RD Sharma Solutions Class 11 Maths Chapter 8 Ex 8.1

Transformation Formulae Ex 8.1 Q1

(i) $2\sin 3\theta\cos\theta$

$= \sin(3\theta+\theta)+\sin(3\theta-\theta)$ $\quad [\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)]$

$= \sin 4\theta+\sin 2\theta$

(ii) $2\cos 3\theta\sin 2\theta$

$\because$ $2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$\Rightarrow$ $2\cos 3\theta\sin 2\theta = \sin(3\theta+2\theta)-\sin(3\theta-2\theta)$

$=$ $\sin 5\theta-\sin\theta$

(iii) $2\sin 4\theta\sin 3\theta$

$\because$ $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow$ $2\sin 4\theta\sin 3\theta = \cos(4\theta-3\theta)-\cos(4\theta+3\theta)$

$= \cos\theta-\cos 7\theta$

(iv) $2\cos 7\theta\cos 3\theta$

$\because$ $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$\Rightarrow$ $2\cos 7\theta\cos 3\theta = \cos(7\theta+3\theta)+\cos(7\theta-3\theta)$

$= \cos 10\theta+\cos 4\theta$

Transformation Formulae Ex 8.1 Q2

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\because$ $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow$ $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)-\cos\left(\frac{6\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)-\cos\left(\frac{\pi}{2}\right)$

$= \frac{1}{2}-0 = \frac{1}{2} = \text{RHS}$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

$\because$ $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$= \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \cos(\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{3}\right)$

$= 0+\frac{1}{2} = \frac{1}{2} = \text{RHS}$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$\because$ $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$= \left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \sin\frac{\pi}{2}+\sin\frac{\pi}{3}$

$= 1+\frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \text{RHS}$ (Taking LCM)

Transformation Formulae Ex 8.1 Q3(i)

$\sin 50°\cos 85° = \frac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$

$\text{LHS} = \sin 50°\cos 85° = \frac{2\sin 50°\cos 85°}{2}$

$\because$ $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$\Rightarrow$ $\frac{2\sin 50°\cos 85°}{2} = \frac{1}{2}[\sin(50°+85°)+\sin(50°-85°)]$

$= \frac{1}{2}[\sin 135°+\sin(-35°)]$

$= \frac{1}{2}[\sin(90°+45°)-\sin 35°]$ $\quad [\because \sin(-\theta) = -\sin\theta]$

$= \frac{1}{2}[\cos 45°-\sin 35°]$ $\quad [\because \sin(90°+\theta) = \cos\theta]$

Now

Now,

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} - \sin 35°\right]$$

$$= \frac{1 - \sqrt{2}\sin 35°}{2\sqrt{2}}$$

Transformation Formulae Ex 8.1 Q3(ii)

$$\text{LHS} = \sin 25° \cos 115°$$

$$= \frac{2\sin 25° \cos 115°}{2}$$

We Know that

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$= \frac{1}{2}\left[\sin(25° + 115°) + \sin(25° - 115°)\right]$$

$$= \frac{1}{2}\left[\sin 140° + \sin(-90°)\right]$$

$$\sin(-\theta) = -\sin\theta$$

And, $\sin(90° + \theta) = \cos\theta$

$$\Rightarrow \quad \frac{1}{2}\left[\sin(90° + 50°) - \sin 90°\right]$$

$$= \frac{1}{2}\left[\cos 50° - 1\right]$$

Also,

$$\cos\theta = \sin(90° - \theta)$$

$$\cos 50° = \sin(90° - 50°) = \sin 40°$$

$$\frac{1}{2}\left[\sin 40° - 1\right]$$

Transformation Formulae Ex 8.1 Q4.

We have,

$$\text{LHS} = 4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)$$

$$= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right]$$

$$= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta+\frac{\pi}{3}-\theta\right)+\cos\left(\frac{\pi}{3}+\theta-\frac{\pi}{3}+\theta\right)\right]$$

$$= 2\cos\theta\left[\cos\frac{2\pi}{3}+\cos 2\theta\right]$$

$$= 2\cos\theta\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)+\cos 2\theta\right]$$

$$= 2\cos\theta\left[-\sin\frac{\pi}{6}+\cos 2\theta\right]$$

$$= 2\cos\theta\left[-\frac{1}{2}+\cos 2\theta\right]$$

$$= -2\cos\theta\times\frac{1}{2}+2\cos\theta\cos 2\theta$$

$$= -\cos\theta+\left[\cos(\theta+2\theta)+\cos(2\theta-\theta)\right]$$

$$= -\cos\theta+\cos 3\theta+\cos\theta$$

$$= \cos 3\theta$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS Hence proved.

Transformation Formulae Ex 8.1 Q5(i)

$$\cos 10° \cos 30° \cos 50° \cos 70^0 = \frac{3}{16}$$

$$\text{LHS} = \cos 10° \cos 30° \cos 50° \cos 70°$$

$$= \cos 30° \cos 10° \cos 50° \cos 70°$$

$$= \frac{\sqrt{3}}{2}(\cos 10° \cos 50° \cos 70°)$$

$$= \frac{\sqrt{3}}{2}(\cos 10° \cos 50°)\cos 70°$$

$$= \frac{\sqrt{3}}{4}(2\cos 10° \cos 50°)\cos 70° \quad \text{[Multiiplying and dividing by 2]}$$

Also,

$\Rightarrow \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B) \qquad$ ---(i)

$= \frac{\sqrt{3}}{4}\cos 70°\left(\cos(50°+10°) + \cos(10°-50°)\right)$

$= \frac{\sqrt{3}}{4}\cos 70°\left(\cos 60° + \cos(-40°)\right)$

Now,

$\cos(-\theta) = \cos\theta$

$= \frac{\sqrt{3}}{4}\cos 70°\left(\frac{1}{2} + \cos 40°\right) \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \frac{\sqrt{3}}{8}\cos 70° + \frac{\sqrt{3}}{4}\cos 70°\cos 40°$

$= \frac{\sqrt{3}}{8}\cos 70° + \frac{\sqrt{3}}{8}(2\cos 70°\cos 40°)$

$= \frac{\sqrt{3}}{8}\left[\cos 70° + \cos(70°+40°) + \cos(70°-40°)\right] \qquad$ [from (i)]

$= \frac{\sqrt{3}}{8}\left[\cos 70° + \cos 110° + \cos 30°\right]$

$= \frac{\sqrt{3}}{8}\left[\cos 70° + \cos(180°-70°) + \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{8}\left[\cos 70° - \cos 70° + \frac{\sqrt{3}}{2}\right] \qquad \left[\because \cos(180°-\theta) = -\cos\theta\right]$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

= RHS

Transformation Formulae Ex 8.1 Q5(ii)

$\cos 40°\cos 80°\cos 160° = -\frac{1}{8}$

LHS $= \cos 40°\cos 80°\cos 160°$

$= \cos 80°\cos 40°\cos 160°$

Multiplying and dividing by 2

$= \frac{1}{2}\left(\cos 80° \times (2\cos 40°\cos 160°)\right)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$= \frac{1}{2}\left(\cos 80°\left(\cos(40°+160°) + \cos(40°-160°)\right)\right)$

$= \frac{1}{2}\left(\cos 80°\left(\cos 200 + \cos(-120)\right)\right)$

$= \frac{1}{2}\cos 80°\left(\cos(180°+20°) + \cos(180°-60°)\right)$

$= \frac{1}{2}\cos 80°\left(\cos 20° + \cos 60°\right)$

$= \frac{1}{2}\cos 80°\cos 20° + \frac{1}{2}\cos 80° + \cos 60°$

$= -\frac{1}{2}(2\cos 80°\cos 20°) + \frac{1}{2}\cos 80°\cos 60°$

$= -\frac{1}{4}\left[2\cos 80°\cos 20° + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos(80°+20°) + \cos(80°-20°) + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos 100° + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos(180°-80°) + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\left[-\cos 80° + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\cos 60°$

$= -\frac{1}{4} \times \frac{1}{2}$

$= -\frac{1}{8}$ RHS

Transformation Formulae Ex 8.1 Q5(iii)

$\sin 20°\sin 40°\sin 80°$

$= \frac{1}{2}(2\sin 20°\sin 40°)\sin 80°$

$= \frac{1}{2}\left[\cos(40°-20°) - \cos(40°+20°)\right]\sin 80° \qquad \left[\because 2\sin A\sin B = \cos(A-B) - \cos(A+B)\right]$

$= \frac{1}{2}\left[\cos 20° - \cos 60°\right]\sin 80°$

$= \frac{1}{2}\left[\cos 20° - \frac{1}{2}\right]\sin 80°$

$= \frac{1}{2}\left[\cos 20°\sin 80°\right] - \frac{1}{4}\sin 80°$

$= \frac{1}{4}\left[2\cos 20°\sin 80° - \sin 80°\right]$

$= \frac{1}{4}\left[\sin(80° + 20°) + \sin(80° - 20°) - \sin 80°\right]$ $\left[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)\right]$

$= \frac{1}{4}\left[\sin 100° + \sin 60° - \sin 80°\right]$

$= \frac{1}{4}\left[\sin(180° - 80°) + \frac{\sqrt{3}}{2} - \sin 80°\right]$

$= \frac{1}{4}\left[\sin 80° + \frac{\sqrt{3}}{2} - \sin 80°\right]$

$= \frac{\sqrt{3}}{8} \quad = \text{RHS}$

Transformation Formulae Ex 8.1 Q5(iv)

$\cos 20° \cos 40° \cos 80°$

$= \frac{1}{2}(2\cos 20° \cos 40°)\cos 80°$

$= \frac{1}{2}\left[\cos(40° + 20°) + \cos(40° - 20°)\right]\cos 80°$ $\left[\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)\right]$

$= \frac{1}{2}\left[\cos 60° + \cos 20°\right]\cos 80°$

$= \frac{1}{2}\left[\frac{1}{2} + \cos 20°\right]\cos 80°$

$= \frac{1}{2}\left[\cos 80° + 2\cos 20° \cos 80°\right]$

$= \frac{1}{4}\left[\cos 80° + \cos(80° + 20°) + \cos(20° - 80°)\right]$

$= \frac{1}{4}\left[\cos 80° + \cos 100° + \cos 60°\right]$

$= \frac{1}{4}\left[\cos 80° + \cos(180° - 80°) + \cos 60°\right]$

$= \frac{1}{4}\left[\cos 80° - \cos 80° + \cos 60°\right]$

$= \frac{1}{4}\left[\frac{1}{2}\right] = \frac{1}{8} = \text{RHS}$

Transformation Formulae Ex 8.1 Q5(V)

$\tan 20° \tan 40° \tan 60° \tan 80°$

$= (\tan 20° \tan 40° \tan 80°)\sqrt{3}$ $\left[\because \tan 60° = \sqrt{3}\right]$

$= \left(\frac{\sin 20° \sin 40° \sin 80°}{\cos 20° \cos 40° \cos 80°}\right)\sqrt{3}$

$= \frac{(2\sin 20° \sin 40°)\sin 80° \times \sqrt{3}}{(2\cos 20° \cos 40°)\cos 80°}$

Applying

$\Rightarrow \quad 2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$= \frac{\{\cos(40° - 20°) - \cos(40° + 20°)\}\sin 80° \times \sqrt{3}}{\{\cos(20° + 40°) + \cos(40° - 20°)\}\cos 80°}$

$= \frac{(\cos 20° - \cos 60°)\sin 80° \times \sqrt{3}}{(\cos 60° + \cos 20°)\cos 80°}$

$= \frac{\left(\cos 20° - \frac{1}{2}\right)\sin 80° \times \sqrt{3}}{\left(\frac{1}{2} + \cos 20°\right)\cos 80°}$

$= \frac{(2\sin 80° \cos 20° - \sin 80°)\sqrt{3}}{\cos 80° + 2\cos 20° \cos 80°}$

$\Rightarrow \quad 2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$= \frac{\{\sin(80° + 20°) + \sin(80° - 20°) - \sin 80°\}\sqrt{3}}{\cos 80° + \{\cos(20° + 80°) + \cos(80° - 20°)\}}$

$= \frac{(\sin 100° + \sin 60° - \sin 80°)\sqrt{3}}{\cos 80° + \cos 100° + \cos 60°}$

$= \frac{\left(\sin(180° - 80°) + \frac{\sqrt{3}}{2} - \sin 80°\right)\sqrt{3}}{\cos 80° + \cos(180° - 80°) + \cos 60°}$

$= \frac{\left(\sin 80° + \frac{\sqrt{3}}{2} - \sin 80°\right)\sqrt{3}}{\cos 80° - \cos 80° + \cos 60°}$

$= \frac{\frac{3}{2}}{\frac{1}{2}} = 3 = \quad \text{RHS}$

Transformation Formulae Ex 8.1 Q5(vi)

$\tan 20° \tan 30° \tan 40° \tan 80°$

$\frac{1}{\sqrt{3}}(\tan 20° \tan 40° \tan 80°)$ $\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$= \frac{(\sin 20° \sin 40° \sin 80°)}{(\cos 20° \cos 40° \cos 80°)\sqrt{3}}$

$= \frac{(2\sin 20° \sin 40°)\sin 80°}{\sqrt{3}(2\cos 20° \cos 40°)\cos 80°}$

Applying

$$\Rightarrow \quad 2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{\left(\cos(40^\circ-20^\circ) - \cos(20^\circ+40^\circ)\right)\sin 80^\circ}{\cos(20^\circ+40^\circ) + \cos(40^\circ-20^\circ)\cos 80^\circ\sqrt{3}}$$

$$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$$

$$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ}$$

$$= \frac{2\sin 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + 2\cos 20^\circ\cos 80^\circ)}$$

Now,

$$\Rightarrow \quad 2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$= \frac{\sin(80^\circ+20^\circ) + \sin(80^\circ-20^\circ) - \sin 80^\circ}{\sqrt{3}\left(\cos 80^\circ + \cos(20^\circ+80^\circ) + \cos(80^\circ-20^\circ)\right)}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin(80^\circ - 100^\circ)}{\sqrt{3}\left(\cos 80^\circ + \cos(1800^\circ - 80^\circ) + \sin 60^\circ\right)}$$

$$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)} = 1 = \quad \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(vii)

$$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$$

LHS

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ \frac{\sqrt{3}}{2} \qquad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$$

$$= \sin(90^\circ-80^\circ)\sin(90^\circ-40^\circ)\sin(90^\circ-20^\circ)\frac{\sqrt{3}}{2}$$

$$= \cos 80^\circ \cos 40^\circ \cos 20^\circ \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2\times 2}(2\cos 40^\circ \cos 20^\circ)\cos 80^\circ \qquad [\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)]$$

$$= \frac{\sqrt{3}}{2\times 2}\left[\cos(40^\circ+20^\circ) + \cos(40^\circ-20^\circ)\right]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{2\times 2}[\cos 60^\circ + \cos 20^\circ]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{2\times 2}\left[\frac{1}{2} + \cos 20^\circ\right]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 80^\circ + \cos 20^\circ \cos 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos(80^\circ+20^\circ) + \cos(80^\circ-20^\circ)]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos 100^\circ + \cos 60^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos(180^\circ-80^\circ) + \cos 60^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 60^\circ] = \frac{\sqrt{3}}{16} = \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(viii)

$$\text{LHS} = \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

$$= \sin 20^\circ \sin 40^\circ \sin 80^\circ \times \frac{\sqrt{3}}{2} \qquad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2}(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4}\left[\cos(40^\circ-20^\circ) - \cos(40^\circ+20^\circ)\right]\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4}[\cos 20^\circ - \cos 60^\circ]\sin 80^\circ$$

$$= \frac{\sqrt{3}}{4}\left[\cos 20^\circ \sin 80^\circ - \frac{1}{2}\sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8}[2\cos 20^\circ \sin 80^\circ - \sin 80^\circ]$$

$$= \frac{\sqrt{3}}{8}\left[\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8}\left[\sin 100^\circ + \sin 60^\circ - \sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8}\left[\sin 80^\circ + \sin 60^\circ - \sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8} \times \sin 60^\circ = \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$$

3 RHS

Transformation Formulae Ex 8.1 Q6(i)

We have,

$$\text{LHS} = \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B)$$

$$= \frac{1}{2}\left[2 \sin A \sin(B - C) + 2 \sin B \sin(C - A) + 2 \sin C \sin(A - B)\right]$$

$$= \frac{1}{2}\begin{bmatrix}\cos(A - B + C) - \cos(A + B - C) + \cos(B - C + A) - \cos(B + C - A) \\ + \cos(C - A + B) - \cos(C + A - B)\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\cos(A - B + C) - \cos(A - B + C) - \cos(A + B - C) + \cos(A + B - C) \\ - \cos(B + C - A) + \cos(B + C - A)\end{bmatrix}$$

$$= \frac{1}{2} \times 0$$

$$= 0$$

$$= \text{RHS}$$

$\therefore \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q6(ii)

We have,

$$\text{LHS} = \sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D)$$

$$= \frac{1}{2}\left[2 \sin(B - C)\cos(A - D) + 2 \sin(C - A)\cos(B - D) + 2 \sin(A - B)\cos(C - D)\right]$$

$$= \frac{1}{2}\begin{bmatrix}\sin(B - C + A - D) + \sin(B - C - A + D) + \sin(C - A + B - D) + \sin(C - A - B + D) \\ + \sin(A - B + C - D) + \sin(A - B - C + D)\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\sin(A + B - C - D) + \sin(B + D - C - A) + \sin(B + C - A - D) + \sin(C + D - A - B) \\ + \sin(A + C - B - D) + \sin(A + D - B - C)\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\sin(A + B - C - D) + \sin(B + D - C - A) + \sin\{-(A + D - B - C)\} + \sin\{-(A + B - C - D)\} \\ + \sin\{-(B + D - A - C)\} + \sin(A + D - B - C)\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\sin(A + B - C - D) + \sin(B + D - C - A) - \sin(A + D - B - C) - \sin(A + B - C - D) \\ - \sin(B + D - A - C) + \sin(A + D - B - C)\end{bmatrix}$$

$$= \frac{1}{2} \times 0 \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= 0$$

$$= \text{RHS}$$

$\therefore \sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q7

We have,

$$\text{LHS} = \tan\theta \tan(60^\circ - \theta)\tan(60^\circ + \theta)$$

$$= \frac{\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta)}{\cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta)}$$

$$= \frac{2 \sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta)}{2 \cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta)}$$

$$= \frac{\sin\theta\left[2 \sin(60^\circ - \theta)\sin(60^\circ + \theta)\right]}{\cos\theta\left[2 \cos(60^\circ - \theta)\cos(60^\circ + \theta)\right]}$$

$$= \frac{\sin\theta\left[\cos\{(60^\circ - \theta) - (60^\circ + \theta)\} - \cos\{(60^\circ - \theta) + (60^\circ + \theta)\}\right]}{\cos\theta\left[\cos\{(60^\circ - \theta) + (60^\circ + \theta)\} + \cos\{(60^\circ - \theta) - (60^\circ + \theta)\}\right]}$$

$$= \frac{\sin\theta\left[\cos(-2\theta) - \cos 120^\circ\right]}{\cos\theta\left[\cos 120^\circ + \cos(-2\theta)\right]}$$

$$= \frac{\sin\theta\left[\cos 2\theta - \cos 120^\circ\right]}{\cos\theta\left[\cos 120^\circ + \cos 2\theta\right]} \qquad [\because \cos(-\theta) = \cos\theta]$$

$$= \frac{\sin\theta\left[\cos 2\theta - \cos(90^\circ + 30^\circ)\right]}{\cos\theta\left[\cos(90^\circ + 30^\circ) + \cos 2\theta\right]}$$

$$= \frac{\sin\theta[\cos 2\theta + \sin 30°]}{\cos\theta[-\sin 30° + \cos 2\theta]}$$ [∵ cos is negative in IInd quadrant]

$$= \frac{\sin\theta\left[\cos 2\theta + \frac{1}{2}\right]}{\cos\theta\left[\frac{-1}{2} + \cos 2\theta\right]}$$

$$= \frac{\sin\theta\cos 2\theta + \frac{1}{2}\sin\theta}{\frac{-1}{2}\cos\theta + \cos\theta\cos 2\theta}$$

Transformation Formulae Ex 8.1 Q8

Let $y = \cos\alpha\cos\beta$ then,

$$y = \frac{1}{2}(2\cos\alpha\cos\beta)$$

$$= \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$= \frac{1}{2}[\cos 90° + \cos(\alpha-\beta)]$$ $[\because \alpha+\beta = 90°]$

$$= \frac{1}{2}[0 + \cos(\alpha-\beta)]$$

$$= \frac{1}{2}\cos(\alpha-\beta)$$

$$\Rightarrow \quad y = \frac{1}{2}\cos(\alpha-\beta)$$

Now,

$$-1 \le \cos(\alpha-\beta) \le 1$$

$$\Rightarrow \quad \frac{-1}{2} \le \frac{1}{2}\cos(\alpha-\beta) \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le y \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le \cos\alpha\cos\beta \le \frac{1}{2}$$

Hence, the maximum values of $\cos\alpha\cos\beta$ is $\frac{1}{2}$.