RD Sharma Solutions Class 11 Maths Chapter 7 Ex 7.1

Trigonometric Ratios of Compound Angles Ex 7.1 Q1

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5}\times\frac{5}{13}+\frac{3}{5}\times\frac{12}{13}$$

$$= \frac{20}{65}+\frac{36}{65}$$

$$= \frac{20+36}{65}$$

$$= \frac{56}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{3}{5}\times\frac{5}{13}-\frac{4}{5}\times\frac{12}{13}$$

$$= \frac{15}{65}-\frac{48}{65}$$

$$= \frac{15-48}{65}$$

$$= \frac{-33}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}} \text{ and } \sin B = \sqrt{1-\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \text{ and } \sin B = \sqrt{\frac{169-25}{169}}$$

$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}}$ and $\sin B = \sqrt{\frac{144}{169}}$

$\Rightarrow \quad \cos A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$

Now,

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$= \frac{4}{5}\times\frac{5}{13} - \frac{3}{5}\times\frac{12}{13}$$
$$= \frac{20}{65} - \frac{36}{65}$$
$$= \frac{20-36}{65}$$
$$= -\frac{16}{65}$$

We have,

$\sin A = \frac{4}{5}$ and $\cos B = \frac{5}{13}$

$\therefore \quad \cos A = \sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$

$\Rightarrow \quad \cos A = \sqrt{1-\left(\frac{4}{5}\right)^2}$ and $\sin B = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$\Rightarrow \quad \cos A = \sqrt{1-\frac{16}{25}}$ and $\sin B = \sqrt{1-\frac{25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}}$ and $\sin B = \sqrt{\frac{169-25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}}$ and $\sin B = \sqrt{\frac{144}{169}}$

$\Rightarrow \quad \cos A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$

Now,

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$= \frac{3}{5}\times\frac{5}{13} + \frac{4}{5}\times\frac{12}{13}$$
$$= \frac{15}{65} + \frac{48}{65}$$
$$= \frac{15+48}{65}$$
$$= \frac{63}{65}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q2

We have,

$\sin A = \frac{12}{13}$ and $\sin B = \frac{4}{5}$

$\therefore \quad \cos A = -\sqrt{1-\sin^2 A}$ and $\cos B = \sqrt{1-\sin^2 B}$

[$\because$ In the second quadrant $\cos\theta$ is negative]

$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{12}{13}\right)^2}$ and $\cos B = \sqrt{1-\left(\frac{4}{5}\right)^2}$

$\Rightarrow \quad \cos A = -\sqrt{1-\frac{144}{169}}$ and $\cos B = \sqrt{1-\frac{16}{25}}$

$\Rightarrow \quad \cos A = -\sqrt{\frac{25}{169}}$ and $\cos B = \sqrt{\frac{9}{25}}$

$\Rightarrow \quad \cos A = \frac{-5}{13}$ and $\cos B = \frac{3}{5}$

Now,

(i)

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$= \frac{12}{13}\times\frac{3}{5} - \frac{5}{13}\times\frac{4}{5}$$
$$= \frac{36}{65} - \frac{20}{65}$$
$$= \frac{16}{65}$$

(ii)

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$= \frac{-5}{13}\times\frac{3}{5} - \frac{12}{13}\times\frac{4}{5}$$
$$= \frac{-15}{65} - \frac{48}{65}$$
$$= \frac{-63}{65}$$

We have,

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{-12}{13}$$

$\therefore \quad \cos A = -\sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$

[$\because$ In the second quadrant $\cos\theta$ is negative]

$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{3}{5}\right)^2}$ and $\sin B = \sqrt{1-\left(\frac{-12}{13}\right)^2}$

$\Rightarrow \quad \cos A = -\sqrt{1-\frac{9}{25}}$ and $\sin B = \sqrt{1-\frac{144}{169}}$

$\Rightarrow \quad \cos A = -\sqrt{\frac{16}{25}}$ and $\sin B = \sqrt{\frac{25}{169}}$

$\Rightarrow \quad \cos A = -\frac{4}{5}$ and $\sin B = \frac{5}{13}$

Now,

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$= \frac{3}{5}\times\left(\frac{-12}{13}\right) - \frac{4}{5}\times\frac{5}{13}$$
$$= -\frac{36}{65} - \frac{20}{65}$$
$$= -\frac{56}{65}$$

$\therefore \quad \sin(A+B) = -\frac{56}{65}$

Trigonometric Ratios of Compound Angles Ex 7.1 Q3

We have,

$$\cos A = -\frac{24}{25} \text{ and } \cos B = \frac{3}{5}$$

$\therefore \quad \sin A = -\sqrt{1-\cos^2 A}$ and $\sin B = -\sqrt{1-\cos^2 B}$

[$\because$ In the 3rd and 4th quadrant $\sin\theta$ is negative]

$\Rightarrow \quad \sin A = -\sqrt{1-\left(-\frac{24}{25}\right)^2}$ and $\sin B = -\sqrt{1-\left(\frac{3}{5}\right)^2}$

$\Rightarrow \quad \sin A = -\sqrt{1-\frac{576}{625}}$ and $\sin B = -\sqrt{1-\frac{9}{25}}$

$\Rightarrow \quad \sin A = -\sqrt{\frac{49}{625}}$ and $\sin B = -\sqrt{\frac{16}{25}}$

$\Rightarrow \quad \sin A = -\frac{7}{25}$ and $\sin B = -\frac{4}{5}$

Now,

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$= -\frac{7}{25}\times\frac{3}{5} - \frac{24}{25}\times\left(-\frac{4}{5}\right)$$
$$= -\frac{21}{25} + \frac{96}{125}$$
$$= \frac{75}{125}$$
$$= \frac{3}{5}$$

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= -\frac{24}{25}\times\frac{3}{5} - \left(-\frac{7}{25}\right)\times\left(-\frac{4}{5}\right)$$
$$= -\frac{72}{125} - \frac{28}{125}$$
$$= \frac{-72-28}{125}$$
$$= \frac{-100}{125} = \frac{-4}{5}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q4

We have,

$$\tan A = \frac{3}{4}, \text{ and } \cos B = \frac{9}{41}$$

$\therefore \quad \sin B = \sqrt{1-\cos^2 B}$

$\therefore \quad \sin B = \sqrt{1-\cos^2 B}$

$$= \sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= \sqrt{1-\frac{81}{1681}}$$

$$= \sqrt{\frac{1600}{1681}}$$

$$= \frac{40}{41}$$

$$\therefore \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{40}{41}}{\frac{9}{41}} = \frac{40}{9}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{3}{4}+\frac{40}{9}}{1-\frac{3}{4}\times\frac{40}{9}}$$

$$= \frac{\frac{27+160}{36}}{\frac{36-120}{36}}$$

$$= \frac{\frac{187}{36}}{\frac{-84}{36}}$$

$$= -\frac{187}{84}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = -\sqrt{1-\cos^2 B}$$

$\left[\because \text{cosine is negative in second quadrant and sine is negative in fourth quadrant}\right]$

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = -\sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$$

$$\text{Now,} \quad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}}-\left(\frac{-5}{12}\right)}{1+\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{-5}{12}\right)}$$

$$= \frac{\frac{-1}{\sqrt{3}}+\frac{5}{12}}{1+\frac{5}{12\sqrt{3}}}$$

$$= \frac{\frac{-12+5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3}+5}{12\sqrt{3}}}$$

$$= \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

$$\Rightarrow \quad \tan(A-B) = \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

$\left[\because \text{cosine is negative in second quadrant}\right]$

$[\because \text{cosine is negative in second quadrant}]$

$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2}$ and $\sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}}$ and $\sin B = \sqrt{1-\frac{3}{4}}$

$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = \sqrt{\frac{1}{4}}$

$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = \frac{1}{2}$

$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

and, $\quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)}$$

$$= 0$$

$\therefore \quad \tan(A+B) = 0$

We have,

$\sin A = \frac{1}{2}$ and $\cos B = \frac{\sqrt{3}}{2}$

$\therefore \quad \cos A = -\sqrt{1-\sin^2 A}$ and $\sin B = \sqrt{1-\cos^2 B}$ $[\because \text{cosine is negative in second quadrant}]$

$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2}$ and $\sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}}$ and $\sin B = \sqrt{1-\frac{3}{4}}$

$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = \sqrt{\frac{1}{4}}$

$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = \frac{1}{2}$

$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

and, $\quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Now, $\quad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-3}{\sqrt{3}}$$

$$= \frac{-\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$\therefore \quad \tan(A-B) = -\sqrt{3}$

Trigonometric Ratios of Compound Angles Ex 7.1 Q7

(i)

$\sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ \qquad [\sin(A-B) = \sin A \cos B - \cos A \sin B]$

$= \sin(78^\circ - 18^\circ)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

(ii)

$\cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ \qquad [\cos(A+B) = \cos A \cos B - \sin A \sin B]$

$= \cos(47^\circ + 13^\circ)$

$= \cos 60^\circ$

$= \frac{1}{2}$

(iii)

$$\sin 36° \cos 9° + \cos 36° \sin 9° \qquad [\sin(A+B) = \sin A \cos B + \cos A \sin B]$$
$$= \sin(36° + 9°)$$
$$= \sin 45°$$
$$= \frac{1}{\sqrt{2}}$$

(iv)

$$\cos 80° \cos 20° + \sin 80° \sin 20° \qquad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$$
$$= \cos(80° - 20°)$$
$$= \cos 60°$$
$$= \frac{1}{2}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$
$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$
$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right)$$
$$= \frac{-120}{325} + \frac{84}{325}$$
$$= \frac{-120+84}{325}$$
$$= \frac{-36}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$
$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$
$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$= \left(\frac{-12}{13}\right)\times\left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right)\times\left(\frac{-7}{25}\right)$$
$$= \frac{288}{325} + \frac{35}{325}$$
$$= \frac{323}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$
$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$
$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\therefore \quad \tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
$$= \frac{\frac{-5}{12}+\frac{7}{24}}{1-\left(\frac{-5}{12}\right)\times\frac{7}{24}}$$
$$= \frac{\frac{-10+7}{24}}{1+\frac{35}{288}}$$
$$= \frac{\frac{-3}{24}}{\frac{288+35}{288}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q9

LHS: $\cos 105° + \cos 15°$

$$= \cos(90° + 15°) + \cos(90° - 75°)$$
$$= -\sin 15° + \sin 75° \qquad \left[\begin{array}{l}\because \cos(90+\theta) = -\sin\theta \\ \text{and } \cos(90-\theta) = \sin\theta\end{array}\right]$$
$$= \sin 75° - \sin 15°$$

$$\therefore \quad \cos 105° + \cos 15° = \sin 75° - \sin 15°$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q10

LHS: $\dfrac{\tan A + \tan B}{\tan A - \tan B}$

$$= \frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$
$$= \frac{\frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}{\frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}}$$
$$= \frac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B}$$
$$\sin(A+B) \qquad [\because \quad \sin(A+B) = \sin A\cos B + \cos A\sin B]$$

$$= \frac{}{\sin(A-B)} \qquad \left[\text{and, } \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$$

$$\therefore \quad \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q11

LHS: $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

Dividing numerator and denominator by $\cos 11°$, we get

$$\frac{\frac{\cos 11°}{\cos 11°} + \frac{\sin 11°}{\cos 11°}}{\frac{\cos 11°}{\cos 11°} - \frac{\sin 11°}{\cos 11°}}$$

$$= \frac{1 + \tan 11°}{1 - \tan 11°} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\tan 45° + \tan 11°}{1 - \tan 45° \times \tan 11°} \qquad \left[\tan 45° = 1\right]$$

$$= \tan(45° + 11°) \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}\right]$$

$$= \tan 56°$$

$$\therefore \quad \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$$

Hence proved.

LHS: $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

$$\frac{\frac{\cos 9°}{\cos 9°} + \frac{\sin 9°}{\cos 9°}}{\frac{\cos 9°}{\cos 9°} - \frac{\sin 9°}{\cos 9°}} \qquad \left[\begin{array}{l}\text{Dividing numerator and}\\ \text{denominator by } \cos 9°\end{array}\right]$$

$$= \frac{1 + \tan 9°}{1 - \tan 9°} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\tan 45° + \tan 9°}{1 - \tan 45° \times \tan 9°} \qquad \left[\tan 45° = 1\right]$$

$$= \tan(45° + 9°) \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}\right]$$

$$= \tan 54°$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

LHS: $\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°}$

$$\frac{\frac{\cos 8°}{\cos 8°} - \frac{\sin 8°}{\cos 8°}}{\frac{\cos 8°}{\cos 8°} + \frac{\sin 8°}{\cos 8°}} \qquad \left[\begin{array}{l}\text{Dividing numerator and}\\ \text{denominator by } \cos 8°\end{array}\right]$$

$$= \frac{1 - \tan 8°}{1 + \tan 8°} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\tan 45° - \tan 8°}{1 + \tan 45° \times \tan 8°} \qquad \left[\tan 45° = 1\right]$$

$$= \tan(45° - 8°) \qquad \left[\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}\right]$$

$$= \tan 37°$$
$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q12

LHS: $\sin(60° - \theta)\cos(30° + \theta) + \cos(60° - \theta) \times \sin(30° + \theta)$

$= \sin[(60° - \theta) + (30° + \theta)]$ $\quad [\sin(A + B) = \sin A\cos B + \cos A\sin B]$

$= \sin[60° - \theta + 30° + \theta]$

$= \sin(90°)$

$= 1$

$=$ RHS

$\therefore$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q13

LHS: $\dfrac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°}$

$= \tan(69° + 66°)$ $\quad \left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}\right]$

$= \tan(135°)$

$= \tan(90° + 45°)$

$= -\cot 45°$ $\quad [\because \tan\theta$ is negative in second quadrant$]$

$= -1$

$=$ RHS

$\therefore$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q14

We have,

$\tan A = \dfrac{5}{6}$ and $\tan B = \dfrac{1}{11}$

Now,

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$= \dfrac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$

$= \dfrac{\frac{55 + 6}{66}}{1 - \frac{5}{66}}$

$= \dfrac{\frac{61}{66}}{\frac{66 - 5}{66}}$

$= \dfrac{\frac{61}{66}}{\frac{61}{66}}$

$= \dfrac{61}{66} \times \dfrac{66}{61}$

$= 1$

$= \tan\dfrac{\pi}{4}$ $\quad \left[\because \tan\dfrac{\pi}{4} = 1\right]$

$\Rightarrow \tan(A + B) = \tan\dfrac{\pi}{4}$

$\Rightarrow A + B = \dfrac{\pi}{4}$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \text{ and } \tan B = \frac{1}{2m-1}$$

Now, $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{1 + \frac{m}{(m-1)(2m-1)}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{\frac{(m-1)(2m-1)+(m)}{(m-1)(2m-1)}}$$

$$= \frac{m(2m-1)-(m-1)}{(m-1)(2m-1)+(m)}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - 2m + 1}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

$$= 1$$

$\therefore \quad \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$\Rightarrow \quad \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad A - B = \left(\frac{\pi}{4}\right)$

Trigonometric Ratios of Compound Angles Ex 7.1 Q15

LHS: $\cos^2 45° - \sin^2 15°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15° \qquad \left[\because \cos 45 = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{2} - \left(\frac{1 - \cos 2 \times 15°}{2}\right) \qquad \left[\because \cos 2\theta = 1 - \sin^2\theta\right]$

$= \frac{1}{2} - \left(\frac{1 - \cos 30°}{2}\right)$

$= \frac{1 - 1 + \cos 30°}{2}$

$= \frac{\cos 30°}{2}$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4}$

= RHS

$\therefore$ LHS = RHS

Hence proved.

We have,

LHS $\sin^2(n+1)A - \sin^2 nA$

$= \sin[(n+1)A + nA]\sin[(n+1)A - nA]$

$\left[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)\right]$

$= \sin[nA + A + nA]\sin[nA + A - nA]$

$= \sin(2nA + A)\sin(A)$

$= \sin(2n+1)A \sin A$

= RHS

$\because$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q16

We have

We have,

$$LHS = \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B} \qquad \left[\begin{array}{l} \because \quad 2\sin A\cos B = \sin(A+B)+\sin(A-B) \\ \text{and, } 2\cos A\cos B = \cos(A+B)+\cos(A-B) \end{array}\right]$$

$$= \frac{\sin A}{\cos A}$$

$= \tan A$

$= RHS$

$\therefore$ LHS = RHS

Hence proved.

$$LHS = \frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}$$

$$= \frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C-\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A-\cos C\sin A}{\cos C\cos A}$$

$$= \frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C}{\cos B\cos C}-\frac{\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A}{\cos C\cos A}-\frac{\cos C\sin A}{\cos C\cos A}$$

$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$

$= 0$

$= RHS$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$LHS = \frac{\sin(A-B)}{\sin A\sin B}+\frac{\sin(B-C)}{\sin B\sin C}+\frac{\sin(C-A)}{\sin C\sin A}$$

$$= \frac{\sin A\cos B-\cos A\sin B}{\sin A\sin B}+\frac{\sin B\cos C-\cos B\sin C}{\sin B\sin C}+\frac{\sin C\cos A-\cos C\sin A}{\sin C\sin A}$$

$$= \frac{\sin A\cos B}{\sin A\sin B}-\frac{\cos A\sin B}{\sin A\sin B}+\frac{\sin B\cos C}{\sin B\sin C}-\frac{\cos B\sin C}{\sin B\sin C}+\frac{\sin C\cos A}{\sin C\sin A}-\frac{\cos C\sin A}{\sin C\sin A}$$

$$= \frac{\cos B}{\sin B}-\frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}-\frac{\cos B}{\sin B}+\frac{\cos A}{\sin A}-\frac{\cos C}{\sin C}$$

$$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$= 0$

$= RHS$

$\therefore$ LHS = RHS

Hence proved.

We have,

$RHS = \sin^2 A + \sin^2(A-B) - 2\sin A\cos B\sin(A-B)$

$= \sin^2 A + \sin(A-B)\left[\sin(A-B) - 2\sin A\cos B\right]$

$= \sin^2 A + \sin(A-B)\left[\sin A\cos B - \cos A\sin B - 2\sin A\cos B\right]$

$= \sin^2 A + \sin(A-B)\left[-\sin A\cos B - \cos A\sin B\right]$

$= \sin^2 A - \sin(A-B)\left(\sin A\cos B + \cos A\sin B\right)$

$= \sin^2 A - \sin(A-B)\left(\sin(A+B)\right)$

$= \sin^2 A - \sin(A-B)\sin(A+B)$

$= \sin^2 A - \left(\sin^2 A - \sin^2 B\right) \qquad \left[\because \sin(A-B)\sin(A+B) = \sin^2 A - \sin^2 B\right]$

$= \sin^2 A - \sin^2 A + \sin^2 B$

$= \sin^2 B$

$= LHS$

$\therefore$ LHS = RHS

Hence proved.

$RHS = \cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A+B)$

$= \cos^2 A + \left(1 - \sin^2 B\right) - 2\cos A\cos B\cos(A+B)$

$= \left[\cos^2 A - \sin^2 B\right] - 2\cos A\cos B\cos\left(A+B\right) + 1$

$= \left[\cos\left(A+B\right)\cos\left(A-B\right)\right] - 2\cos A\cos B\cos\left(A+B\right) + 1$

$= \cos\left(A+B\right)\left[\cos\left(A-B\right) - 2\cos A\cos B\right] + 1$

$= \cos\left(A+B\right)\left[\cos A\cos B + \sin A\sin B - 2\cos A\cos B\right] + 1$

$= \cos\left(A+B\right)\left[-\cos A\cos B + \sin A\sin B\right] + 1$

$= -\cos\left(A+B\right)\left[\cos A\cos B - \sin A\sin B\right] + 1$

$= -\cos\left(A+B\right)\left[\cos\left(A+B\right)\right] + 1$

$= -\cos^2\left(A+B\right) + 1$

$= 1 - \cos^2\left(A+B\right)$

$= \sin^2\left(A+B\right) \qquad \left[\sin^2\theta = 1 - \cos^2\theta\right]$

= RHS

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\tan\left(A+B\right)}{\cot\left(A-B\right)}$$

$$= \frac{\tan\left(A+B\right)}{\frac{1}{\tan\left(A-B\right)}} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$= \tan\left(A+B\right)\tan\left(A-B\right)$$

$$= \left[\frac{\tan A + \tan B}{1 - \tan A\tan B}\right]\left[\frac{\tan A - \tan B}{1 + \tan A\tan B}\right]$$

$$= \frac{\left(\tan A + \tan B\right)\left(\tan A - \tan B\right)}{\left(1 - \tan A\tan B\right)\left(1 + \tan A\tan B\right)}$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - \left(\tan A\tan B\right)^2} \qquad \left[\because \left(a-b\right)\left(a+b\right) = a^2 - b^2\right]$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A\tan^2 B}$$

= RHS

$\therefore$ LHS = RHS

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q17

We have,

$8\theta = 6\theta + 2\theta$

$\Rightarrow \tan 8\theta = \tan\left(6\theta + 2\theta\right)$

$\Rightarrow \tan 8\theta = \dfrac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta\tan 2\theta} \qquad \left[\because \tan\left(A+B\right) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}\right]$

$\Rightarrow \tan 8\theta\left(1 - \tan 6\theta\tan 2\theta\right) = \tan 6\theta + \tan 2\theta$

$\Rightarrow \tan 8\theta - \tan 8\theta\tan 6\theta\tan 2\theta = \tan 6\theta + \tan 2\theta$

$\Rightarrow \tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta\tan 6\theta\tan 2\theta$

Hence proved.

We have,

$45° = 30° + 15°$

$\Rightarrow \tan 45° = \tan\left(30° + 15°\right)$

$\Rightarrow 1 = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30°\tan 15°} \qquad \left[\because \tan\left(A+B\right) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}\right]$

$\Rightarrow 1 - \tan 30°\tan 15° = \tan 15° + \tan 30°$

$\Rightarrow 1 = \tan 15° + \tan 30° + \tan 30°\tan 15°$

$\Rightarrow \tan 15° + \tan 30° + \tan 15°\tan 30° = 1$

Hence proved.

We have,

$$45° = 9° + 36°$$

$$\Rightarrow \quad \tan 45° = \tan(9° + 36°)$$

$$\Rightarrow \quad 1 = \frac{\tan 9° + \tan 36°}{1 - \tan 9° \tan 36°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad 1 - \tan 9° \tan 36° = \tan 9° + \tan 36°$$

$$\Rightarrow \quad 1 = \tan 9° + \tan 36° + \tan 9° \tan 36°$$

$$\Rightarrow \quad \tan 9° + \tan 36° + \tan 9° \tan 36° = 1$$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \quad \tan 13\theta = \tan(9\theta + 4\theta)$$

$$\Rightarrow \quad \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad \tan 13\theta (1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q18

We have,

$$\begin{aligned} \text{RHS} &= \tan 3\theta \tan \theta \\ &= \tan(2\theta + \theta) \times \tan(2\theta - \theta) \\ &= \left[\frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}\right] \times \left[\frac{\tan 2\theta - \tan\theta}{1 + \tan 2\theta \tan\theta}\right] \\ &= \frac{(\tan 2\theta + \tan\theta)(\tan 2\theta - \tan\theta)}{(1 - \tan 2\theta \tan\theta)(1 + \tan 2\theta \tan\theta)} \\ &= \frac{\tan^2 2\theta - \tan^2\theta}{1 - \tan^2 2\theta \tan^2\theta} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right] \\ &= \text{LHS} \end{aligned}$$

$\therefore \quad$ LHS = RHS

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q19

$$\frac{\sin x.\cos y + \sin y.\cos x}{\sin x.\cos y - \sin y.\cos x} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x.\cos y + \sin y.\cos x + \sin x.\cos y - \sin y.\cos x}{\sin x.\cos y + \sin y.\cos x - \sin x.\cos y + \sin y.\cos x} = \frac{a+b+a-b}{a+b-a+b} \text{[Using Componendo and Dividendo]}$$

$$\Rightarrow \frac{2\sin x.\cos y}{2\sin y.\cos x} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

Hence Proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x \frac{\sin B}{\cos B} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\Rightarrow \quad \sin A \cos B = x \cos A \sin B \qquad \text{---(i)}$$

Now,

$$\begin{aligned} \frac{\sin(A-B)}{\sin(A+B)} &= \frac{\sin A \cos B - \sin B \cos A}{\sin A \cos B + \cos A \sin B} \\ &= \frac{x \cos A \sin B - \cos A \sin B}{x \cos A \sin B + \cos A \sin B} \qquad \text{[Using equation (i)]} \\ &= \frac{\cos A \sin B (x-1)}{\cos A \sin B (x+1)} \\ &= \frac{x-1}{} \end{aligned}$$

$x+1$

$$\therefore \quad \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q21

We have,

$$\tan(A+B) = x \text{ and } \tan(A-B) = y$$

Now, $\tan 2A = \tan[(A+B)+(A-B)]$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

$$\therefore \quad \tan 2A = \frac{x+y}{1-xy}$$

Now, $\tan 2B = \tan[(A+B)-(A-B)]$

$$= \frac{\tan(A+B)-\tan(A-B)}{1+\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x-y}{1+xy}$$

$$\therefore \quad \tan 2B = \frac{x-y}{1+xy}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q22

We have,

$$\cos A + \sin B = m \text{ and } \sin A + \cos B = n$$

Now, $m^2 + n^2 - 2$

$= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$

$= \cos^2 A + \sin^2 B + 2\cos A \sin B + \sin^2 A + \cos^2 B + 2\sin A \cos B - 2$

$= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 1 + 1 + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 2 + 2(\sin A \cos B + \cos A \sin B) - 2$

$= 2(\sin A \cos B + \cos A \sin B)$

$= 2\sin(A+B) \qquad [\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$

$$\therefore \quad 2\sin(A+B) = m^2 + n^2 - 2$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q23

We have,

$$\tan A + \tan B = a \text{ and } \cot A + \cot B = b$$

Now, $\cot A + \cot B = b$

$\Rightarrow \quad \frac{1}{\tan A} + \frac{1}{\tan B} = b \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

$\Rightarrow \quad \frac{\tan B + \tan A}{\tan A \tan B} = b$

$\Rightarrow \quad \frac{a}{\tan A \tan B} = b \qquad [\because \tan A + \tan B = a]$

$\Rightarrow \quad \frac{a}{b} = \tan A \tan B$

$\because \quad \cot(A+B) = \frac{1}{\tan(A+B)}$

$$= \frac{1}{\frac{\tan A + \tan B}{1-\tan A \tan B}}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \frac{a}{b}}{a} \qquad \left[\because \tan A \tan B = \frac{a}{b}\right]$$

$$= \frac{b - a}{ab}$$

$$= \frac{b}{ab} - \frac{a}{ab}$$

$$= \frac{1}{a} - \frac{1}{b}$$

$$\therefore \quad \cot(A + B) = \frac{1}{a} - \frac{1}{b}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q24

We have,

$$\cos\theta = \frac{8}{17}$$

$$\therefore \quad \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$= \frac{15}{17}$$

Now, $\cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right)$

$$= \left[\cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta\right] + \left[\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta\right]$$

$$+ \left[\cos\frac{2\pi}{3}\cos\theta + \sin\frac{2\pi}{3}\sin\theta\right]$$

$$= \left[\cos\frac{\pi}{6} + \cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\right]\cos\theta + \sin\theta\left[-\sin\frac{\pi}{6} + \sin\frac{\pi}{4} + \sin\frac{2\pi}{3}\right]$$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\right]$$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \sin\frac{\pi}{6}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \cos\frac{\pi}{6}\right]$$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right]$$

$[\because \cos A$ is negative in second quadrant$]$

$$= \left[\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right] \times \frac{8}{17} + \frac{15}{17} \times \left[\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right]$$

$$= \left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{8}{17} + \frac{15}{17}\right)$$

$$= \left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{8 + 15}{17}\right)$$

$$= \left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right) \times \frac{23}{17}$$

$$\therefore \quad \cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right) = \left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right) \times \frac{23}{17}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \tan\frac{\pi}{3}}{1 - \tan x \tan\frac{\pi}{3}}\right] + \left[\frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan x \tan\frac{2\pi}{3}}\right] = 3$$

$$\left[1 - \tan x \tan \frac{\pi}{3}\right] \quad \left[1 - \tan x \tan \frac{\pi}{3}\right]$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}\right] + \left[\frac{\tan x + \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}{1 - \tan x \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}\right] = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \cot\frac{\pi}{3}}{1 + \tan x \cot\frac{\pi}{3}} = 3 \qquad \left[\because \tan\theta \text{ is negative in second quadrant}\right]$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = 3$$

$$\Rightarrow \quad \tan x + \frac{\left(\tan x + \sqrt{3}\right)\left(1 + \sqrt{3}\tan x\right) + \left(\tan x - \sqrt{3}\right)\left(1 - \sqrt{3}\tan x\right)}{\left(1 - \sqrt{3}\tan x\right)\left(1 + \sqrt{3}\tan x\right)} = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}\tan^2 x + \sqrt{3} + 3\tan x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1 - \left(\sqrt{3}\tan x\right)^2} = 3$$

$$\Rightarrow \quad \tan x + \frac{8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x\left(1 - 3\tan^2 x\right) + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x - 3\tan^3 x + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{9\tan x - 3\tan^3 x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\left(3\tan x - \tan^3 x\right)}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 1$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q26

We have,

$$\sin\left(\alpha + \beta\right) = 1$$

$$\Rightarrow \quad \sin\left(\alpha + \beta\right) = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{\pi}{2} \qquad \text{---(i)}$$

and, $\sin\left(\alpha - \beta\right) = \frac{1}{2}$

$$\Rightarrow \quad \sin\left(\alpha - \beta\right) = \sin\frac{\pi}{6}$$

$$\Rightarrow \quad \alpha - \beta = \frac{\pi}{6} \qquad \text{---(ii)}$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{3}$$

Putting $\alpha = \frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \quad \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \quad \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \quad \beta = \frac{\pi}{6}$$

Now, $\tan\left(\alpha + 2\beta\right) = \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right)$

$$= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$$

$$= \tan\frac{2\pi}{3}$$

$= \tan\frac{\ }{3}$

$= \tan\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$

$= -\cot\frac{\pi}{6}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\sqrt{3}$

$\therefore \quad t\left(\alpha+2\beta\right) = -\sqrt{3}$

and, $\tan\left(2\alpha+\beta\right) = \tan\left(2\times\frac{\pi}{3}+\frac{\pi}{6}\right)$

$= \tan\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)$

$= \tan\left(\frac{4\pi+\pi}{6}\right)$

$= \tan\left(\frac{5\pi}{6}\right)$

$= \tan\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

$= -\cot\frac{\pi}{3}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\frac{1}{\sqrt{3}}$

$\therefore \quad \tan\left(2\alpha+\beta\right) = \frac{-1}{\sqrt{3}}$

Trigonometric Ratios of Compound Angles Ex 7.1 Q27

We have,

$6\cos\theta + 8\sin\theta = 9$ ---(i)

$\Rightarrow 8\sin\theta = 9 - 6\cos\theta$

$\Rightarrow (8\sin\theta)^2 = (9-6\cos\theta)^2$ $[\because \text{Squaring both sides}]$

$\Rightarrow 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow 64\left(1-\cos^2\theta\right) = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow 64 - 64\cos^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow 36\cos^2\theta + 64\cos^2\theta - 108\cos\theta + 81 - 64 = 0$

$\Rightarrow 100\cos^2\theta - 108\cos\theta + 17 = 0$ ---(ii)

Since α, β are roots of equation (ii).

Therefore, $\cos\alpha$ and $\cos\beta$ are roots of equation (ii)

$\therefore \quad \cos\alpha + \cos\beta = \frac{17}{100}$ ---(iii)

Again, $6\cos\theta + 8\sin\theta = 9$

$\Rightarrow 6\cos\theta = 9 - 8\sin\theta$

$\Rightarrow (6\cos\theta)^2 = (9-8\sin\theta)^2$ $[\because \text{Squaring both sides}]$

$\Rightarrow 36\cos^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow 36\left(1-\sin^2\theta\right) = 81 + 64\sin^2\theta - 144\cos\theta$

$\Rightarrow 36 - 36\sin^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow 64\sin^2\theta + 36\sin^2\theta - 144\sin\theta + 81 - 36 = 0$

$\Rightarrow 100\sin^2\theta - 144\sin\theta + 45 = 0$ ---(iv)

$\therefore \quad \sin\alpha \times \sin\beta = \frac{45}{100}$ ---(v)

Now, $\cos\left(\alpha+\beta\right) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{17}{100} - \frac{45}{100}$ [Using equation (iii) and (v)]

$= -\frac{28}{100}$

$$= -\frac{7}{25}$$

Now, $\sin(\alpha+\beta) = \sqrt{1-(\cos\theta)^2}$

$$= \sqrt{1-\left(-\frac{7}{25}\right)^2}$$

$$= \sqrt{1-\frac{49}{625}}$$

$$= \sqrt{\frac{625-49}{625}}$$

$$= \sqrt{\frac{576}{625}}$$

$$= \frac{24}{25}$$

$\therefore \quad \sin(\alpha+\beta) = \frac{24}{25}$

Trigonometric Ratios of Compound Angles Ex 7.1 Q28

$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$

$$b^2+a^2 = (\cos\alpha+\cos\beta)^2 + (\sin\alpha+\sin\beta)^2$$

$$\Rightarrow \quad b^2+a^2 = (\cos^2\alpha+\sin^2\alpha)+(\cos^2\beta+\sin^2\beta)+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2+a^2 = 1+1+2\cos(\alpha-\beta) = 2+2\cos(\alpha-\beta)$$

and, $b^2-a^2 = (\cos\alpha+\cos\beta)^2-(\sin\alpha+\sin\beta)^2$

$$b^2-a^2 = \cos^2\alpha+\cos^2\beta-\sin^2\alpha-\sin^2\beta+2(\cos\alpha\cos\beta-\sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2-a^2 = (\cos^2\alpha-\sin^2\beta)+(\cos^2\beta-\sin^2\alpha)+2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\cos(\alpha-\beta)+\cos(\beta+\alpha)\cos(\beta-\alpha)+2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = 2\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos(\alpha+\beta)$$

$$\left[\because \cos(\beta-\alpha) = \cos\{-(\alpha-\beta)\} = \cos(\alpha-\beta)\right]$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\{2\cos(\alpha-\beta)+2\}$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)(b^2+a^2) \quad \text{[Using (i)]}$$

Thus, $b^2-a^2 = (b^2+a^2)\cos(\alpha+\beta)$

$$\Rightarrow \quad \cos(\alpha+\beta) = \frac{b^2-a^2}{b^2+a^2}$$

$$\Rightarrow \quad \sin(\alpha+\beta) = \sqrt{1-\left(\frac{b^2-a^2}{b^2-a^2}\right)^2} = \sqrt{\frac{4a^2b^2}{(a^2+b^2)^2}} = \frac{2ab}{a^2+b^2}$$

$$b^2+a^2 = (\cos\alpha+\cos\beta)^2 + (\sin\alpha+\sin\beta)^2$$

$$\Rightarrow \quad b^2+a^2 = (\cos^2\alpha+\sin^2\alpha)+(\cos^2\beta+\sin^2\beta)+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2+a^2 = 1+1+2\cos(\alpha-\beta) = 2+2\cos(\alpha-\beta)$$

and, $b^2-a^2 = (\cos\alpha+\cos\beta)^2-(\sin\alpha+\sin\beta)^2$

$$b^2-a^2 = \cos^2\alpha+\cos^2\beta-\sin^2\alpha-\sin^2\beta+2(\cos\alpha\cos\beta-\sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2-a^2 = (\cos^2\alpha-\sin^2\beta)+(\cos^2\beta-\sin^2\alpha)+2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\cos(\alpha-\beta)+\cos(\beta+\alpha)\cos(\beta-\alpha)+2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = 2\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos(\alpha+\beta)$$

$$\left[\because \cos(\beta-\alpha) = \cos\{-(\alpha-\beta)\} = \cos(\alpha-\beta)\right]$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\{2\cos(\alpha-\beta)+2\}$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)(b^2+a^2) \quad \text{[Using (i)]}$$

Thus, $b^2-a^2 = (b^2+a^2)\cos(\alpha+\beta)$

$$\Rightarrow \quad \cos(\alpha+\beta) = \frac{b^2-a^2}{b^2+a^2}$$

b + a

Trigonometric Ratios of Compound Angles Ex 7.1 Q29

LHS $\frac{1}{\sin(x-a)\sin(x-b)}$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)}-\frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\cot(x-a)-\cot(x-b)\right]$

$= \frac{\cot(x-a)-\cot(x-b)}{\sin(a-b)}$

= RHS

∴ LHS= RHS

Hence proved

LHS $\frac{1}{\sin(x-a)\cos(x-b)}$

$= \frac{1}{\cos(a-b)}\left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}\right]$

$= \frac{1}{\cos(a-b)}\left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)}\right]$

$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$

$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)}+\frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$

$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-a)}{\sin(x-a)}+\frac{\sin(x-b)}{\cos(x-b)}\right]$

$= \frac{1}{\cos(a-b)}\left[\cot(x-a)+\tan(x-b)\right]$

$= \frac{\cot(x-a)+\tan(x-b)}{\cos(a-b)}$

= RHS

∴ LHS= RHS

Hence proved

LHS $\frac{1}{\cos(x-a)\cos(x-b)}$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)}-\frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)}{\cos(x-b)}-\frac{\sin(x-a)}{\cos(x-a)}\right]$

$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-s)\right]$

$= \frac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$

= RHS

∴ LHS = RHS

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q30

We have,

$\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$

$\Rightarrow \quad -(\cos\alpha \cos\beta - \sin\alpha \sin\beta) = -1$

$\Rightarrow \quad \cos(\alpha + \beta) = 1 \qquad ---(i)$

$\therefore \quad \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$

$= \sqrt{1 - 1^2}$

$= 0$

$\Rightarrow \quad \sin(\alpha + \beta) = 0 \qquad ---(ii)$

Now,

$$1 + \cot\alpha \tan\beta = 1 + \frac{\cos\alpha}{\sin\alpha} \times \frac{\sin\beta}{\cos\beta}$$

$$= \frac{\sin\alpha \times \cos\beta + \cos\alpha \times \sin\beta}{\sin\alpha \times \cos\beta}$$

$$= \frac{\sin(\alpha + \beta)}{\sin\alpha \times \cos\beta}$$

$$= \frac{0}{\sin\alpha \times \cos\beta} \qquad [\text{Using equation (ii)}]$$

$$= 0$$

$\therefore \quad 1 + \cot\alpha \tan\beta = 0$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q31

We have,

$\tan\alpha = x + 1$ and $\tan\beta = x - 1$

Now, $2\cot(\alpha - \beta)$

$$= \frac{2}{\tan(\alpha - \beta)}$$

$$= \frac{2}{\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}}$$

$$= \frac{2(1 + \tan\alpha \tan\beta)}{\tan\alpha - \tan\beta}$$

$$= \frac{2[1 + (x+1)(x-1)]}{x + 1 - (x - 1)}$$

$$= \frac{2[1 + x^2 - 1]}{x + 1 - x + 1}$$

$$= \frac{2 \times x^2}{2} = x^2$$

$\therefore \quad 2\cot(\alpha - \beta) = x^2$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q32

Let the two parts of the angle be θ and θ − Ø.

$\tan(\theta - Ø) = \lambda \tan Ø \quad$ *[According to question]*

$$\Rightarrow \frac{\tan(\theta - Ø)}{\tan Ø} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - Ø)}{\tan Ø} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - Ø) + \tan Ø}{\tan(\theta - Ø) - \tan Ø} = \frac{\lambda + 1}{\lambda - 1} \quad \textit{[Using Componendo and Dividendo]}$$

$$\Rightarrow \frac{\frac{\tan\theta - \tan\emptyset}{1+\tan\theta.\tan\emptyset} + \tan\emptyset}{\frac{\tan\theta - \tan\emptyset}{1+\tan\theta.\tan\emptyset} - \tan\emptyset} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\frac{\tan\theta - \tan\emptyset + \tan\emptyset(1+\tan\theta.\tan\emptyset)}{1+\tan\theta.\tan\emptyset}}{\frac{\tan\theta - \tan\emptyset - \tan\emptyset(1+\tan\theta.\tan\emptyset)}{1+\tan\theta.\tan\emptyset}} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan\theta - \tan\emptyset + \tan\emptyset + \tan\theta.\tan^2\emptyset}{\tan\theta - \tan\emptyset - \tan\emptyset - \tan\theta.\tan^2\emptyset} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan\theta + \tan\theta.\tan^2\emptyset}{\tan\theta - 2\tan\emptyset - \tan\theta.\tan^2\emptyset} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan}{}$$

Trigonometric Ratios of Compound Angles Ex 7.1 33

$$\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - 1}{\tan\alpha + 1}\text{[Dividing both Numerator and Denominator by }\cos\alpha\text{]}$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1+\tan\frac{\pi}{4}.\tan\alpha}$$

$$\Rightarrow \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \theta = \alpha - \frac{\pi}{4}\quad\text{[Removing tan from both sides]}$$

$$\Rightarrow \cos\theta = \cos\left(\alpha - \frac{\pi}{4}\right)\text{[Taking cos on both sides]}$$

$$\Rightarrow \cos\theta = \cos\alpha.\cos\frac{\pi}{4} + \sin\alpha.\sin\frac{\pi}{4}$$

$$\Rightarrow \cos\theta = \cos\alpha.\frac{1}{\sqrt{2}} + \sin\alpha.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\theta = \frac{\cos\alpha + \sin\alpha}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}\cos\theta = \sin\alpha + \cos\alpha$$

Hence Proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q34

RHS,

$$\frac{p+q}{1-pq}$$

$$= \frac{\tan(A+B) + \tan(A-B)}{1-\tan(A+B).\tan(A-B)}$$

$$= \frac{\frac{\tan A + \tan B}{1-\tan A.\tan B} + \frac{\tan A - \tan B}{1+\tan A.\tan B}}{1 - \frac{\tan A+\tan B}{1-\tan A.\tan B}.\frac{\tan A - \tan B}{1+\tan A.\tan B}}$$

$$= \frac{\frac{(\tan A+\tan B)(1+\tan A.\tan B) + (\tan A - \tan B)(1-\tan A.\tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}{\frac{(1-\tan A.\tan B)(1+\tan A.\tan B) - (\tan A+\tan B).(\tan A - \tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}$$

$$= \frac{\tan A + \tan B + \tan^2 A.\tan B + \tan A.\tan^2 B + \tan A - \tan B - \tan^2 A.\tan B + \tan A.\tan^2 B}{1-\tan^2 A.\tan^2 B - \tan^2 A + \tan^2 B}$$

$$= \frac{2\tan A + 2\tan A.\tan^2 B}{(1-\tan^2 A)(1+\tan^2 B)} = \frac{2\tan A(1+\tan^2 B)}{(1-\tan^2 A)(1+\tan^2 B)} = \frac{2\tan A}{1-\tan^2 A} = \tan 2A = LHS$$

Hence Proved