\text{LHS} = \sin^6\theta + \cos^6\theta$$
$$= (\sin^2\theta)^3 + (\cos^2\theta)^3$$
$$= (\sin^2\theta+\cos^2\theta)\left[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right] \qquad (\because a^3+b^3 = (a+b)(a^2-ab+b^2))$$
$$= \quad (\sin^2\theta)^2 + (\cos\ \theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$$

[adding and subtracting $2\sin^2\theta\cos^2\theta$ and using identity $\sin^2\theta + \cos^2\theta = 1$]

$$= (\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$
$$= (\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$
$$= 1^2 - 3\sin^2\theta\cos^2\theta \quad (\because \sin^2\theta+\cos^2\theta = 1)$$
$$= 1 - 3\sin^2\theta\cos^2\theta$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q3

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos^2\theta.\sin^2\theta.1}{\sin^2\theta.\cos^2\theta} \left(\begin{array}{l}\because \sin^2\theta+\cos^2\theta = 1 \\ \Rightarrow 1-\sin^2\theta = \cos^2\theta, \text{and} \\ \qquad 1-\cos^2\theta = \sin^2\theta\end{array}\right)$$

$$= 1$$
$$= \text{RHS}$$

Proved

Trigonometric Functions Ex 5.1 Q4

$$\text{LHS} = \operatorname{cosec}\theta(\sec\theta - 1) - \cot\theta(1-\cos\theta)$$

$$= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right) - \frac{\cos\theta}{\sin\theta}(1-\cos\theta) \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}\cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{(1-\cos\theta)}{\sin\theta\cos\theta} - \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$$

$$= \frac{(1-\cos\theta) - \cos^2\theta(1-\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)\sin^2\theta}{\sin\theta\cos\theta} \quad (\because 1-\cos^2\theta = \sin^2\theta)$$

$$= (1 - \cos\theta)\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \sin\theta$$

$$= \tan\theta - \sin\theta \quad (\because \tan\theta = \sin\theta - \cos\theta)$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q5

$$\text{LHS} = \frac{1 - \sin A\cos A}{\cos A(\sec A - \operatorname{cosec} A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$$

$$= \frac{1 - \sin A\cos A}{\cos A\left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)} \cdot \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A\cos A)}$$

$$\begin{bmatrix} \text{Using } a^2 - b^2 = (a-b)(a+b) \\ \text{and } a^3 + b^3 = (a+b)(a^2 + b^2 - ab) \end{bmatrix}$$

$$= \frac{(1 - \sin A\cos A)}{\cos A\left(\frac{\sin A - \cos A}{\cos A\sin A}\right)} \cdot \frac{(\sin A - \cos A)}{(1 - \sin A\cos A)} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{\cos A\sin A}{\cos A}$$

$$= \sin A$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q6

$$\text{LHS} = \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{(\sin A / \cos A)}{\left(1 - \frac{\cos A}{\sin A}\right)} + \frac{(\cos A / \sin A)}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A\frac{(\sin A - \cos A)}{\sin A}} + \frac{\cos A}{\sin A\frac{(\cos A - \sin A)}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos A\sin A(\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A)}{\cos A\sin A(\sin A - \cos A)} \quad \left[\text{Using } a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$$

$$= \frac{1 + \sin A\cos A}{\sin A\cos A} \quad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= \frac{1}{\sin A\cos A} + \frac{\sin A\cos A}{\sin A\cos A}$$

$$= \sec A\operatorname{cosec} A + 1 \quad \left[\because \frac{1}{\cos A} = \sec A, \frac{1}{\sin A} = \operatorname{cosec} A\right]$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q7

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{} + \frac{\sin^3 A - \cos^3 A}{}$$

$\quad \sin A + \cos A \qquad \sin A - \cos A$

$$= \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \cos^2}{\sin A - \cos A}$$

$$\left(\text{Using } a^3 + b^3 = (a+b)(a^2 + b^2 - ab) \text{ and } a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right)$$

$= (1 - \sin A \cos A) + (1 + \sin A \cos A) \left(\because \sin^2 A + \cos^2 A = 1\right)$

$= 2$

$=$ RHS

Trigonometric Functions Ex 5.1 Q8

LHS $= (\sec A \sec B + \tan A \tan B)^2 - (\sec A \tan B + \tan A \sec B)^2$

$= (\sec A \sec B)^2 + (\tan A \tan B)^2 + 2 \sec A \sec B \tan A \tan B$

$- \left((\sec A \tan B)^2 + (\tan A \sec B)^2 + 2 \sec A \tan B \tan A \sec B\right) \quad \left[\text{Using} (a+b)^2 = a^2 + b^2 + 2ab\right]$

$= \sec^2 A \sec^2 B + \tan^2 A \tan^2 B + 2 \sec A \sec B \tan A \tan B$

$- \sec^2 A \tan^2 B - \tan^2 A \sec^2 B - 2 \sec A \sec B \tan A \tan B \left[\text{Using } (ab)^2 = a^2 b^2\right]$

$= \sec^2 A \sec^2 B - \sec^2 A \tan^2 B + \tan^2 A \tan^2 B - \tan^2 A \sec^2 B$

$= \sec^2 A(\sec^2 B - \tan^2 B) + \tan^2 A(\tan^2 B - \sec^2 B)$

$= \sec^2 A\, 1 - \tan^2 A.1 \qquad \left[\begin{array}{l} \because \sec^2\theta = 1 + \tan^2\theta \\ \Rightarrow \sec^2\theta - \tan^2\theta = 1 \end{array}\right]$

$= 1 + \tan^2 A - \tan^2 A$

$= 1$

$=$ RHS

Proved

Trigonometric Functions Ex 5.1 Q9

RHS $= \dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta}$

$$= \frac{((1+\cos\theta)+\sin\theta)}{(1+\cos\theta) - \sin\theta} \times \frac{((1+\cos\theta) + \sin\theta)}{(1 + \cos\theta + \sin\theta)}$$

$$= \frac{((1+\cos\theta) + \sin\theta)^2}{(1+\cos\theta)^2 - \sin^2\theta} \qquad \left(\begin{array}{l}\text{Using } (a+b)(a+b) = (a+b)^2 \\ \& (a+b)(a-b) = a^2 b^2\end{array}\right)$$

$$= \frac{(1+\cos\theta)^2 + \sin^2\theta + 2\sin\theta(1+\cos\theta)}{1 + \cos^2\theta + 2\cos\theta - \sin^2\theta} \qquad \left(\text{Using } (a+b)^2 = a^2 + b^2 + 2ab\right)$$

$$= \frac{1 + \cos^2\theta + 2.1\cos\theta + \sin^2\theta + 2\sin\theta(1+\cos\theta)}{1 + \cos^2 + 2\cos\theta - (1 - \cos^2\theta)} \qquad \left(\text{Using } \sin^2\theta = 1 - \cos^2\theta\right)$$

$$= \frac{1 + 1 + 2\cos\theta + 2\sin\theta(1+\cos\theta)}{1 - 1 + \cos^2\theta + \cos^2\theta + 2\cos\theta} \qquad \left(\text{Using } \sin^2\theta + \cos^2\theta = 1\right)$$

$$= \frac{2 + 2\cos\theta + 2\sin\theta(1+\cos\theta)}{2\cos^2\theta + 2\cos\theta}$$

$$= \frac{2(1+\cos\theta) + 2\sin\theta(1+\cos\theta)}{2\cos\theta(\cos\theta + 1)}$$

$$= \frac{(1+\cos\theta)(2 + 2\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta}{}$$

$-\overline{1-\sin\theta}$

Trigonometric Functions Ex 5.1 Q10

$$\text{LHS} = \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)} + \frac{\cos^3\theta}{\sin^3\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)} \qquad \left(\because\ \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right)$$

$$= \frac{\sin^3\theta\cos^2\theta}{\cos^3\theta\left(\cos^2\theta+\sin^2\theta\right)} + \frac{\cos^3\theta\sin^2\theta}{\sin^3\theta\left(\sin^2\theta+\cos^2\theta\right)}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} \qquad \left(\because\ \cos^2\theta+\sin^2\theta = 1\right)$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad \left(\text{adding and subtracting } 2\sin^2\theta\cos^2\theta\right)$$

$$= \frac{\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad \left(\because\ \sin^2\theta+\cos^2\theta = 1\right)$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q11

$$\text{LHS} = 1-\frac{\sin^2\theta}{1+\cot\theta}-\frac{\cos^2\theta}{1+\tan\theta}$$

$$= 1-\frac{\sin^2\theta}{1+\frac{\cos\theta}{\sin\theta}}-\frac{\cos^2\theta}{1+\frac{\sin\theta}{\cos\theta}} \qquad \left(\because\ \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= 1-\frac{\sin^2\theta}{\frac{\sin\theta+\cos\theta}{\sin\theta}}-\frac{\cos^2\theta}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$= 1-\frac{\sin^3\theta}{\sin\theta+\cos\theta}-\frac{\cos^3\theta}{\cos\theta+\sin\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin^3+\cos^3\theta\right)}{\sin\theta+\cos\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin\theta+\cos\theta\right)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta}$$

$$\left(\text{Using } a^3+b^3 = (a+b)\left(a^2+b^2-ab\right)\right)$$

$$\frac{\left(\sin\theta+\cos\theta\right)\left(1-\left(1-\sin\theta\cos\theta\right)\right)}{\sin\theta+\cos\theta} \qquad \left(\text{Using } \sin^2\theta+\cos^2\theta = 1\right)$$

$= \sin\theta\cos\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q12

$$\text{LHS} = \left(\frac{1}{sec^2\theta - cos^2\theta} + \frac{1}{cosec^2\theta - sin^2\theta}\right) sin^2\theta\, cos^2\theta$$

$$= \left(\frac{1}{\frac{1}{cos^2\theta} - cos^2\theta} + \frac{1}{\left(\frac{1}{sin^2\theta} - sin^2\theta\right)}\right) sin^2\theta\, cos^2\theta$$

$$= \left(\frac{1}{\frac{1-cos^4\theta}{cos^2\theta}} + \frac{1}{\frac{1-sin^4\theta}{sin^2\theta}}\right) sin^2\theta\, cos^2\theta$$

$$= \left(\frac{cos^2\theta}{\left(1-cos^2\theta\right)\left(1+cos^2\theta\right)} + \frac{sin^2\theta}{\left(1-sin^2\theta\right)\left(1+sin^2\theta\right)}\right) sin^2\theta\, cos^2\theta$$

$$\left(\begin{aligned} \text{Using } 1-a^4 &= 1-\left(a^2\right)^2 \\ &= \left(1-a^2\right)\left(1+a^2\right) \end{aligned}\right)$$

$$= \left(\frac{cos^2\theta}{sin^2\theta\left(1+cos^2\theta\right)} + \frac{sin^2\theta}{cos^2\theta\left(1+sin^2\right)}\right) sin^2\theta\, cos^2\theta \qquad \left(\begin{aligned} &\text{Using } 1-cos^2\theta = sin^2\theta \\ &\quad \& \quad 1-sin^2\theta = cos^2\theta \end{aligned}\right)$$

$$= \left(\frac{cos^4\theta\left(1+sin^2\theta\right) + sin^4\theta\left(1+cos^2\theta\right)}{sin^2\theta\, cos^2\theta\left(1+cos^2\theta\right)\left(1+sin^2\theta\right)}\right).\, sin^2\theta\, cos^2\theta$$

$$= \frac{cos^4\theta + sin^2\theta\, cos^4\theta + sin^4\theta + cos^2\theta\, sin^4\theta}{\left(1+cos^2\theta\right)\left(1+sin^2\theta\right)}$$

$$= \frac{\left(cos^2\theta\right)^2 + \left(sin^2\theta\right)^2 + 2cos^2\theta\, sin^2\theta - 2cos^2\theta\, sin^2\theta + sin^2\theta\, cos^4\theta + cos^2\theta\, sin^4\theta}{\left(1+cos^2\theta\right)\left(1+sin^2\theta\right)}$$

$$\left(\text{adding and subtracting } 2cos^2\theta\, sin^2\theta\right)$$

$$= \frac{\left(cos^2\theta + sin^2\theta\right)^2 - 2cos^2\theta\, sin^2\theta + sin^2\theta\, cos^2\theta\left(cos^2\theta + sin^2\theta\right)}{1 + sin^2\theta + cos^2\theta + sin^2\theta\, cos^2\theta}$$

$$= \frac{1^2 - 2cos^2\theta\, sin^2\theta + sin^2\theta\, cos^2\theta.1}{1 + 1 + sin^2\theta\, cos^2\theta}$$

$$= \frac{1 - sin^2\theta\, cos^2\theta}{2 + sin^2\theta\, cos^2\theta}$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q13

$$\text{LHS} = \left(1 + tan\,\alpha\, tan\,\beta\right)^2 + \left(tan\,\alpha - tan\,\beta\right)^2$$

$$= 1 + \left(tan\,\alpha + tan\,\beta\right)^2 + 2.1\, tan\,\alpha\, tan\,\beta + \left(tan\,\alpha\right)^2 + \left(tan\,\beta\right)^2 - 2\, tan\,\alpha.\, tan\,\beta$$

$$\left(\text{Using } \left(a+b\right)^2 = a^2 + b^2 + 2ab \text{ and } \left(a-b\right)^2 = a^2 + b^2 - 2ab\right)$$

$$= 1 + tan^2\alpha + tan^2\beta + 2\, tan\,\alpha\, tan\,\beta + tan^2\alpha + tan^2\beta - 2\, tan\,\alpha\, tan\,\beta$$

$$= 1 + tan^2\alpha + tan^2\alpha + tan^2\beta + tan^2\beta$$

$$= sec^2\alpha + tan^2\beta\left(1 + tan^2\alpha\right) \qquad \left(\because 1 + tan^2\alpha = sec^2\alpha\right)$$

$$= sec^2\alpha + tan^2\beta.\, sec^2\alpha$$

$$= sec^2\alpha\left(1 + tan^2\beta\right)$$

$= sec^2\alpha . sec^2\beta$ $\left(\because 1+tan^2\beta = sec^2\beta\right)$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q14

$$\text{LHS} = \frac{(1+cot\,\theta+tan\,\theta)(sin\,\theta-cos\,\theta)}{sec^3\theta-cosec^3\theta}$$

$$= \frac{\left(1+\frac{cos\,\theta}{sin\,\theta}+\frac{sin\,\theta}{cos\,\theta}\right)}{\left(\frac{1}{cos^3\theta}-\frac{1}{sin^3\theta}\right)} \qquad \left(\begin{array}{l}\because cot\,\theta = \frac{cos\,\theta}{sin\,\theta}, tan\,\theta = \frac{sin\,\theta}{cos\,\theta}\\ sec\,\theta = \frac{1}{cos\,\theta}, cosec\theta = \frac{1}{sin\,\theta}\end{array}\right)$$

$$= \left(\frac{1+\frac{cos^2\theta+sin^2\theta}{sin\,\theta\,cos\,\theta}}{\frac{sin^3\theta-cos^3\theta}{cos^3\theta\,sin^3\theta}}\right)(sin\,\theta-cos\,\theta)$$

$$= \frac{(sin\,\theta\,cos\,\theta+1)\,sin^3\theta\,cos^3\theta}{sin\,\theta\,cos\,\theta.\left(sin^3\theta-cos^3\theta\right)}.(sin\,\theta-cos\,\theta) \qquad \left(\because sin^2\theta+cos^2\theta=1\right)$$

$$= \frac{(1+sin\,\theta\,cos\,\theta)\,sin^2\theta\,cos^2\theta.(sin\,\theta-cos\,\theta)}{(sin\,\theta-cos\,\theta)\left(sin^2\theta+cos^2\theta+sin\,\theta\,cos\,\theta\right)} \qquad \left(\because a^3-b^3=(a-b)\left(a^2+b^2+ab\right)\right)$$

$$= \frac{(1+sin\,\theta\,cos\,\theta).\,sin^2\theta\,cos^2\theta}{(1+sin\,\theta\,cos\,\theta)}$$

$= sin^2\theta\,cos^2\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q15

$$\text{LHS} = \frac{2\,sin\,\theta\,cos\,\theta-cos\,\theta}{1-sin\,\theta+sin^2\theta-cos^2\theta}$$

$$= \frac{cos\,\theta\,(2\,sin\,\theta-1)}{1-cos^2\theta+sin^2\theta-sin\,\theta}$$

$$= \frac{cos\,\theta\,(2\,sin\,\theta-1)}{sin^2\theta+sin^2\theta-sin\,\theta} \qquad \left(\because 1-cos^2\theta = sin^2\theta\right)$$

$$= \frac{cos\,\theta\,(2\,sin\,\theta-1)}{2\,sin^2\theta-sin\,\theta}$$

$$= \frac{cos\,\theta\,(2\,sin\,\theta-1)}{sin\,\theta\,(2\,sin\,\theta-1)}$$

$$= \frac{cos\,\theta}{sin\,\theta}$$

$= cot\,\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q16

$\text{LHS} = cos\,\theta\,(tan\,\theta+2)(2\,tan\,\theta+1)$

$$= cos\,\theta\left(\frac{sin\,\theta}{cos\,\theta}+2\right)\left(\frac{2\,sin\,\theta}{cos\,\theta}+1\right)\left(\because tan\,\theta = \frac{sin\,\theta}{cos\,\theta}\right)$$

$$= cos\,\frac{(sin\,\theta+2\,cos\,\theta)(2\,sin\,\theta+cos\,\theta)}{cos\,\theta.\,cos\,\theta}$$

$(2\,sin^2\theta+sin\,\theta\,cos\,\theta+4\,sin\,\theta\,cos\,\theta+2\,cos^2\theta)$

$$= \frac{[illegible]}{\cos\theta}$$

$$= \frac{2\left(\sin^2\theta + \cos^2\theta\right) + 5\sin\theta\cos\theta}{\cos\theta}$$

$$= \frac{2 + 5\sin\theta\cos\theta}{\cos\theta} \quad \left(\because \sin^2\theta + \cos^2\theta\right) = 1$$

$$= \frac{2}{\cos\theta} + \frac{5\sin\theta\cos\theta}{\cos\theta}$$

$= 2\sec\theta + 5\sin\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q17

$$\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\cos\theta+\sin\theta)(1-\cos\theta+\sin\theta)} = x \quad [\textit{Rationalizing the denominator}]$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\sin\theta)^2-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta-2\sin\theta\cos\theta+2\sin^2\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin^2\theta+2\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin\theta(1+\sin\theta)} = x$$

$$\Rightarrow \frac{1+\cos\theta-\sin\theta}{1+\sin\theta} = x \ [\textit{Cancelling the } 2\sin\theta \textit{ in both Numerator and Denominator}]$$

Hence Proved

Trigonometric Functions Ex 5.1 Q18

Now, $\cos\theta = \sqrt{1-\sin^2\theta}$

$$= \sqrt{\frac{1-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}} \qquad \left[\because \sin\theta = \frac{a^2-b^2}{a^2+b^2}\right]$$

$$= \sqrt{\frac{\left(a^2+b^2\right)^2-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}}$$

$$= \sqrt{\frac{\left(a^2+b^2+a^2-b^2\right)\left(a^2+b^2-a^2+b^2\right)}{a^2+b^2}} \quad \left(\text{Using } x^2-y^2 = (x-y)(x+y)\right)$$

$$= \sqrt{\frac{2a^2\times 2b^2}{a^2+b^2}}$$

$$= \frac{2ab}{a^2+b^2} \ \ldots\ldots\ldots\ldots (ii)$$

Now $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\frac{a^2-b^2}{a^2+b^2}}{\frac{2ab}{a^2+b^2}}$$

$$= \frac{a^2-b^2}{2ab}$$

[illegible]

$\sec\theta = \frac{\quad}{\cos\theta} = \frac{\quad}{2ab}$ (from (ii))

and $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{a^2+b^2}{a^2-b^2}$ (from (i))

Trigonometric Functions Ex 5.1 Q19

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$= \sqrt{\frac{\frac{a}{b}+1}{\frac{a}{b}-1}} + \sqrt{\frac{\frac{a}{b}-1}{\frac{a}{b}+1}} \text{ [Dividing both Numerator and denominator by } b\text{]}$$

$$= \sqrt{\frac{\tan\theta+1}{\tan\theta-1}} + \sqrt{\frac{\tan\theta-1}{\tan\theta+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1}} + \sqrt{\frac{\frac{\sin\theta}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}} + \sqrt{\frac{\frac{\sin\theta-\cos\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}} + \sqrt{\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}}$$

$$= \frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

$$= \frac{2\sin\theta}{\sqrt{\sin^2\theta - \cos^2\theta}}$$

Trigonometric Functions Ex 5.1 Q20

Given $= \tan\theta = \frac{a}{b}$

To show: $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Since, $\tan\theta = \frac{a}{b}$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$\Rightarrow \quad b\sin\theta = a\cos\theta = \lambda$ (say)

$\Rightarrow \quad \sin\theta = \frac{\lambda}{b}$ and $\cos\theta = \frac{\lambda}{a}$

how $\quad \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{\frac{a.\lambda}{b} - \frac{b.\lambda}{a}}{\frac{a.\lambda}{b} + \frac{b.\lambda}{a}}$

$$= \frac{\lambda\left(\frac{a}{b} - \frac{b}{a}\right)}{\lambda\left(\frac{a}{b} + \frac{b}{a}\right)}$$

$$= \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{b} + \frac{b}{a}}$$

$$= \frac{\frac{a^2-b^2}{ab}}{\frac{a^2+b^2}{ab}}$$

$= \frac{a^2-b^2}{\quad}$

$$= a^2 + b^2$$

Proved

Trigonometric Functions Ex 5.1 Q21

Given, $\operatorname{cosec}\theta - \sin\theta = a^3, \sec\theta - \cos\theta = b^3$

To show: $a^2b^2\left(a^2+b^2\right) = 1$

Since, $\operatorname{cosec}\theta - \sin\theta = a^3$

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = a^3 \qquad \left(\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right)$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = a^3 \qquad \left(\because 1-\sin^2\theta = \cos^2\theta\right)$$

$$\Rightarrow \quad a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$

$$\text{Since, } \frac{1}{\cos\theta} - \cos\theta = b^3 \qquad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$$

$$\Rightarrow \quad \frac{1-\cos^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} = b^3 \qquad \left(\because 1-\cos^2\theta = \sin^2\theta\right)$$

$$\Rightarrow \quad b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

$$\text{Now, } a^2b^2\left(a^2+b^2\right) = \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left(\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$$

$$= \cos^{2/3}\theta \times \sin^{2/3}\theta \, \frac{\left(\cos^{6/3}\theta + \sin^{6/3}\theta\right)}{\sin^{2/3}\theta . \cos^{2/3}\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

Proved

Trigonometric Functions Ex 5.1 Q22

Let,

$$\cot\theta\left(1+\sin\theta\right) = 4m \qquad ---(i)$$

and, $$\cot\theta\left(1-\sin\theta\right) = 4n \qquad ---(ii)$$

To show: $\left(m^2-n^2\right)^2 = mn$

From (i) and (ii), we get

$$m = \frac{\cot\theta\left(1+\sin\theta\right)}{4} \quad \& \quad n = \frac{\cot\theta\left(1-\sin\theta\right)}{4}$$

$$\text{LHS} = \left(m^2-n^2\right)^2$$

$$= \left(\left(m+n\right)\left(m-n\right)\right)^2$$

$$= \left(m+n\right)^2\left(m-n\right)^2$$

$$= \left(\frac{\cot\theta\left(1+\sin\theta\right)+\cot\theta\left(1-\sin\theta\right)}{4}\right)^2\left(\frac{\cot\theta\left(1+\sin\theta\right)-\cot\theta\left(1-\sin\theta\right)}{4}\right)^2$$

$$= \left(\frac{\cot\theta\left(1+\sin\theta+1-\sin\theta\right)}{4}\right)^2 \times \left(\frac{\cot\theta\left(1+\sin\theta-1+\sin\theta\right)}{4}\right)^2$$

$\cot^2\theta \quad \cot^2\theta$

$$= \frac{\cot^2\theta}{16} \times 4 \times \frac{\cot^2\theta}{16} \times 4\sin^2\theta$$

$$= \frac{\cot^2\theta}{16} \times \frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\cot\theta}{4} \times \frac{\cot\theta}{4} \times \left(1 - \sin^2\theta\right) \qquad \left[\because \cos^2\theta = 1 - \sin^2\theta\right]$$

$$= \frac{\cot\theta\left(1+\sin\theta\right)}{4} \times \frac{\cot\theta\left(1-\sin\theta\right)}{4}$$

$$= mn$$

Trigonometric Functions Ex 5.1 Q23

To show: $sin^6\theta + cos^6\theta = \dfrac{4 - 3\left(m^2-1\right)^2}{4}$, where $m^2 \le 2$

Since, $sin\,\theta + cos\,\theta = m \qquad \ldots(i)$

$\Rightarrow \quad \left(sin\,\theta + cos\,\theta\right)^2 = m^2$

$\Rightarrow \quad sin^2\theta + cos^2\theta + 2\,sin\,\theta\cos\theta = m^2$

$\Rightarrow \quad 1 + 2\,sin\,\theta\cos\theta = m^2 \quad \left(\because sin^2\theta + cos^2\theta = 1\right)$

$\Rightarrow \quad 2\,sin\,\theta\cos\theta = m^2 - 1$

$\Rightarrow \quad sin\,\theta\cos\theta = \dfrac{m^2-1}{2} \qquad \ldots(ii)$

$\therefore$ LHS $= sin^6\theta + cos^2\theta$

$$= \left(sin^2\theta\right)^3 + \left(cos^2\theta\right)^3$$

$$= \left(sin^2\theta + cos^2\theta\right)\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 - sin^2\theta\,cos^2\theta\right)$$

$$= 1.\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 + 2\,sin^2\theta\,cos^2\theta - 2\,sin^2\theta\,cos^2\theta - sin^2\theta\,cos^2\theta\right)$$

$\left(\text{adding and subtracting } 2\,sin^2\theta\,cos^2\theta\right)$

$$= \left(sin^2\theta + cos^2\theta\right)^2 - 3\,sin^2\theta\,cos^2\theta$$

$$= 1 - 3\,sin^2\theta\,cos^2\theta$$

$$= 1 - 3\left(sin\,\theta\,cos\,\theta\right)^2$$

$$= 1 - 3\frac{\left(m^2-1\right)^2}{4}\left(\text{from (ii)}\right)$$

$$= \frac{4 - 3\left(m^2-1\right)^2}{4}, \text{ where } m^2 \le 2$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q24

$$LHS = ab + a - b + 1$$

$$= (sec\,\theta - \tan\,\theta)(cosec\; + cot\;) + sec\,\theta - \tan\,\theta - cosec\,\theta - cot\,\theta + 1$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} - \tan\theta \times cot\,\theta + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\sin\theta} - \frac{1}{\cos\theta} - 1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1 - \sin^2\theta - \cos^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1 - (\cos^2\theta + \sin^2\theta)}{\sin\theta.\cos\theta}$$

$$= \frac{1-1}{\sin\theta.\cos\theta} = 0 = RHS.\ Hence\ Proved$$

Trigonometric Functions Ex 5.1 Q25

$$LHS = \left|\sqrt{\frac{1-sin\,\theta}{\ldots}} \quad \sqrt{\frac{1+sin\,\theta}{\ldots}}\right|$$

LHS = $\left|\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}\right|$

$= \left|\dfrac{\left(\sqrt{1-\sin\theta}\right)^2 + \left(\sqrt{1+\sin\theta}\right)^2}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}\right|$

$= \left|\dfrac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}\right|$

$= \left|\dfrac{2}{|\cos\theta|}\right| \qquad \left(\because\ 1-\sin^2\theta = \cos^2\theta \Rightarrow \sqrt{1-\sin^2\theta} = \cos\theta\right)$

$= \dfrac{-2}{\cos\theta} \qquad \left(\because \dfrac{\pi}{2} < \theta < \pi \Rightarrow \cos\theta < 0\right)$

= RHS