RD Sharma Solutions Class 11 Maths Chapter 7 Ex 7.2

Trigonometric Ratios of Compound Angles Ex 7.2 Q1

Let $f(\theta) = 12\sin\theta - 5\cos\theta$

We know that

$$-\sqrt{(12)^2+(-5)^2} \le f(\theta) \le \sqrt{(12)^2+(-5)^2}$$
$$\Rightarrow \quad -\sqrt{144+25} \le f(\theta) \le \sqrt{144+25}$$
$$\Rightarrow \quad -\sqrt{169} \le f(\theta) \le \sqrt{169}$$
$$\Rightarrow \quad -13 \le f(\theta) \le 13$$

Hence, minimum and maximum values of $12\sin\theta - 5\cos\theta$ are -13 and 13 respectively.

Let $f(\theta) = 12\cos\theta + 5\sin\theta + 4$

We know that

$$-\sqrt{(12)^2+(5)^2} \le 12\cos\theta + 5\sin\theta \le \sqrt{(12)^2+(-5)^2}$$
$$\Rightarrow \quad -\sqrt{144+25} \le 12\cos\theta + 5\sin\theta \le \sqrt{144+25}$$
$$\Rightarrow \quad -\sqrt{169} \le 12\cos\theta + 5\sin\theta \le \sqrt{169}$$
$$\Rightarrow \quad -13 \le 12\cos\theta + 5\sin\theta \le 13$$
$$\Rightarrow \quad -13+4 \le 12\cos\theta + 5\sin\theta + 4 \le 13+4$$
$$\Rightarrow \quad -9 \le 12\cos\theta + 5\sin\theta + 4 \le 17$$
$$\Rightarrow \quad -9 \le f(\theta) \le 17$$

Hnece, minimum and maximum values of $12\cos\theta + 5\sin\theta + 4$ are -9 and 17 respectively.

Let $f(\theta) = 5\cos\theta + 3\sin\left(\frac{\pi}{6} - \theta\right) + 4$

Then, $f(\theta) = 5\cos\theta + 3\left[\sin\frac{\pi}{6}\cos\theta - \cos\frac{\pi}{6}\sin\theta\right] + 4$

$$= 5\cos\theta + 3\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] + 4$$
$$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$
$$= \left(5+\frac{3}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$
$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$
$$= \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4$$

We know that

$$-\sqrt{\left(\frac{13}{2}\right)^2+\left(\frac{-3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2+\left(\frac{-3\sqrt{3}}{2}\right)^2}$$
$$\Rightarrow \quad -\sqrt{\frac{169}{4}+\frac{27}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{169}{4}+\frac{27}{4}}$$
$$\Rightarrow \quad -\sqrt{\frac{196}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{196}{4}}$$
$$\Rightarrow \quad -\frac{14}{2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \frac{14}{2}$$
$$\Rightarrow \quad -7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$$
$$\Rightarrow \quad -7+4 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4 \le 7+4$$
$$\Rightarrow \quad -3 \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4 \le 11$$
$$\Rightarrow \quad -3 \le f(\theta) \le 11$$

Let $f(\theta) = \sin\theta - \cos\theta + 1$. Then,

$$f(\theta) = \sin\theta + (-1)\cos\theta + 1$$
$$= (-1)\cos\theta + \sin\theta + 1$$

We know that

We know that

$$-\sqrt{(-1)^2+(1)^2} \le -\cos\theta+\sin\theta \le \sqrt{(-1)^2+(1)^2}$$

$\Rightarrow \quad -\sqrt{1+1} \le -\cos\theta+\sin\theta \le \sqrt{1+1}$

$\Rightarrow \quad -\sqrt{2} \le -\cos\theta+\sin\theta \le \sqrt{2}$

$\Rightarrow \quad -\sqrt{2}+1 \le -\cos\theta+\sin\theta+1 \le \sqrt{2}+1$

$\Rightarrow \quad 1-\sqrt{2} \le f(\theta) \le 1+\sqrt{2}$

Hence, minimum and maximum values of $\sin\theta-\cos\theta+1$ are $1-\sqrt{2}$ and $1+\sqrt{2}$ respectively.

Trigonometric Ratios of Compound Angles Ex 7.2 Q2

Let $f(\theta) = \sqrt{3}\sin\theta - \cos\theta$

Multiplying and dividing by $\sqrt{(\sqrt{3})^2+(-1)^2}$, we get

$$f(\theta) = \sqrt{(\sqrt{3})^2+(-1)^2}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{(\sqrt{3})^2+(-1)^2}} - \frac{\cos\theta}{\sqrt{(\sqrt{3})^2+(-1)^2}}\right]$$

$$= \sqrt{3+1}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{3+1}} - \frac{\cos\theta}{\sqrt{3+1}}\right]$$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}\sin\theta}{2} - \frac{\cos\theta}{2}\right] \qquad ---(i)$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}}{2}\times\sin\theta - \frac{1}{2}\times\cos\theta\right]$

$$= 2\left[\cos\frac{\pi}{6}\times\sin\theta - \sin\frac{\pi}{6}\times\cos\theta\right]$$

$$= 2\left[\sin\theta\times\cos\frac{\pi}{6} - \cos\theta\times\sin\frac{\pi}{6}\right]$$

$$= 2\sin\left(\theta-\frac{\pi}{6}\right) \qquad [\because \sin(A-B) = \sin A\cos B - \cos A\sin B]$$

$\Rightarrow \quad f(\theta) = 2\sin\left(\theta-\frac{\pi}{6}\right)$

Again,

$$f(\theta) = 2\left[\frac{\sqrt{3}}{2}\sin\theta - \frac{\cos\theta}{2}\right]$$

$$= -2\left[\frac{1}{2}\times\cos\theta - \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= -2\left[\cos\frac{\pi}{3}\times\cos\theta - \sin\frac{\pi}{3}\times\sin\theta\right]$$

$$= -2\cos\left(\frac{\pi}{3}+\theta\right)$$

Let $f(\theta) = \cos\theta - \sin\theta$

Multiplying and dividing by $\sqrt{1^2+1^2}$, we get

$$f(\theta) = \sqrt{1^2+1^2}\left[\frac{\cos\theta}{\sqrt{1^2+1^2}} - \frac{\sin\theta}{\sqrt{1^2+1^2}}\right]$$

$\Rightarrow \quad f(\theta) = \sqrt{2}\left[\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right] \qquad ---(i)$

Now, $\quad f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$

$$= \sqrt{2}\left[\sin\frac{\pi}{4}\times\cos\theta - \cos\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right) \qquad [\because \sin(A-B) = \sin A\cos B - \cos A\sin B]$$

$\Rightarrow \quad f(\theta) = \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)$

Again,

$$f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4}\times\cos\theta - \sin\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right) \qquad [\because \cos(A+B) = \cos A\cos B - \sin A\sin B]$$

$\Rightarrow \quad f(\theta) = \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)$

Let $f(\theta) = 24\cos\theta + 7\sin\theta$

Multiplying and dividing by $\sqrt{(24)^2 + (7)^2}$, we get

$$f(\theta) = \sqrt{(24)^2 + 7^2}\left[\frac{24\cos\theta}{\sqrt{24^2 + 7^2}} + \frac{7\sin\theta}{\sqrt{24^2 + 7^2}}\right]$$

$$= \sqrt{576 + 49}\left[\frac{24\cos\theta}{\sqrt{576 + 49}} + \frac{7\sin\theta}{\sqrt{576 + 49}}\right]$$

$$= \sqrt{625}\left[\frac{24\cos\theta}{\sqrt{625}} + \frac{7\sin\theta}{\sqrt{625}}\right]$$

$$= 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$\Rightarrow \quad f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right] \quad ---(i)$$

Now, $f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$

$$= 25\left[\sin\alpha\times\cos\theta + \cos\alpha\times\sin\theta\right]$$

where $\sin\alpha = \frac{24}{25}$ and $\cos\alpha = \frac{7}{25}$

$$\Rightarrow \quad f(\theta) = 25\sin(\alpha + \theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{24}{7}$$

Again,

$$f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$= 25\left[\cos\alpha\times\cos\theta + \sin\alpha\times\sin\theta\right], \text{ where } \cos\alpha = \frac{24}{25} \text{ and } \sin\alpha = \frac{7}{25}$$

$$\Rightarrow \quad f(\theta) = 25\cos(\alpha - \theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{7}{24}$$

Trigonometric Ratios of Compound Angles Ex 7.2 Q3

We have,

$\sin 100° - \sin 10°$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\times\sin 100° - \frac{1}{\sqrt{2}}\times\cos 100°\right)$ [Multiplying and dividing by $\sqrt{1^2 + 1^2}$ ie., by $\sqrt{2}$]

$= \sqrt{2}\left(\cos 45°\times\sin 100° - \sin 45°\times\cos 100°\right)$

$= \sqrt{2}\left(\sin 100°\times\cos 45° - \cos 100°\times\sin 45°\right)$

$= \sqrt{2}\left(\sin(100° - 45°)\right)$

$= \sqrt{2}\sin 55°$, which is positive real number.

[$\because \sin\theta$ is positive in first quadrant]

Trigonometric Ratios of Compound Angles Ex 7.2 Q4

$(2\sqrt{3}+3)\sin\theta + 2\sqrt{3}\cos\theta$

assume $a = 2\sqrt{3}+3$, $b = 2\sqrt{3}$

$\sqrt{a^2+b^2} = \sqrt{12+9+12\sqrt{3}+12} = \sqrt{33+12\sqrt{3}}$

Dividing and multiplying the above equation with above value

we get, $\sqrt{33+12\sqrt{3}}\left(\frac{2\sqrt{3}+3}{\sqrt{33+12\sqrt{3}}}\sin\theta + \frac{2\sqrt{3}}{\sqrt{33+12\sqrt{3}}}\cos\theta\right)$

Assume $\tan\phi = \frac{a}{b}$, we have $\sin\phi = \frac{a}{\sqrt{a^2+b^2}}$, $\cos\phi = \frac{b}{\sqrt{a^2+b^2}}$

so above expressions changes to $\sqrt{33+12\sqrt{3}}(\sin\phi\sin\theta + \cos\phi\cos\theta)$

which is equal to $\sqrt{33+12\sqrt{3}}\cos(\theta-\phi)$

We know that maximum and minimum value of any cosine term is +1 and -1

$\sqrt{33+12\sqrt{3}} = \sqrt{15+12+6+12\sqrt{3}}$

we know that $12\sqrt{3}+6 < 12\sqrt{5}$ becasue value of $\sqrt{5}-\sqrt{3}$ is more than 0.5

so if we replace $12\sqrt{3}+6$ with $12\sqrt{5}$ the above inequality still holds

So range of above expression can be $\sqrt{15+12+12\sqrt{5}} = 2\sqrt{3}+\sqrt{15}$

$-(2\sqrt{3}+\sqrt{15}) < \sqrt{33+12\sqrt{3}}\cos(\theta-\phi) < 2\sqrt{3}+\sqrt{15}$