

RD Sharma
Solutions
Class 11 Maths
Chapter 10
Ex 10.1

p>Sine and Cosine Formulae and their Applications Ex-10.1 Q1

$$\angle A = 45^\circ, \angle B = 60^\circ \text{ and } \angle C = 75^\circ$$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin 45} = \frac{b}{\sin 60} = \frac{c}{\sin 75} = k$$

$$\frac{a}{\sqrt{2}} = \frac{b}{\sqrt{3}} = \frac{c}{\sqrt{3}+1} = k$$

$$a : b : c = 2 : \sqrt{6} : (\sqrt{3}+1)$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q2

$$\angle C = 105^\circ, \angle B = 45^\circ, a = 2$$

From here we can calculate that

$$\angle A = 30^\circ$$

$$a \sin B = b \sin A$$

$$\Rightarrow 2 \sin 45 = b \sin 30$$

$$\Rightarrow 2 \times \frac{1}{\sqrt{2}} = b \times \frac{1}{2}$$

$$\Rightarrow \sqrt{2} = \frac{b}{2}$$

$$\Rightarrow b = 2\sqrt{2}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q3

$$a = 18, b = 24, c = 30, \angle C = 90^\circ$$

$$\text{let } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{\sin 90}{30}$$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{1}{30}$$

$$\frac{\sin A}{18} = \frac{1}{30} \Rightarrow \sin A = \frac{18}{30} = \frac{3}{5}$$

$$\frac{\sin B}{24} = \frac{1}{30} \Rightarrow \sin B = \frac{24}{30} = \frac{4}{5}$$

$$\therefore \sin A = \frac{3}{5}, \sin B = \frac{4}{5}, \sin C = 1$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q4

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

Let $a = k \sin A, b = k \sin B$ (Using sine rule)

LHS

$$= \frac{a-b}{a+b}$$

$$= \frac{k \sin A - k \sin B}{k \sin A + k \sin B}$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}{2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = \text{RHS}$$

$$\tan\left(\frac{A+B}{2}\right)$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q5

$$(a-b)\cos\frac{C}{2} = c\sin\left(\frac{A-B}{2}\right)$$

$$\text{Let } a = k \sin A, b = k \sin B, c = k \sin C$$

LHS

$$(a-b)\cos\frac{C}{2}$$

$$= k(\sin A - \sin B) \cdot \cos\frac{C}{2}$$

$$= 2k \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \cdot \cos\frac{C}{2}$$

$$= 2k \cos\left(\frac{\pi-C}{2}\right) \sin\left(\frac{A-B}{2}\right) \cdot \cos\frac{C}{2}$$

$$= 2k \sin\left(\frac{C}{2}\right) \cdot \cos\frac{C}{2} \cdot \sin\left(\frac{A-B}{2}\right) \quad \left[\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta\right]$$

$$= k \sin C \cdot \sin\left(\frac{A-B}{2}\right)$$

$$= c \cdot \sin\left(\frac{A-B}{2}\right) = \text{RHS}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q6

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)}$$

LHS

$$\frac{c}{a-b}$$

$$= \frac{k \sin C}{k \sin A - k \sin B}$$

$$= \frac{\sin C}{\sin A - \sin B}$$

$$= \frac{2 \sin\frac{C}{2} \cos\frac{C}{2}}{\sin A - \sin B}$$

$$= \frac{2 \sin\frac{C}{2} \cos\frac{C}{2}}{2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\frac{C}{2} \cos\left(\frac{\pi-(A+B)}{2}\right)}{\cos\left(\frac{\pi-C}{2}\right) \sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\frac{C}{2} \sin\left(\frac{A+B}{2}\right)}{\sin\frac{C}{2} \sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A+B}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) + \sin\left(\frac{B}{2}\right) \cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) - \sin\left(\frac{B}{2}\right) \cos\left(\frac{A}{2}\right)}$$

$$= \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)} \quad \left[\text{Dividing both Numerator and Denominator by } \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right)\right]$$

$$= \text{RHS}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q7

$$\frac{c}{a+b} = \frac{1 - \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}{1 + \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}$$

LHS

$$\begin{aligned}
&= \frac{c}{a+b} \\
&= \frac{k \sin C}{k \sin A + k \sin B} \\
&= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)} \\
&= \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\sin \left(\frac{\pi - C}{2} \right) \cos \left(\frac{A-B}{2} \right)} \\
&= \frac{\sin \left(\frac{\pi - (A+B)}{2} \right) \cos \frac{C}{2}}{\cos \left(\frac{C}{2} \right) \cos \left(\frac{A-B}{2} \right)} \\
&= \frac{\cos \left(\frac{A+B}{2} \right)}{\cos \left(\frac{A-B}{2} \right)} \\
&= \frac{\cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2} + \sin \frac{A}{2} \sin \frac{B}{2}} \\
&= \frac{1 - \tan \frac{A}{2} \tan \frac{B}{2}}{1 + \tan \frac{A}{2} \tan \frac{B}{2}} \left[\text{Dividing both Numerator and Denominator by } \cos \left(\frac{A}{2} \right) \cos \left(\frac{B}{2} \right) \right] \\
&= RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q8

$$\frac{a+b}{c} = \frac{\cos \left(\frac{A-B}{2} \right)}{\sin \frac{C}{2}}$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

LHS

$$\begin{aligned}
&= \frac{k \sin A + k \sin B}{k \sin C} \\
&= \frac{\sin A + \sin B}{\sin C} \\
&= \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{C}{2} \cos \frac{C}{2}} \\
&= \frac{\sin \left(\frac{\pi - C}{2} \right) \cos \frac{A-B}{2}}{\sin \frac{C}{2} \cos \frac{C}{2}} \\
&= \frac{\cos \frac{A-B}{2}}{\sin \frac{C}{2}} = RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q9

$$\sin \left(\frac{B-C}{2} \right) = \frac{b-c}{a} \cos \frac{A}{2}$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

RHS

$$\begin{aligned}
&= \frac{b-c}{a} \cos \frac{A}{2} \\
&= \frac{k \sin B - k \sin C}{k \sin A} \cos \frac{A}{2} \\
&= \frac{\sin B - \sin C}{\sin A} \cos \frac{A}{2}
\end{aligned}$$

$$\begin{aligned}
& \frac{\cos \frac{A}{2}}{\sin A} \cdot \cos \frac{A}{2} \\
&= \frac{2 \cos \frac{B+C}{2} \cdot \sin \frac{B-C}{2}}{2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}} \cos \frac{A}{2} \\
&= \frac{\cos \frac{\pi-A}{2} \sin \frac{B-C}{2}}{\sin \frac{A}{2}} \\
&= \frac{\sin \frac{A}{2} \sin \frac{B-C}{2}}{\sin \frac{A}{2}} = \sin \frac{B-C}{2} = RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q10

$$\text{let } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

LHS,

$$\begin{aligned}
& \frac{a^2 - c^2}{b^2} \\
&= \frac{k^2 \sin^2 A - k^2 \sin^2 C}{k^2 \sin^2 B} \\
&= \frac{k^2 (\sin^2 A - \sin^2 C)}{k^2 \sin^2 B} \\
&= \frac{(\sin^2 A - \sin^2 C)}{\sin^2 (\pi - (A+C))} \\
&= \frac{\sin(A+C) \sin(A-C)}{\sin^2(A+C)} \\
&= \frac{\sin(A-C)}{\sin(A+C)} = RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q11

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

RHS,

$$\begin{aligned}
& a \sin(B-C) \\
&= a \sin B \cos C - a \sin C \cos B \\
&= a(bk) \left(\frac{a^2 + b^2 - c^2}{2ab} \right) - a(ck) \left(\frac{a^2 + c^2 - b^2}{2ac} \right) \\
&= k \cdot \frac{(a^2 + b^2 - c^2)}{2} - k \cdot \frac{(a^2 + c^2 - b^2)}{2} \\
&= 2k \cdot \frac{(b^2 - c^2)}{2} \\
&= b(kb) - c(kc) \\
&= b(\sin B) - c(\sin C)
\end{aligned}$$

LHS

Sine and Cosine Formulae and their Applications Ex-10.1 Q12

$$a^2 \sin(B-C) = (b^2 - c^2) \sin A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

LHS,

$$\begin{aligned}
& a^2 \sin(B-C) \\
&= a^2 (\sin B \cos C - \sin C \cos B) \\
&= a^2 kb \cdot \frac{a^2 + b^2 - c^2}{2ab} - a^2 ck \cdot \frac{a^2 + c^2 - b^2}{2ac} \quad [\text{Using cos rule and sine rule}] \\
&= a^2 k \cdot \frac{a^2 + b^2 - c^2}{2a} - a^2 k \cdot \frac{a^2 + c^2 - b^2}{2a} \\
&= a^2 k \cdot \left(\frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2a} \right)
\end{aligned}$$

$$\begin{aligned}
 &= a^2 k \cdot \left(\frac{2b^2 - 2c^2}{2a} \right) \\
 &= ak \cdot (b^2 - c^2) \\
 &= \sin A (b^2 - c^2) = RHS \\
 &\text{Hence Proved}
 \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q13

$$\begin{aligned}
 \frac{\sqrt{\sin A} - \sqrt{\sin B}}{\sqrt{\sin A} + \sqrt{\sin B}} &= \frac{a + b - 2\sqrt{ab}}{a - b} \\
 &\text{RHS} \\
 \frac{a + b - 2\sqrt{ab}}{a - b} \\
 &= \frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}}{(\sqrt{a})^2 - (\sqrt{b})^2} \\
 &= \frac{(\sqrt{a} - \sqrt{b})^2}{(\sqrt{a})^2 - (\sqrt{b})^2} \\
 &= \frac{(\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b})} \\
 &= \frac{(\sqrt{k \sin A} - \sqrt{k \sin B})}{(\sqrt{k \sin A} + \sqrt{k \sin B})} \\
 &= \frac{(\sqrt{\sin A} - \sqrt{\sin B})}{(\sqrt{\sin A} + \sqrt{\sin B})} \text{ [taking } k \text{ common and cancelling them]} \\
 &= LHS \\
 &\text{Hence Proved}
 \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q14

$$\begin{aligned}
 &LHS, \\
 &a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) \\
 &= a \sin B - a \sin C + b \sin C - b \sin A + c \sin A - c \sin B \\
 &= b \sin A - c \sin A + c \sin B - b \sin A + c \sin A - c \sin B \text{ [} \because b \sin A = a \sin B, b \sin C = c \sin B, c \sin A = a \sin C \text{]} \\
 &= 0 = RHS \\
 &\text{Hence Proved}
 \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q15

$$\begin{aligned}
 \frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} &= 0 \\
 \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} &= k \\
 &LHS \\
 \frac{a^2 \sin(B-C)}{\sin A} + \frac{b^2 \sin(C-A)}{\sin B} + \frac{c^2 \sin(A-B)}{\sin C} \\
 &= ak \sin(B-C) + bk \sin(C-A) + ck \sin(A-B) \\
 &= \sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) \\
 &= \sin(\pi - (B+C)) \sin(B-C) + \sin(\pi - (C+A)) \sin(C-A) \\
 &\quad + \sin(\pi - (A+B)) \sin(A-B) \\
 &= \sin(B+C) \sin(B-C) + \sin(C+A) \sin(C-A) \\
 &\quad + \sin(A+B) \sin(A-B) \\
 &= \sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B = 0 = RHS
 \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q16

$$\begin{aligned}
 a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) &= 0 \\
 &LHS
 \end{aligned}$$

$$\begin{aligned}
&= a'(1 - \sin^2 B - 1 + \sin^2 C) + b'(1 - \sin^2 C - 1 + \sin^2 A) \\
&+ c'(1 - \sin^2 A - 1 + \sin^2 B) \\
&= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2(\sin^2 B - \sin^2 A) \\
&= a^2(k^2c^2 - k^2b^2) + b^2(k^2a^2 - k^2c^2) + c^2(k^2b^2 - k^2a^2) \\
&= k^2(a^2c^2 - a^2b^2 + b^2a^2 - b^2c^2 + b^2c^2 - a^2c^2) \\
&= k^2 \times 0 = 0 = RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q17

Let $a = k \sin A, b = k \sin B, c = k \sin C$

LHS

$$\begin{aligned}
&b \cos B + c \cos C \\
&= k \sin B \cos B + k \sin C \cos C \\
&= \frac{k}{2} (2 \sin B \cos B + 2 \sin C \cos C) \\
&= \frac{k}{2} (\sin 2B + \sin 2C) \\
&= \frac{k}{2} 2 \sin(B+C) \cos(B-C) \\
&= k \sin(\pi - A) \cos(B-C) \\
&= k \sin A \cos(B-C) \\
&= a \cos(B-C) = RHS
\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q18

$$\begin{aligned}
\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} &= \frac{1}{a^2} - \frac{1}{b^2} \\
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} &= k
\end{aligned}$$

LHS

$$\begin{aligned}
&= \frac{1 - 2 \sin^2 A}{a^2} - \frac{1 - 2 \sin^2 B}{b^2} \\
&= \frac{1}{a^2} - \frac{1}{b^2} - 2 \left(\frac{\sin^2 A}{a^2} - \frac{\sin^2 B}{b^2} \right) \\
&= \frac{1}{a^2} - \frac{1}{b^2} - 2(k^2 - k^2) \text{ [Using sine rule]} \\
&= \frac{1}{a^2} - \frac{1}{b^2} = RHS
\end{aligned}$$

hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q19

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} = 0$$

LHS

$$\begin{aligned}
&\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} \\
&= \frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} \\
&= \frac{1 - \sin^2 B - 1 + \sin^2 C}{b+c} + \frac{1 - \sin^2 C - 1 + \sin^2 A}{c+a} + \frac{1 - \sin^2 A - 1 + \sin^2 B}{a+b} \\
&= \frac{\sin^2 C - \sin^2 B}{b+c} + \frac{\sin^2 A - \sin^2 C}{c+a} + \frac{\sin^2 B - \sin^2 A}{a+b} \\
&= \frac{k^2c^2 - k^2b^2}{b+c} + \frac{k^2a^2 - k^2c^2}{c+a} + \frac{k^2b^2 - k^2a^2}{a+b} \\
&= k^2 \left(\frac{c^2 - b^2}{b+c} + \frac{a^2 - c^2}{c+a} + \frac{b^2 - a^2}{a+b} \right) \\
&= k^2(c-b+a-c+b-a) \text{ [Using } b^2 - a^2 = (b-a)(b+a)]} \\
&= 0 = RHS
\end{aligned}$$

Hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q20

We know $a \sin B = b \sin A, c \sin B = b \sin C, a \sin C = c \sin A$

$$a \sin \frac{A}{2} \sin \left(\frac{B-C}{2} \right) + b \sin \frac{B}{2} \sin \left(\frac{C-A}{2} \right) + c \sin \frac{C}{2} \sin \left(\frac{A-B}{2} \right) = 0$$

LHS

$$\begin{aligned} &= a \sin \left(\frac{\pi - (B+C)}{2} \right) \sin \left(\frac{B-C}{2} \right) + b \sin \left(\frac{\pi - (C+A)}{2} \right) \sin \left(\frac{C-A}{2} \right) \\ &+ c \sin \left(\frac{\pi - (A+B)}{2} \right) \sin \left(\frac{A-B}{2} \right) \\ &= a \cos \left(\frac{B+C}{2} \right) \sin \left(\frac{B-C}{2} \right) + b \cos \left(\frac{C+A}{2} \right) \sin \left(\frac{C-A}{2} \right) \\ &+ c \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \\ &= a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) \\ &= a \sin B - a \sin C + b \sin C - b \sin A + c \sin A - c \sin B \\ &= b \sin A - a \sin C + b \sin C - b \sin A + a \sin C - b \sin C \\ &= 0 = RHS \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q21

$$\frac{b \sec B + c \sec C}{\tan B + \tan C} = \frac{c \sec C + a \sec A}{\tan C + \tan A} = \frac{a \sec A + b \sec B}{\tan A + \tan B}$$

$$\frac{b \sec B + c \sec C}{\tan B + \tan C}$$

$$= \frac{k \sin B \sec B + k \sin C \sec C}{\tan B + \tan C}$$

$$= \frac{k \sin B \frac{1}{\cos B} + k \sin C \frac{1}{\cos C}}{\tan B + \tan C}$$

$$= \frac{k \tan B + k \tan C}{\tan B + \tan C} = \frac{k(\tan B + \tan C)}{\tan B + \tan C} = k$$

$$\text{Similarly, } \frac{c \sec C + a \sec A}{\tan C + \tan A} = k$$

$$\text{Similarly, } \frac{a \sec A + b \sec B}{\tan A + \tan B} = k$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q22

$$a \cos A + b \cos B + c \cos C = 2b \sin A \sin C = 2c \sin A \sin B$$

LHS

$$a \cos A + b \cos B + c \cos C$$

$$= k \sin A \cos A + k \sin B \cos B + k \sin C \cos C$$

$$= \frac{k}{2} (\sin 2A + \sin 2B + \sin 2C)$$

$$= \frac{k}{2} (2 \sin(A+B) \cos(A-B) + 2 \sin C \cos C)$$

$$= \frac{2k}{2} (\sin(\pi - C) \cos(A-B) + \sin C \cos C)$$

$$= k(\sin C \cos(A-B) + \sin C \cos C)$$

$$= k \sin C (\cos(A-B) + \cos C)$$

$$= k \sin C \cdot 2 \cos \left(\frac{A-B+C}{2} \right) \cos \left(\frac{A-B-C}{2} \right)$$

$$= k \sin C \cdot 2 \cos \left(\frac{\pi - 2B}{2} \right) \cos \left(\frac{A - \pi + A}{2} \right)$$

$$= k \sin C \cdot 2 \sin B \cos \left(\frac{2A - \pi}{2} \right)$$

$$= k \sin C \cdot 2 \sin B \cos \left(\frac{\pi - 2A}{2} \right)$$

$$= k \sin C \cdot 2 \sin B \sin A$$

$$= 2 \sin B \sin C (k \sin A) = 2a \sin B \sin C$$

= RHS

$$\text{Similarly, } a \cos A + b \cos B + c \cos C = 2c \sin A \sin B$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q23

$$a(\cos B \cos C + \cos A) = b(\cos A \cos C + \cos B) = c(\cos A \cos B + \cos C)$$

$$a(\cos B \cos C - \cos(\pi - (B + C)))$$

$$= a(\cos B \cos C - \cos(B + C))$$

$$= a(\cos B \cos C - \cos B \cos C + \sin B \sin C)$$

$$= a \sin B \sin C$$

$$= k \sin A \sin B \sin C$$

$$\text{Similarly, } b(\cos A \cos C + \cos B) = k \sin A \sin B \sin C$$

$$\text{Similarly, } c(\cos A \cos B + \cos C) = k \sin A \sin B \sin C$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q24

$$\text{Let } a = k \sin A$$

$$a(\cos C - \cos B) = 2(b - c) \cos^2 \frac{A}{2}$$

LHS

$$= a(\cos C - \cos B)$$

$$= a 2 \sin \frac{C+B}{2} \cdot \sin \frac{B-C}{2}$$

$$= 2k \sin A \sin \frac{\pi - A}{2} \cdot \sin \frac{B-C}{2}$$

$$= 2k 2 \sin \frac{A}{2} \cos \frac{A}{2} \cos \frac{A}{2} \sin \frac{B-C}{2}$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \sin \frac{A}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \sin \frac{\pi - (B+C)}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} \left(2 \sin \frac{B-C}{2} \cdot \cos \frac{B+C}{2} \right)$$

$$= 2k \cos^2 \frac{A}{2} (\sin B - \sin C)$$

$$= 2 \cos^2 \frac{A}{2} (k \sin B - k \sin C)$$

$$= 2 \cos^2 \frac{A}{2} (b - c) = \text{RHS}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q25

$$b \cos \theta = c \cos(A - \theta) + a \cos(C + \theta)$$

$$\text{Let } a \sin C = c \sin A \text{ [Using sine rule]}$$

RHS

$$= c \cos(A - \theta) + a \cos(C + \theta)$$

$$= c \cos A \cos \theta + c \sin A \cos \theta + a \cos C \cos \theta - a \sin C \sin \theta$$

$$= k \sin C \cos A \cos \theta + k \sin C \sin A \cos \theta + k \sin A \cos C \cos \theta$$

$$- k \sin A \sin C \sin \theta$$

$$= k \sin C \cos A \cos \theta + k \sin A \cos C \cos \theta$$

$$= k \cos \theta (\sin C \cos A + \sin A \cos C)$$

$$= k \cos \theta \sin(C + A)$$

$$= k \cos \theta \sin(\pi - B)$$

$$= k \cos \theta \sin B$$

$$= k \sin B \cos \theta = b \cos \theta = \text{LHS}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q26

$$\text{Let } \sin A = ak, \sin B = bk, \sin C = ck$$

$$\sin^2 A + \sin^2 B = \sin^2 C$$

$$\Rightarrow k^2 a^2 + k^2 b^2 = k^2 c^2 \text{ [Using sine rule]}$$

$$\Rightarrow a^2 + b^2 = c^2$$

Since the triangle satisfies the Pythagoras theorem, therefore it is right angled.

Sine and Cosine Formulae and their Applications Ex-10.1 Q27

a^2, b^2, c^2 are in A.P.

$\Rightarrow -2a^2, -2b^2, -2c^2$ are in A.P.

$\Rightarrow (a^2 + b^2 + c^2) - 2a^2, (a^2 + b^2 + c^2) - 2b^2, (a^2 + b^2 + c^2) - 2c^2$ are in A.P.

$\Rightarrow (b^2 + c^2 - a^2), (c^2 + a^2 - b^2), (b^2 + a^2 - c^2)$ are in A.P.

$\Rightarrow \frac{(b^2 + c^2 - a^2)}{2abc}, \frac{(c^2 + a^2 - b^2)}{2abc}, \frac{(b^2 + a^2 - c^2)}{2abc}$ are in A.P.

$\Rightarrow \frac{1}{a} \frac{(b^2 + c^2 - a^2)}{2bc}, \frac{1}{b} \frac{(c^2 + a^2 - b^2)}{2ac}, \frac{1}{c} \frac{(b^2 + a^2 - c^2)}{2ab}$ are in A.P.

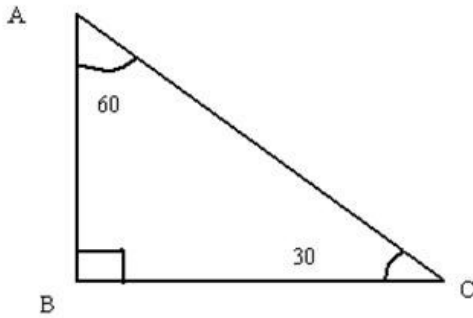
$\Rightarrow \frac{1}{a} \cos A, \frac{1}{b} \cos B, \frac{1}{c} \cos C$ are in A.P.

$\Rightarrow \frac{k}{a} \cos A, \frac{k}{b} \cos B, \frac{k}{c} \cos C$ are in A.P.

$\Rightarrow \frac{\cos A}{\sin A}, \frac{\cos B}{\sin B}, \frac{\cos C}{\sin C}$ are in A.P.

$\Rightarrow \cot A, \cot B, \cot C$ are in A.P.

Sine and Cosine Formulae and their Applications Ex-10.1 Q28



$BC = 15m, AB = h$

From the diagram we can calculate, $\angle A = 60^\circ$

Using sine rule,

$$\frac{\sin A}{15} = \frac{\sin C}{h}$$

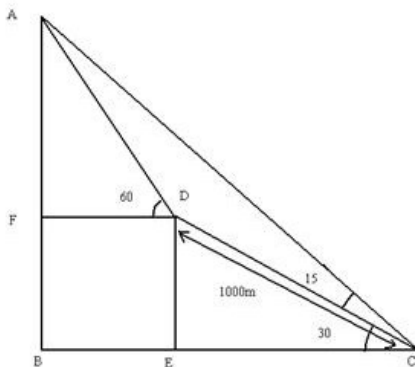
$$\Rightarrow \frac{\sin 60}{15} = \frac{\sin 30}{h}$$

$$\Rightarrow \frac{\sqrt{3}}{2 \times 15} = \frac{1}{2 \times h}$$

$$\Rightarrow \frac{\sqrt{3}}{15} = \frac{1}{h}$$

$$\Rightarrow h = \frac{15}{\sqrt{3}} \Rightarrow h = 5\sqrt{3}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q29



$$DE = 1000 \sin 30 = 1000 \times \frac{1}{2} = 500m = FB$$

$$FC = 1000 \cos 30 = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}m$$

$$\frac{1000 \cos 30^\circ}{2} = 500\sqrt{3}m$$

Let $AF = x$ m

$$DF = \frac{x}{\sqrt{3}}m = BE$$

We know,

From $\triangle ABC$,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{AF + FB}{BE + EC}$$

$$\Rightarrow 1 = \frac{x + 500}{\frac{x}{\sqrt{3}} + 500\sqrt{3}}$$

$$\Rightarrow \frac{x}{\sqrt{3}} + 500\sqrt{3} = x + 500$$

$$\Rightarrow x + 1500 = x\sqrt{3} + 500\sqrt{3}$$

$$\Rightarrow 1500 - 500\sqrt{3} = x\sqrt{3} - x$$

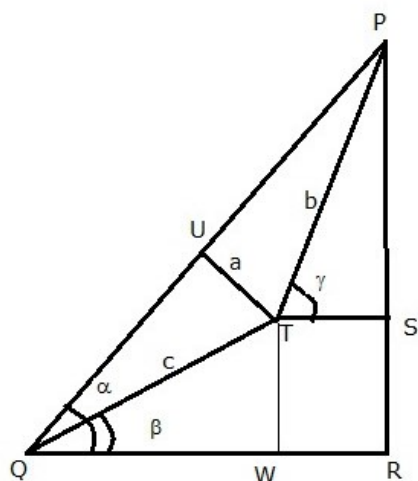
$$\Rightarrow 500\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} - 1)$$

$$\therefore x = 500\sqrt{3}m$$

The height of the triangle is $AB = AF + FB = 500(\sqrt{3} + 1)m$

Sine and Cosine Formulae and their Applications Ex-10.1 Q30

Consider the following figure.



The person is observing the peak P from the point Q .

The distance he travelled is $QT = c$ metres and the angle of inclination of QT is β .

He is observing the peak from the point and the angle of inclination is γ .

Now consider the triangle $\triangle QUT$.

$$\angle TQU = \beta - \alpha$$

$$\text{Thus, } \sin(\beta - \alpha) = \frac{a}{c}$$

$$\Rightarrow a = c \times \sin(\beta - \alpha) \dots (1)$$

Now consider the triangle $\triangle PQR$.

We know that $\angle QPR = 90^\circ - \alpha$

In triangle $\triangle PTS$, $\angle TPS = 90^\circ - \gamma$

Thus, $\angle TPU = \angle QPR - \angle TPS$

$$\Rightarrow \angle TPU = (90^\circ - \alpha) - (90^\circ - \gamma)$$

$$\Rightarrow \angle TPU = \gamma - \alpha$$

Now consider the $\triangle TPU$,

$$\text{Thus, } \sin(\gamma - \alpha) = \frac{a}{b}$$

$$\Rightarrow b = \frac{a}{\sin(\gamma - \alpha)}$$

Substituting the value of a from equation (1), we have,

$$b = \frac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)} \dots(2)$$

We need to find the total height of the peak PR .

Here, $PR = PS + SR \dots(3)$

From the triangle PST ,

$$\sin \gamma = \frac{PS}{PT} = \frac{PS}{b}$$

$$\Rightarrow PS = b \sin \gamma \dots(4)$$

From the triangle QTW ,

$$\sin \beta = \frac{TW}{QT} = \frac{TW}{c}$$

$$\Rightarrow TW = SR = c \sin \beta \dots(5)$$

Substituting the values of PS and SR from equations (4) and (5)

in equation (3), we have

$$PR = PS + SR$$

$$\Rightarrow PR = b \sin \gamma + c \sin \beta$$

$$\Rightarrow PR = \frac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)} \sin \gamma + c \sin \beta \quad [\text{from equation (2)}]$$

$$\Rightarrow PR = \frac{c \times \sin(\alpha - \beta) \times \sin \gamma + c \sin \beta \times \sin(\gamma - \alpha)}{\sin(\gamma - \alpha)}$$

$$\Rightarrow PR = c \left[\frac{\sin \alpha \times \cos \beta \times \sin \gamma - \cos \alpha \times \sin \beta \times \sin \gamma + \sin \beta \times \sin \gamma \times \cos \alpha - \sin \beta \times \sin \alpha \times \cos \gamma}{\sin(\gamma - \alpha)} \right]$$

$$\Rightarrow PR = c \left[\frac{\sin \alpha \times \cos \beta \times \sin \gamma - \sin \beta \times \sin \alpha \times \cos \gamma}{\sin(\gamma - \alpha)} \right]$$

$$\Rightarrow PR = \frac{c \sin \alpha \times (\cos \beta \times \sin \gamma - \sin \beta \times \cos \gamma)}{\sin(\gamma - \alpha)}$$

$$\Rightarrow PR = \frac{c \sin \alpha \times \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q31

If the sides a, b, c of a ΔABC are in H.P.

$\therefore \frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in AP

$$\therefore \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{a-b}{ba} = \frac{b-c}{ca}$$

$$\Rightarrow \frac{\sin A - \sin B}{\sin B \sin A} = \frac{\sin B - \sin C}{\sin C \sin B} \dots\dots\dots [\text{Using sine rule}]$$

$$\Rightarrow \frac{2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}}{\sin A} = \frac{2 \sin \frac{B-C}{2} \cos \frac{B+C}{2}}{\sin C}$$

But $A + B + C = \pi$

$$A + B = \pi - C$$

$$\cos \frac{A+B}{2} = \cos \left(\frac{\pi - C}{2} \right) = \sin \frac{C}{2}$$

$$\sin^2 \frac{C}{2} \cos \frac{C}{2} \sin \frac{A-B}{2} = \sin \frac{B-C}{2} \cos \frac{A}{2} \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{C}{2} \sin \frac{A+B}{2} \sin \frac{A-B}{2} = \sin \frac{B-C}{2} \cos \frac{B+C}{2} \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{C}{2} \left[\sin^2 \frac{A}{2} - \sin^2 \frac{B}{2} \right] = \sin^2 \frac{A}{2} \left[\sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} \right]$$

$$\sin^2 \frac{C}{2} \sin^2 \frac{A}{2} - \sin^2 \frac{C}{2} \sin^2 \frac{B}{2} = \sin^2 \frac{A}{2} \sin^2 \frac{B}{2} - \sin^2 \frac{A}{2} \sin^2 \frac{C}{2}$$

$$\frac{1}{\sin^2 \frac{B}{2}} - \frac{1}{\sin^2 \frac{A}{2}} = \frac{1}{\sin^2 \frac{C}{2}} - \frac{1}{\sin^2 \frac{B}{2}}$$

Hence $\frac{\sin^2 A}{2}, \frac{\sin^2 B}{2}, \frac{\sin^2 C}{2}$ are in AP.

$\therefore \sin^2 A, \sin^2 B, \sin^2 C$ are in HP.