RD Sharma Solutions Class 11 Maths Chapter 5 Ex 5.2

Trigonometric Functions Ex 5.2 Q 1

We have,

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \pm\sqrt{1+\cot^2\theta}$$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$

$$= -\sqrt{1+\left(\frac{12}{5}\right)^2} \qquad \left[\because \quad \cot\theta = \frac{12}{5}\right]$$

$$= -\sqrt{1+\frac{144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\therefore \quad \operatorname{cosec}\theta = -\frac{13}{5}$$

Now, $\tan\theta = \dfrac{1}{\cot\theta}$

$$= \frac{1}{\frac{12}{5}}$$

$$= \frac{5}{12}$$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

In the 2^{nd} quadrant $\sin\theta$ is positive and $\tan\theta$ is negative

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(-\frac{1}{2}\right)^2} \qquad \left[\because \quad \cos\theta = -\frac{1}{2}\right]$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

Now, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{1}{2}} = -2$$

and $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$

Hence, $\sin\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = -\sqrt{3}$,

$\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$, $\sec\theta = -2$ and $\cot\theta = \frac{-1}{\sqrt{3}}$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$
$$= -\sqrt{1+\left(\frac{4}{3}\right)^2}$$
$$= -\sqrt{1+\frac{16}{9}}$$
$$= -\sqrt{\frac{25}{9}}$$
$$= -\frac{5}{3}$$

Now, $\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\frac{-5}{3}} = \frac{-3}{5}$

and, $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{-5}{4}} = \frac{-4}{5}$

Hence, $\sin\theta = \frac{-3}{5}$, $\cos\theta = \frac{-4}{5}$,

$\operatorname{cosec}\theta = -\frac{5}{3}$, $\sec\theta = \frac{-5}{4}$ and $\cot\theta = \frac{4}{3}$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$
$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 1st quadrant $\cos\theta$ is positive and $\tan\theta$ is also positive

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$$
$$= \sqrt{1-\left(\frac{3}{5}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{3}{5}\right]$$
$$= \sqrt{1-\frac{9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

and, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

Now, $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

and, $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

Hence, $\cos\theta = \frac{4}{5}$, $\operatorname{cosec}\theta = \frac{5}{3}$, $\tan\theta = \frac{3}{4}$,

$\sec\theta = \frac{5}{4}$, and $\cot\theta = \frac{4}{3}$

Trigonometric Functions Ex 5.2 Q 2

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$
$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 2^{st} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-\left(\frac{12}{13}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{12}{13}\right]$$

$$= -\sqrt{1-\frac{144}{169}}$$

$$= -\sqrt{\frac{25}{169}}$$

$$= -\frac{5}{13}$$

and, $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{-5}{13}} = -\frac{13}{5}$$

Now, $$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-5}{13}} = -\frac{13}{5}$$

$$\therefore \quad \sec\theta + \tan\theta = -\frac{13}{5} - \frac{12}{5}$$

$$= \frac{-13-12}{5}$$

$$= -\frac{25}{5}$$

$$= -5$$

$$\Rightarrow \quad \sec\theta + \tan\theta = -5$$

Trigonometric Functions Ex 5.2 Q 3

We have,

$$\sin\theta = \frac{3}{5}, \quad \tan\phi = \frac{1}{2} \quad \text{and} \frac{\pi}{2} < \theta < \pi < \frac{3\pi}{2}$$

$\Rightarrow$ θ lies in the second quadrant and ϕ lies in the third quadrant.

Now, $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 2^{st} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= -\sqrt{1-\frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$$\Rightarrow \quad \cos\theta = -\frac{4}{5}$$

and, $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{\frac{-4}{5}} = -\frac{3}{4} - - - - - \text{(i)}$$

Now, $\sec^2\phi - \tan^2\phi = 1$

$$\Rightarrow \quad \sec^2\phi = 1 + \tan^2\phi$$

$$\Rightarrow \quad \sec\phi = \pm\sqrt{1+\tan^2\phi}$$

In the third quadrant $\sec\phi$ is negative

$$\therefore \quad \sec\phi = -\sqrt{1+\left(\frac{1}{2}\right)^2}$$

$$= -\sqrt{1+\frac{1}{4}}$$

$$= -\sqrt{\frac{5}{4}}$$

$\Rightarrow \quad \sec\phi = -\frac{\sqrt{5}}{2}$ ------- (ii)

$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi$

$$= 8 \times \left(\frac{-3}{4}\right) - \sqrt{5} \times \left(-\frac{\sqrt{5}}{2}\right) \quad [\text{by equations (i) and (ii)}]$$

$$= -2 \times 3 + \frac{5}{2}$$

$$= -6 + \frac{5}{2}$$

$$= \frac{-12+5}{2}$$

$$= \frac{-7}{2}$$

$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi = -\frac{7}{2}$

Trigonometric Functions Ex 5.2 Q 4

We have,

$$\sin\theta + \cos\theta = 0$$

$\Rightarrow \quad \sin\theta = -\cos\theta$ ------- (i)

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = -1$

$\Rightarrow \quad \tan\theta = -1$

We know that,

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow \quad \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \quad \sec\theta = \pm\sqrt{1+\tan^2\theta}$

In the 4^{th} quadrant $\sec\theta$ is positive.

$\therefore \quad \sec\theta = \sqrt{1+\tan^2\theta}$

$$= \sqrt{1+(-1)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

$\therefore \quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{2}}$

putting $\cos\theta = \frac{1}{\sqrt{2}}$ in equation (i), we get,

$\sin\theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$

Hence, $\sin\theta = -\frac{1}{\sqrt{2}}$ and $\cos\theta = \frac{1}{\sqrt{2}}$.

Chapter 5 Trigonometric Functions Ex 5.2 Q 5.

We have,

$$\cos\theta = -\frac{3}{5}, \quad \text{and } \pi < \theta < \frac{3\pi}{2}$$

$\Rightarrow \quad \theta$ lies in the 3^{rd} quadrant

We know that,

$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$

In the 3^{rd} quadrant $\sin\theta$ is negative and $\tan\theta$ is positive.

$\therefore \quad \sin\theta = -\sqrt{1-\cos^2\theta}$

$$= -\sqrt{1-\left(-\frac{3}{5}\right)^2} \quad \left[\because \quad \cos\theta = -\frac{3}{5}\right]$$

$$= -\sqrt{1-\frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$\Rightarrow \quad \sin\theta = -\frac{4}{5}$

and, $\quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{5}$

Now, $\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-4}{5}} = \frac{-5}{4}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-3}{5}} = \frac{-5}{3}$$

and, $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

$\therefore \quad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}}$

$$= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}}$$

$$= \frac{-\frac{2}{4}}{-\frac{9}{3}}$$

$$= \frac{2}{4} \times \frac{3}{9}$$

$$= \frac{1}{6}$$

$\therefore \quad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{1}{6}$