

RD Sharma
Solutions
Class 11 Maths
Chapter 10
Ex 10.2

Sine and Cosine Formulae and their Applications Ex-10.2 Q1

The area of a triangle ABC is given by

$$\begin{aligned}\Delta &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2} \times 5 \times 6 \sin 60^\circ \\ &= \frac{15\sqrt{3}}{2} \text{ sq. unit}\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q2

The area of a triangle ABC is given by

$$\begin{aligned}\Delta &= \frac{1}{2}ab \sin C \\ \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{2 + 3 - 5}{2\sqrt{6}} \\ &= 0\end{aligned}$$

$$\begin{aligned}\sin C &= \sqrt{1 - \cos^2 C} \\ &= 1\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2}\sqrt{6}\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q3

We have, $a = 4, b = 6$ and $c = 8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{7}{8}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{11}{16}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{4}$$

$$\begin{aligned}8\cos A + 16\cos B + 4\cos C &= 8 \times \frac{7}{8} + 16 \times \frac{11}{16} + 4 \times \left(-\frac{1}{4}\right) \\ &= 17\end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q4

In any $\triangle ABC$, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

we have,

$$a = 18, b = 24, c = 30$$

Therefore,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1152}{1440} = \frac{4}{5}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{648}{1080} = \frac{3}{5}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{0}{864} = 0$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q5

$$b(c \cos A - a \cos C) = c^2 - a^2$$

RHS

$$\begin{aligned} &= c^2 - a^2 \\ &= k^2 \sin^2 C - k^2 \sin^2 A \\ &= k^2 (\sin^2 C - \sin^2 A) \\ &= k^2 \sin(C+A) \sin(C-A) \\ &= k^2 \sin(\pi - B) \sin(C-A) \\ &= k^2 \sin B \sin(C-A) \\ &= k \sin B \cdot k \sin(C-A) \\ &= bk \sin(C-A) \\ &= bk(\sin C \cos A - \sin A \cos C) \\ &= b(k \sin C \cos A - k \sin A \cos C) \\ &= b(c \cos A - a \cos C) = LHS \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q6

$$c(a \cos B - b \cos A)$$

$$\begin{aligned} &= ac \cos B - bc \cos A \\ &= ac \cdot \frac{a^2 + c^2 - b^2}{2ac} - bc \cdot \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{a^2 + c^2 - b^2}{2} - \frac{b^2 + c^2 - a^2}{2} \\ &= \frac{a^2 + c^2 - b^2 - b^2 - c^2 + a^2}{2} \\ &= \frac{2a^2 - 2b^2}{2} = (a^2 - b^2) = RHS \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q7

$$2(bc \cos A + ca \cos B + ab \cos C) = a^2 + b^2 + c^2$$

LHS

$$\begin{aligned} &= 2bc \cos A + 2ca \cos B + 2ab \cos C \\ &= 2bc \frac{b^2 + c^2 - a^2}{2bc} + 2ca \frac{a^2 + c^2 - b^2}{2ca} + 2ab \frac{a^2 + b^2 - c^2}{2ab} \\ &= b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2 \\ &= a^2 + b^2 + c^2 = RHS \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q8

For any $\triangle ABC$, we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

therefore,

$$\begin{aligned} (c^2 + b^2 - a^2) \tan A &= (c^2 + b^2 - a^2) \frac{\sin A}{\cos A} \\ &= (c^2 + b^2 - a^2) \frac{ka}{\frac{b^2 + c^2 - a^2}{2bc}} \\ &= 2kabc \end{aligned}$$

Also,

$$\begin{aligned} (a^2 + c^2 - b^2) \tan B &= (a^2 + c^2 - b^2) \frac{\sin B}{\cos B} \\ &= (a^2 + c^2 - b^2) \frac{kb}{\frac{a^2 + c^2 - b^2}{2ac}} \\ &= 2kabc \end{aligned}$$

Now,

$$\begin{aligned} (a^2 + b^2 - c^2) \tan C &= (a^2 + b^2 - c^2) \frac{\sin C}{\cos C} \\ &= (a^2 + b^2 - c^2) \frac{kc}{\frac{a^2 + b^2 - c^2}{2ab}} \\ &= 2kabc \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q9

$$\begin{aligned} \frac{c - b \cos A}{b - c \cos A} &= \frac{\cos B}{\cos C} \\ \text{LHS} &= \frac{c - b \cos A}{b - c \cos A} \\ &= \frac{k \sin C - k \sin B \cos A}{k \sin B - k \sin C \cos A} \\ &= \frac{\sin(\pi - (A + B)) - \sin B \cos A}{\sin(\pi - (A + C)) - \sin C \cos A} \\ &= \frac{\sin(A + B) - \sin B \cos A}{\sin(A + C) - \sin C \cos A} \\ &= \frac{\sin A \cos B + \cos A \sin B - \sin B \cos A}{\sin A \cos C + \cos A \sin C - \sin C \cos A} \\ &= \frac{\sin A \cos B}{\sin A \cos C} \\ &= \frac{\cos B}{\cos C} = \text{RHS} \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q10

In any $\triangle ABC$, we have

$$a = b \cos C + c \cos B$$

$$b = c \cos A + a \cos C$$

$$c = a \cos B + b \cos A$$

Therefore,

$$\begin{aligned} L.H.S &= a(\cos B + \cos C - 1) + b(\cos C + \cos A - 1) + c(\cos A + \cos B - 1) \\ &= a \cos B + a \cos C - a + b \cos C + b \cos A - b + c \cos A + c \cos B - c \\ &= c - b \cos A + a \cos C - a + a - c \cos B + b \cos A - b + b - a \cos C + c \cos B - c \\ &= 0 \\ &= R.H.S \end{aligned}$$

Hence proved.

Sine and Cosine Formulae and their Applications Ex-10.2 Q11

By sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$k \sin A = a, k \sin B = b, k \sin C = c$$

$$\begin{aligned} a \cos A + b \cos B + c \cos C &= k \sin A \cos A + k \sin B \cos B + k \sin C \cos C \\ &= \left(\frac{1}{2}\right)k[2 \sin A \cos A + 2 \sin B \cos B + 2 \sin C \cos C] \\ &= \left(\frac{1}{2}\right)k[\sin 2A + \sin 2B + \sin 2C] \\ &= k[\sin(A + B) \cos(A - B) + \sin C \cos C] \\ &= k[\sin(\pi - C) \cos(A - B) + \sin C \cos(\pi - (A + B))] \\ &= k[\sin C \cos(A - B) - \sin C \cos(A + B)] \\ &= k[\sin C(\cos(A - B) - \cos(A + B))] \\ &= k \sin C[2 \sin A \sin B] \\ &= 2 \sin C(k \sin A) \sin B \\ &= 2a \sin B \sin C \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q12

We know that by cosine rule

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ \Rightarrow 2bc \cos A &= b^2 + c^2 - a^2 \\ \Rightarrow a^2 &= b^2 + c^2 - 2bc \cos A \end{aligned}$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc \left(2 \cos^2 \frac{A}{2} - 1 \right)$$

$$\Rightarrow a^2 = b^2 + c^2 + 2bc - 4bc \cos^2 \frac{A}{2}$$

$$\Rightarrow a^2 = (b+c)^2 - 4bc \cos^2 \frac{A}{2}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q13

$$4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right) = (a+b+c)^2$$

LHS,

$$4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right)$$

$$= 2 \left(bc \cdot 2 \cos^2 \frac{A}{2} + ca \cdot 2 \cos^2 \frac{B}{2} + ab \cdot 2 \cos^2 \frac{C}{2} \right)$$

$$= 2 (bc(1 - \cos A) + ca(1 - \cos B) + ab(1 - \cos C))$$

$$= 2bc - 2bc \cos A + 2ca - 2ca \cos B + 2ab - 2ab \cos C$$

$$= 2bc - 2bc \frac{b^2 + c^2 - a^2}{2bc} + 2ca - 2ca \frac{a^2 + c^2 - b^2}{2ca} + 2ab$$

$$- 2ab \frac{b^2 + a^2 - c^2}{2ab} \text{ [cos rule]}$$

$$= 2bc - b^2 - c^2 + a^2 + 2ca - a^2 - c^2 + b^2 + 2ab - b^2 - a^2 + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (a+b+c)^2 = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q14

$$\sin^3 A \cos(B-C) + \sin^3 B \cos(C-A) + \sin^3 C \cos(A-B)$$

$$= \sin^2 A \sin A \cos(B-C) + \sin^2 B \sin B \cos(C-A) + \sin^2 C \sin C \cos(A-B)$$

$$= \sin^2 A \sin(\pi - (B+C)) \cos(B-C) + \sin^2 B \sin(\pi - (A+C)) \cos(C-A)$$

$$+ \sin^2 C \sin(\pi - (A+B)) \cos(A-B)$$

$$= \sin^2 A \sin(B+C) \cos(B-C) + \sin^2 B \sin(C+A) \cos(C-A)$$

$$+ \sin^2 C \sin(A+B) \cos(A-B)$$

$$= \sin^2 A (\sin 2B + \sin 2C) + \sin^2 B (\sin 2C + \sin 2A) + \sin^2 C (\sin 2A + \sin 2B)$$

$$= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A)$$

$$+ \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B)$$

$$= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A)$$

$$+ \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B)$$

$$= \sin^2 A \cdot 2 \sin B \cos B + \sin^2 A \cdot 2 \sin C \cos C + \sin^2 B \cdot 2 \sin C \cos C$$

$$+ \sin^2 B \cdot 2 \sin A \cos A + \sin^2 C \cdot 2 \sin A \cos A + \sin^2 C \cdot 2 \sin B \cos B$$

$$= k^2 a^2 \cdot 2kb \cos B + k^2 a^2 \cdot 2kc \cos C + k^2 b^2 \cdot 2ka \cos C$$

$$+ k^2 b^2 \cdot 2ka \cos A + k^2 c^2 \cdot 2ka \cos A + k^2 c^2 \cdot 2kb \cos B$$

$$= k^3 ab(a \cos B + b \cos A) + k^3 ac(a \cos C + c \cos A) + k^3 bc(c \cos B + b \cos C)$$

$$= k^3 abc + k^3 acb + k^3 bca$$

$$= k^3 3abc$$

$$= 3(k \sin A \cdot k \sin B \cdot k \sin C)$$

$$= 3abc = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q15

$$\text{Let } \frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = \lambda \text{ (say)}$$

$$b+c = 12\lambda, c+a = 13\lambda, a+b = 15\lambda$$

$$(b+c+c+a+a+b) = 12\lambda + 13\lambda + 15\lambda$$

$$2(a+b+c) = 40\lambda$$

$$a+b+c = 20\lambda$$

$$b+c = 12\lambda \text{ and } a+b+c = 20\lambda \Rightarrow a = 8\lambda$$

$$c+a = 13\lambda \text{ and } a+b+c = 20\lambda \Rightarrow b = 7\lambda$$

$$a+b = 15\lambda \text{ and } a+b+c = 20\lambda \Rightarrow c = 5\lambda$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{49\lambda^2 + 25\lambda^2 - 64\lambda^2}{2 \cdot 7\lambda \cdot 5\lambda} = 1$$

$$\cos A = \frac{2bc}{a^2 + c^2 - b^2} = \frac{70\lambda^2}{64\lambda^2 + 25\lambda^2 - 49\lambda^2} = \frac{7}{2}$$

$$\cos B = \frac{2ac}{a^2 + c^2 - b^2} = \frac{80\lambda^2}{64\lambda^2 + 25\lambda^2 - 49\lambda^2} = \frac{11}{2}$$

$$\cos C = \frac{2ab}{a^2 + b^2 - c^2} = \frac{112\lambda^2}{64\lambda^2 + 49\lambda^2 - 25\lambda^2} = \frac{11}{14}$$

$$\cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{11}{14} = 2 : 7 : 11$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q16

We have, $\angle B = 60^\circ$

$$\cos B = \frac{1}{2} \Rightarrow \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow a^2 + c^2 - b^2 = ac$$

$$\Rightarrow a^2 + c^2 - ac = b^2 \quad \dots(i)$$

$$(a+b+c)(a-b+c) = 3ca$$

$$a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2 = 3ac$$

$$a^2 + c^2 - b^2 + 2ac - 3ac = 0$$

$$a^2 + c^2 - ac = b^2$$

which is given.

Sine and Cosine Formulae and their Applications Ex-10.2 Q17

Consider the given equation:

$$\cos^2 A + \cos^2 B + \cos^2 C = 1$$

$$\Rightarrow 1 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$$

$$\Rightarrow 3 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q18

Let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$. Then, $\sin A = ka$, $\sin B = kb$, $\sin C = kc$

$$\text{Now, } \cos C = \frac{\sin A}{2 \sin B}$$

$$2 \sin B \cos C = \sin A$$

$$2 \left(\frac{a^2 + b^2 - c^2}{2ab} \right) kb = ka$$

$$a^2 + b^2 - c^2 = a^2$$

$$b^2 = c^2$$

$$b = c$$

$\triangle ABC$ is isosceles.

Sine and Cosine Formulae and their Applications Ex-10.2 Q19

Let P and Q be the position of two ships at the end of 3 hours.

Then,

$$OP = 3 \times 24 = 72 \text{ km and } OQ = 3 \times 32 = 96 \text{ km}$$

Using cosine formula in $\triangle OPQ$, we get

$$PQ^2 = OP^2 + OQ^2 - 2OP \times OQ \cos 90^\circ$$

$$PQ^2 = 72^2 + 96^2 - 2 \times 72 \times 96 \cos 90^\circ$$

$$PQ^2 = 14400$$

$$PQ = 120 \text{ km}$$

