

RD Sharma
Solutions
Class 11 Maths
Chapter 5
Ex 5.3

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 i

$$\begin{aligned} \sin \frac{5\pi}{3} &= \sin \left(2\pi - \frac{\pi}{3} \right) \\ &= -\sin \frac{\pi}{3} \quad (\because \sin(2\pi - \theta) = -\sin \theta) \\ &= \frac{-\sqrt{3}}{2} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 ii

$$3060^\circ = 17\pi \quad (\because \pi = 180^\circ)$$

$$\begin{aligned} \therefore \sin 3060^\circ &= \sin 17\pi \\ &= 0 \quad (\because \sin n\pi = 0 \text{ for all } n \in \mathbb{Z}) \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 iii

$$\begin{aligned} \tan \frac{11\pi}{6} &= \tan \left(2\pi - \frac{\pi}{6} \right) \\ &= -\tan \frac{\pi}{6} \quad (\because \tan(2\pi - \theta) = -\tan \theta) \\ &= \frac{-1}{\sqrt{3}} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.iv

$$\begin{aligned} 1125^\circ &= 6\pi + \frac{\pi}{4} \quad (\pi = 180^\circ) \\ \cos(-1125^\circ) &= \cos \left(- \left(6\pi + \frac{\pi}{4} \right) \right) \\ &= \cos \left(6\pi + \frac{\pi}{4} \right) \quad (\because \cos(-\theta) = \cos \theta) \\ &= \cos \left(2 \times 3\pi + \frac{\pi}{4} \right) \\ &= \cos \frac{\pi}{4} \quad (\because \cos(2k\pi + \theta) = \cos \theta, k \in \mathbb{N}) \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.v

$$\begin{aligned} \tan 315^\circ &= \tan\left(2\pi - \frac{\pi}{4}\right) \\ &= -\tan\frac{\pi}{4} && (\because \tan(2\pi - \theta) = -\tan\theta) \\ &= -1 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vi

$$\begin{aligned} \sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\ &= \sin\frac{\pi}{6} && (\because 3\pi - \frac{\pi}{6} \text{ lies in second quadrant}) \\ &= \frac{1}{2} \end{aligned}$$

Alternative solution

$$\begin{aligned} \sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\ &= \sin\left(2\pi + \left(\pi - \frac{\pi}{6}\right)\right) \\ &= \sin\left(\pi - \frac{\pi}{6}\right) && (\because \sin(2\pi + \theta) = \sin\theta, \text{ as sine is periodic with period } 2\pi) \\ &= \sin\frac{\pi}{6} && (\because \sin(\pi - \theta) = \sin\theta) \\ &= \frac{1}{2} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vii

$$\begin{aligned} \cos 570^\circ &= \cos\left(3\pi + \frac{\pi}{6}\right) \\ &= \cos\left(2\pi + \left(\pi + \frac{\pi}{6}\right)\right) \\ &= \cos\left(\pi + \frac{\pi}{6}\right) && (\because \cos(2\pi + \theta) = \cos\theta, \text{ as cosine} \\ &&& \text{is periodic with period } 2\pi) \\ &= -\cos\frac{\pi}{6} && (\because \cos(\pi + \theta) = -\cos\theta) \\ &= \frac{-\sqrt{3}}{2} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.viii

$$\begin{aligned} \sin(-330^\circ) &= \sin\left(-\left(2\pi - \frac{\pi}{6}\right)\right) \\ &= \sin\left(2\pi - \frac{\pi}{6}\right) && (\because \sin(-\theta) = -\sin\theta) \\ &= -\left(-\sin\frac{\pi}{6}\right) && (\because \sin(2\pi - \theta) = -\sin\theta) \\ &= \sin\frac{\pi}{6} \\ &= \frac{1}{2} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.ix

$$\begin{aligned} \operatorname{cosec}(-1200^\circ) &= \operatorname{cosec}\left(-\left(7\pi - \frac{\pi}{3}\right)\right) \\ &= \operatorname{cosec}\left(7\pi - \frac{\pi}{3}\right) && (\because \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta) \\ &= -\operatorname{cosec}\left(2 \times 3\pi + \left(\pi - \frac{\pi}{3}\right)\right) \\ &= -\operatorname{cosec}\left(\pi - \frac{\pi}{3}\right) && (\because \operatorname{cosec} \text{ is periodic of period } 2\pi, \\ &&& \therefore \operatorname{cosec}(2\pi + \theta) = \operatorname{cosec}(2n\pi + \theta) \\ &&& = \operatorname{cosec}\theta \text{ for all } n \in \mathbb{N}) \\ &= -\operatorname{cosec}\frac{\pi}{3} && (\because \operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta) \\ &= \frac{-2}{\sqrt{3}} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.x

$$\begin{aligned} \tan(-585^\circ) &= -\tan(585^\circ) && (\because \tan(-\theta) = -\tan\theta) \\ &= -\tan\left(3\pi + \frac{\pi}{4}\right) \\ &= -\tan\left(2\pi + \left(\pi + \frac{\pi}{4}\right)\right) && (\because \tan(2\pi + \theta) = \tan\theta) \\ &= -\tan\frac{\pi}{4} && (\because \tan(\pi + \theta) = \tan\theta) \\ &= -1 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xi

$$\begin{aligned}
 \cos 855^\circ &= \cos \left(5\pi - \frac{\pi}{4} \right) \\
 &= \cos \left(2 \times 2\pi + \left(\pi - \frac{\pi}{4} \right) \right) \\
 &= \cos \left(\pi - \frac{\pi}{4} \right) && (\because \cos(2k\pi + \theta) = \cos \theta \text{ for all } k \in \mathbb{N}) \\
 &= -\cos \frac{\pi}{4} && (\because \cos(\pi - \theta) = -\cos \theta) \\
 &= \frac{-1}{\sqrt{2}}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xii

$$\begin{aligned}
 \sin 1845^\circ &= \sin \left(10\pi + \frac{\pi}{4} \right) \\
 &= \left(2 \times 5\pi + \frac{\pi}{4} \right) \\
 &= \sin \pi && (\because \sin(2k\pi + \theta) = \sin \theta, \text{ for all } k \in \mathbb{N}) \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiii

$$\begin{aligned}
 \cos 1755^\circ &= \cos \left(10\pi - \frac{\pi}{4} \right) \\
 &= \cos \left(2 \times 5\pi - \frac{\pi}{4} \right) \\
 &= \cos \frac{\pi}{4} && (\because \cos(2k\pi - \theta) = \cos \theta, k \in \mathbb{N}) \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiv

$$\begin{aligned}
 4530^\circ &= \left(25\pi + \frac{\pi}{6} \right) \\
 \therefore \sin 4530 &= \sin \left(25\pi + \frac{\pi}{6} \right) \\
 &= \sin \left(2 \times 12\pi + \left(\pi + \frac{\pi}{6} \right) \right) \\
 &= \sin \left(\pi + \frac{\pi}{6} \right) && (\because \sin(2k\pi + \theta) = \sin \theta, k \in \mathbb{N}) \\
 &= -\sin \frac{\pi}{6} && (\because \sin(\pi + \theta) = -\sin \theta) \\
 &= \frac{-1}{2}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.i

$$\begin{aligned}
 \text{LHS} &= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ \\
 &= \tan \left(\pi + \frac{\pi}{4} \right) \cot \left(2\pi + \frac{\pi}{4} \right) + \tan \left(4\pi + \frac{\pi}{4} \right) \cot \left(4\pi - \frac{\pi}{4} \right) \\
 &= \tan \frac{\pi}{4} \cdot \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \times \left(-\cot \frac{\pi}{4} \right) && (\because \cot \left(4\pi - \frac{\pi}{4} \right) = -\cot \frac{\pi}{4}) \\
 &= 1 \cdot 1 + 1 \cdot (-1) \\
 &= 0 \\
 &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.ii

$$\begin{aligned}
 \text{LHS} &= \sin \frac{8\pi}{3} \cos \frac{23\pi}{6} + \cos \frac{13\pi}{3} \sin \frac{35\pi}{6} \\
 &= \sin \left(3\pi - \frac{\pi}{3} \right) \cos \left(4\pi - \frac{\pi}{6} \right) + \cos \left(4\pi + \frac{\pi}{3} \right) \sin \left(6\pi - \frac{\pi}{6} \right) \\
 &= \sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3} \left(-\sin \frac{\pi}{6} \right) && (\because \sin(6\pi - \theta) = -\sin \theta) \\
 &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \left(\frac{-1}{2} \right) \\
 &= \frac{3}{4} - \frac{1}{4} \\
 &= \frac{2}{4} \\
 &= \frac{1}{2} \\
 &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.iii

$$\text{LHS} = \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$\begin{aligned}
&= \cos 24^\circ + \cos 204^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ \\
&= \cos 24^\circ + \cos (\pi + 24^\circ) + \cos 55^\circ + \cos (\pi - 55^\circ) + \cos \left(2\pi - \frac{\pi}{3} \right) \\
&= \cos 24^\circ - \cos 24^\circ + \cos 55^\circ - \cos 55^\circ + \cos \frac{\pi}{3} \\
&= \cos \frac{\pi}{3} \\
&= \frac{1}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 1 2.iv

$$\begin{aligned}
\text{LHS} &= \tan(-225^\circ) \cot(-405^\circ) - \tan(-765^\circ) \cot(675^\circ) \\
&= -\tan 225^\circ (-\cot 405^\circ) + \tan 765^\circ \cot 765^\circ && \left(\begin{array}{l} \because \tan(-\theta) = -\tan \theta \\ \& \cot(-\theta) = -\cot \theta \end{array} \right) \\
&= \tan \left(\pi + \frac{\pi}{4} \right) \cot \left(2\pi + \frac{\pi}{4} \right) + \tan \left(4\pi + \frac{\pi}{4} \right) \cot \left(4\pi - \frac{\pi}{4} \right) \\
&= \tan \frac{\pi}{4} \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \times \left(-\cot \frac{\pi}{4} \right) && \left(\because \cot(4\pi - \theta) = -\cot \theta \right) \\
&= 1.1 + 1(-1) \\
&= 1 - 1 \\
&= 0 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.v

$$\begin{aligned}
\text{LHS} &= \cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ) \\
&= \cos \left(3\pi + \frac{\pi}{6} \right) \sin \left(3\pi - \frac{\pi}{6} \right) - \sin 330^\circ \cos 390^\circ && \left(\begin{array}{l} \because \sin(-\theta) = -\sin \theta \text{ and} \\ \cos(-\theta) = \cos \theta \end{array} \right) \\
&= -\cos \frac{\pi}{6} \sin \frac{\pi}{6} - \sin \left(2\pi - \frac{\pi}{6} \right) \cos \left(2\pi + \frac{\pi}{6} \right) \\
&= -\sin \frac{\pi}{6} \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cdot \cos \frac{\pi}{6} && \left(\because \sin(2\pi - \theta) = -\sin \theta \right) \\
&= 0 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vi

$$\begin{aligned}
\text{LHS} &= \tan \frac{11\pi}{3} - 2 \sin \frac{4\pi}{6} - \frac{3}{4} \cos \sec^2 \frac{\pi}{4} + 4 \cos^2 \frac{17\pi}{6} \\
&= \tan \left(4\pi - \frac{\pi}{3} \right) - 2 \sin \frac{2\pi}{3} - \frac{3}{4} \times (\sqrt{2})^2 + 4 \cos^2 \left(3\pi - \frac{\pi}{6} \right) \\
&= -\tan \frac{\pi}{3} - 2 \sin \left(\pi - \frac{\pi}{3} \right) - \frac{3}{4} \times 2 + 4 \cos^2 \frac{\pi}{6} \\
&\quad \left(\because \tan \left(4\pi - \frac{\pi}{3} \right) = -\tan \frac{\pi}{3}, \cos \left(3\pi - \frac{\pi}{6} \right) = -\cos \frac{\pi}{6} \right) \\
&= -\sqrt{3} - 2 \sin \frac{\pi}{3} - \frac{3}{2} + 4 \times \left(\frac{\sqrt{3}}{2} \right)^2 \\
&= -\sqrt{3} - 2 \times \frac{\sqrt{3}}{2} - \frac{3}{2} + 4 \times \frac{3}{4} \\
&= -\sqrt{3} - \sqrt{3} - \frac{3}{2} + 3 \\
&= -2\sqrt{3} - \frac{3+6}{2} \\
&= -2\sqrt{3} + \frac{3}{2} \\
&= \frac{3 - 4\sqrt{3}}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vii

$$\begin{aligned}
\text{LHS} &= 3 \sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} \\
&= 3 \times \frac{1}{2} \times 2 - 4 \sin \left(\pi - \frac{\pi}{6} \right) \times 1 \\
&= 3 - 4 \sin \frac{\pi}{6} && \left(\because \sin(\pi - \theta) = \sin \theta \right) \\
&= 3 - 4 \times \frac{1}{2}
\end{aligned}$$

= 3 - 2
 = 1
 = RHS
 Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.i

$$\begin{aligned} \text{LHS} &= \frac{\cos(2\pi + \theta) \operatorname{cosec}(2\pi + \theta) \tan\left(\frac{\pi}{2} + \theta\right)}{\sec\left(\frac{\pi}{2} + \theta\right) \cos \theta \cot(\pi + \theta)} \\ &= \frac{\cos \theta \times \operatorname{cosec} \theta (-\cot \theta)}{-\operatorname{cosec} \theta \cos \theta \cot \theta} \left(\begin{array}{l} \because \tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta \\ \& \sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta \end{array} \right) \\ &= 1 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.ii

$$\begin{aligned} \text{LHS} &= \frac{\operatorname{cosec}(90^\circ + \theta) + \cot(450^\circ + \theta)}{\cos \operatorname{cosec}(90^\circ - \theta) + \tan(180^\circ - \theta)} + \frac{\tan(180^\circ + \theta) + \sec(180^\circ - \theta)}{\tan(360^\circ + \theta) - \sec(-\theta)} \\ &= \frac{\sec \theta + \cot\left(2\pi + \frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta} \\ &\quad \left(\because \operatorname{cosec}(90^\circ + \theta) = \sec \theta, \cos \operatorname{cosec}(90^\circ - \theta) = \sec \theta, \tan(180^\circ - \theta) = -\tan \theta, \sec(-\theta) = \sec \theta \right) \\ &= \frac{\sec \theta + \cot\left(\frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + 1 \quad \left(\because \cot(2\pi + \theta) = \cot \theta \right) \\ &= \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} + 1 \quad \left(\because \cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta \right) \\ &= 1 + 1 \\ &= 2 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iii

$$\begin{aligned} \text{LHS} &= \frac{\sin(180^\circ + \theta) \cos(90^\circ + \theta) \tan(270^\circ - \theta) \cot(360^\circ - \theta)}{\sin(360^\circ - \theta) \cos(360^\circ + \theta) \operatorname{cosec}(-\theta) \sin(270^\circ + \theta)} \\ &= \frac{\sin \theta (-\sin \theta) \cot \theta (-\cot \theta)}{-\sin \theta \cos \theta (-\operatorname{cosec} \theta) (-\cos \theta)} \left(\begin{array}{l} \because \tan(270^\circ - \theta) = \cot \theta \\ \& \sin(270^\circ + \theta) = -\cos \theta \end{array} \right) \\ &= \frac{-\sin \theta \times \sin \theta \times \cos \theta \times \cos \theta \times \sin \theta}{-\sin \theta \times \cos \theta \times \sin \theta \times \sin \theta \times \cos \theta} \left(\begin{array}{l} \because \cot \theta = \frac{\cos \theta}{\sin \theta} \\ \& \operatorname{cosec} \theta = \frac{1}{\sin \theta} \end{array} \right) \\ &= 1 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iv

$$\begin{aligned} \text{LHS} &= \left\{ 1 + \cot \theta - \sec\left(\frac{\pi}{2} + \theta\right) \right\} \left\{ 1 + \cot \theta + \sec\left(\frac{\pi}{2} + \theta\right) \right\} \\ &= \{1 + \cot \theta - (-\operatorname{cosec} \theta)\} \{1 + \cot \theta - \operatorname{cosec} \theta\} \\ &\quad \left(\because \sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta \right) \\ &= \{(1 + \cot \theta) + \operatorname{cosec} \theta\} \{(1 + \cot \theta) - \operatorname{cosec} \theta\} \\ &= (1 + \cot \theta)^2 - \operatorname{cosec}^2 \theta \\ &= 1 + \cot^2 \theta + 2\cot \theta - \operatorname{cosec}^2 \theta \\ &= \operatorname{cosec}^2 \theta + 2\cot \theta - \operatorname{cosec}^2 \theta \quad \left(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \right) \\ &= 2\cot \theta \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 3 v

$$\begin{aligned} \text{LHS} &= \frac{\tan(90^\circ - \theta) \sec(180^\circ - \theta) \sin(-\theta)}{\sin(180^\circ + \theta) \cot(360^\circ - \theta) \operatorname{cosec}(90^\circ - \theta)} \\ &= \frac{\cot \theta \times (-\sec \theta) \times (-\sin \theta)}{-\sin \theta \times (-\cot \theta) \times \sec \theta} \\ &= 1 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 4

$$\begin{aligned} \text{LHS} &= \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} \\ &= \sin^2 \frac{\pi}{18} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} \\ &= \sin^2 \left(\frac{\pi}{2} - \frac{4\pi}{9} \right) + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{9} \right) \quad \left(\because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9} \right) \\ &= \cos^2 \frac{4\pi}{9} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9} \quad \left(\because \sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \right) \\ &= 1 + 1 \quad \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right) \\ &= 2 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 5

$$\begin{aligned} \text{LHS} &= \sec \left(\frac{3\pi}{2} - \theta \right) \sec \left(\theta - \frac{5\pi}{2} \right) + \tan \left(\frac{5\pi}{2} + \theta \right) \tan \left(\theta - \frac{3\pi}{2} \right) \\ &= \sec \left(\frac{3\pi}{2} - \theta \right) \sec \left(- \left(\frac{5\pi}{2} - \theta \right) \right) + \tan \left(\frac{5\pi}{2} + \theta \right) \tan \left(- \left(\frac{3\pi}{2} - \theta \right) \right) \\ &= -\operatorname{cosec} \theta \cdot \sec \left(\frac{5\pi}{2} - \theta \right) - \cot \theta \times (-) \tan \left(\frac{3\pi}{2} - \theta \right) \\ &\quad \left[\begin{array}{l} \because \left(\sec \left(\frac{3\pi}{2} - \theta \right) \right) = -\operatorname{cosec} \theta, \sec(-\theta) = \sec \theta, \tan \left(\frac{5\pi}{2} + \theta \right) = -\cot \theta \\ \& \tan(-\theta) = -\tan \theta \end{array} \right] \\ &= -\operatorname{cosec} \theta \times \operatorname{cosec} \theta - \cot \theta \times (-1) \times \cot \theta \quad \left[\begin{array}{l} \because \sec \left(\frac{5\pi}{2} - \theta \right) = \operatorname{cosec} \theta \\ \& \tan \left(\frac{3\pi}{2} - \theta \right) = \cot \theta \end{array} \right] \\ &= -\operatorname{cosec}^2 \theta + \cot^2 \theta \\ &= -\operatorname{cosec}^2 \theta + \operatorname{cosec}^2 \theta - 1 \quad \left(\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \right) \\ &= -1 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 6

$$\begin{aligned} \text{We have } A + B + C &= \pi \quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right) \\ \Rightarrow A + B &= \pi - C \\ \Rightarrow \cos(A + B) &= \cos(\pi - C) \\ \Rightarrow &= -\cos C \quad \left(\because \cos(\pi - \theta) = -\cos \theta \right) \\ \Rightarrow \cos(A + B) + \cos C &= 0 \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 ii

$$\begin{aligned} \text{We have } A + B + C &= \pi \quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right) \\ \Rightarrow A + B &= \pi - C \\ \Rightarrow \frac{A + B}{2} &= \frac{\pi - C}{2} \\ \Rightarrow \frac{A + B}{2} &= \frac{\pi}{2} - \frac{C}{2} \\ \Rightarrow \cos \left(\frac{A + B}{2} \right) &= \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) \\ \Rightarrow &= \sin \frac{C}{2} \quad \left(\because \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta \right) \\ \text{Hence } \cos \left(\frac{A + B}{2} \right) &= \sin \frac{C}{2} \\ &\text{Proved} \end{aligned}$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 iii

$$\text{We have } A + B + C = \pi \quad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right)$$

$$\Rightarrow A + B = \pi - C$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$= \cot \frac{C}{2} \quad \left(\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta\right)$$

Hence $\tan\left(\frac{A+B}{2}\right) = \cot \frac{C}{2}$
 Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 7

$\because A, B, C, D$ are the angles of a cyclic quadrilateral in order,
 $\therefore A + C = \pi$ & $B + D = \pi$
 $\Rightarrow \pi - A = C$ & $\pi - D = B$
 $\Rightarrow \cos(\pi - A) = \cos C$ (i)
 & $\cos(\pi - D) = \cos B$ (ii)

Now, $\cos(180^\circ - A) + \cos(180^\circ + B) + (180^\circ + C) - \sin(90^\circ + D)$
 $= \cos C + (-\cos B) - \cos C - \cos D$
 $= -\cos B - \cos D$
 $= -\cos B - (-\cos B)$ (using (ii))
 $= -\cos B + \cos B$
 $= 0$
 Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 8i.

$$\cos \operatorname{ec}(90^\circ + \theta) + x \cos \theta \cot(90^\circ + \theta) = \sin(90^\circ + \theta)$$

$$\Rightarrow \sec \theta + x \cos \theta \times (-\tan \theta) = \cos \theta$$

$$\Rightarrow \frac{1}{\cos \theta} + x \cos \theta \times \frac{(-\sin \theta)}{\cos \theta} = \cos \theta$$

$$\Rightarrow \frac{1}{\cos \theta} - x \sin \theta = \cos \theta$$

$$\Rightarrow \frac{1 - x \sin \theta \cos \theta}{\cos \theta} = \cos \theta$$

$$\Rightarrow 1 - x \sin \theta \cos \theta = \cos^2 \theta$$

$$\Rightarrow 1 - \cos^2 \theta = x \sin \theta \cos \theta$$

$$\Rightarrow \sin^2 \theta = x \sin \theta \cos \theta$$

$$\Rightarrow \sin \theta = x \cos \theta$$

$$\Rightarrow x = \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

Hence $x = \tan \theta$

Chapter 5 Trigonometric Functions Ex 5.3 Q 8. ii.

We have $x \cot(90^\circ + \theta) + \tan(90^\circ + \theta) \sin \theta + \cos \operatorname{ec}(90^\circ + \theta) = 0$
 $\Rightarrow x(-\tan \theta) - \cot \theta \times \sin \theta + \sec \theta = 0$
 $\Rightarrow -x \tan \theta - \frac{\cos \theta}{\sin \theta} \times \sin \theta + \frac{1}{\cos \theta} = 0$
 $\Rightarrow -x \frac{\sin \theta}{\cos \theta} - \cos \theta + \frac{1}{\cos \theta} = 0$
 $\Rightarrow \frac{-x \sin \theta - \cos^2 \theta + 1}{\cos \theta} = 0$
 $\Rightarrow -x \sin \theta + 1 - \cos^2 \theta = 0$
 $\Rightarrow -x \sin \theta + \sin^2 \theta = 0$
 $\Rightarrow x \sin \theta = \sin^2 \theta$

$$\Rightarrow x = \frac{\sin^2 \theta}{\sin \theta}$$

$$\Rightarrow x = \sin \theta$$

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. i.

$$\begin{aligned} \text{LHS} &= \tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ \\ &= \tan 4\pi - \cos \left(\frac{3\pi}{2}\right) - \sin \left(\pi \frac{\pi}{6}\right) \cos \left(\frac{\pi}{2} + \frac{\pi}{6}\right) \quad (\because \pi = 180^\circ) \\ &= 0 - 0 - \sin \frac{\pi}{6} \left(-\sin \frac{\pi}{6}\right) \quad \left(\because \tan n\pi = 0 \text{ for all } n \in \mathbb{Z} \text{ \& } \cos \frac{3\pi}{2} = 0\right) \\ &= \sin^2 \frac{\pi}{6} \\ &= \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} \\ &= \text{RHS} \end{aligned}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. ii.

$$\begin{aligned} \text{LHS} &= \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ \\ &= \sin \left(4\pi + \frac{\pi}{3}\right) \sin \left(3\pi - \frac{\pi}{3}\right) + \cos \left(\frac{\pi}{2} + \frac{\pi}{6}\right) \sin \left(\pi - \frac{\pi}{6}\right) \quad (\because \pi = 180^\circ) \\ &= \sin \frac{\pi}{3} \times \sin \frac{\pi}{3} + \left(-\sin \frac{\pi}{6}\right) \sin \frac{\pi}{6} \quad \left(\because \sin \left(4\pi + \frac{\pi}{3}\right) = \sin \frac{\pi}{3}\right. \\ &\quad \left. \& \sin \left(3\pi - \frac{\pi}{3}\right) = \sin \frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \\ &= \text{RHS} \end{aligned}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9. iii.

$$\begin{aligned} \text{LHS} &= \sin 780^\circ \sin 120^\circ + \cos 240^\circ \sin 390^\circ \\ &= \sin \left(4\pi + \frac{\pi}{3}\right) \sin \left(\frac{\pi}{2} + \frac{\pi}{6}\right) + \cos \left(\pi + \frac{\pi}{6}\right) \sin \left(2\pi + \frac{\pi}{6}\right) \\ &= \sin \frac{\pi}{3} \times \cos \frac{\pi}{6} - \cos \frac{\pi}{3} \times \left(+\sin \frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{1}{2} \\ &= \frac{1}{2} \\ &= \text{RHS} \end{aligned}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9.iv.

$$\begin{aligned} \text{LHS} &= \sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ \\ &= \sin \left(3\pi + \frac{\pi}{3}\right) \cos \left(2\pi + \frac{\pi}{6}\right) + \cos \left(3\pi - \frac{\pi}{3}\right) \sin \left(\pi - \frac{\pi}{6}\right) \\ &= -\sin \frac{\pi}{3} \cos \frac{\pi}{6} - \cos \frac{\pi}{3} - \sin \frac{\pi}{6} \quad \left(\because \sin \left(3\pi + \frac{\pi}{3}\right) = -\sin \frac{\pi}{3} \text{ \& } \cos \left(3\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}\right) \\ &= \frac{-\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{-3}{4} - \frac{1}{4} \end{aligned}$$

$$= \frac{-4}{4}$$

$$= -1$$

$$= \text{RHS}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 9.v.

$$\text{LHS} = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan\left(\pi + \frac{\pi}{4}\right) \cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right) \cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan \frac{\pi}{4} \cot \frac{\pi}{4} + \tan \frac{\pi}{4} \left(-\cot \frac{\pi}{4}\right)$$

$$= 1 \cdot 1 + 1 \cdot (-1)$$

$$= 1 - 1$$

$$= 0$$

$$= \text{RHS}$$

Proved