RD Sharma Solutions Class 11 Maths Chapter 15 Ex 15.3

Linear Inequations Ex 15.3 Q1

Consider the first inequation,

$$x + \frac{1}{3} \geq 0$$

$$\therefore e \quad x \geq \frac{-1}{3}.$$

$$\left|x + \frac{1}{3}\right| - \frac{8}{3} > 0$$

$$x + \frac{1}{3} - \frac{8}{3} > 0$$

$$\frac{3x - 7}{3} > 0$$

$$3x - 7 > 0$$

$$x > \frac{7}{3} \qquad \ldots(i)$$

Consider the second inequation,

$$x + \frac{1}{3} < 0 \quad \therefore \quad e. \quad x < -\frac{1}{3}$$

$$\left|x + \frac{1}{3} - \frac{8}{3} > 0\right|$$

$$-x - \frac{1}{3} - \frac{8}{3} > 0$$

$$-3x - 9 > 0$$

$$-3x > 9$$

$$3x < -9$$

$$x < \frac{-9}{3}$$

$$x < -3 \qquad \ldots(ii)$$

From (i) and (ii), $(-\infty, -3) \cup \left(\frac{7}{3}, \infty\right)$ is the solution set of the simultaneous equations.

Linear Inequations Ex 15.3 Q2

We have,

$|4-x|+1-3<0$

$\Rightarrow |4-x|-2<0 \qquad \ldots(i)$

Case I: When $|4-x| \geq 0$

$$|4-x|-2<0$$
$$\Rightarrow \quad 4-x-2<0$$
$$\Rightarrow \quad 2-x<0$$
$$\Rightarrow \quad -x<-2$$
$$\Rightarrow \quad x>2 \qquad \ldots(ii)$$

Case II: When $|4-x|<0$

$$|4-x|-2<0$$
$$\Rightarrow \quad -(4-x)-2<0$$
$$\Rightarrow \quad -4+x-2<0$$
$$\Rightarrow \quad x-6<0$$
$$\Rightarrow \quad x<6 \qquad \ldots(iii)$$

Combining (ii) and (iii) we get $(2,6)$ as the solution set.

Linear Inequations Ex 15.3 Q3

We have,

$$\frac{|3x-4|}{2}-\frac{5}{12} \leq 0$$

Case I: When $|3x-4| \geq 0$

$$\frac{|3x-4|}{2}-\frac{5}{12} \leq 0$$
$$\Rightarrow \quad \frac{|3x-4|}{2}-\frac{5}{12} \leq 0$$
$$\Rightarrow \quad \frac{3x-4}{2}-\frac{5}{12} \leq 0$$
$$\Rightarrow \quad \frac{6(3x-4)-5}{12} \leq 0$$
$$\Rightarrow \quad 18x-24-5 \leq 0$$
$$\Rightarrow \quad 18x-29 \leq 0$$
$$\Rightarrow \quad 18x \leq 29$$
$$\Rightarrow \quad x \leq \frac{29}{18} \qquad \ldots(ii)$$

Case II: When $|3x-4|<0$

$$\frac{|3x-4|}{2}-\frac{5}{12} \leq 0$$

Linear Inequations Ex 15.3 Q4

We have,

$$\frac{|x-2|}{x-2} > 0 \qquad \ldots(i)$$

Case I: When $|x-2| \geq 0$

$x \geq 2$

$\Rightarrow \quad \frac{x-2}{x-2} \geq 0$

$\Rightarrow \quad x-2 \geq 0$

$\Rightarrow \quad x \geq 2 \qquad \ldots(ii)$

Case II: when $|x-2| < 0$

$x < 2$

$\Rightarrow \quad -\frac{(x-2)}{x-2} > 0$

$\Rightarrow \quad -(x-2) > 0$

$\Rightarrow \quad -x+2 < 0$

$\Rightarrow \quad -x < -2$

$\Rightarrow \quad x > 2 \qquad \ldots(iii)$

Combining (ii) and (iii) we get $(2, \infty)$ as the solution set.

Linear Inequations Ex 15.3 Q5

We have,

$$\frac{1}{|x| - 3} - \frac{1}{2} < 0 \qquad ...(i)$$

Case I: when $|x| \geq 0 \Rightarrow x \geq 0$

$$\Rightarrow \quad \frac{1}{x - 3} - \frac{1}{2} < 0$$

$$\Rightarrow \quad \frac{2 - (x - 3)}{2(x - 3)} < 0$$

$$\Rightarrow \quad \frac{2 - x + 3}{2x - 6} < 0$$

$$\Rightarrow \quad \frac{-x + 5}{2x - 6} < 0$$

$$\Rightarrow \quad -x + 5 < 0$$
$$\Rightarrow \quad -x < -5$$
$$\Rightarrow \quad x > 5 \qquad ...(ii)$$

Case II: when $|x| < 0, \quad x < 0$

$$\Rightarrow \quad \frac{1}{-x - 3} - \frac{1}{2} < 0$$
$$\Rightarrow \quad \frac{2 - (-x - 3)}{2(-x - 3)} < 0$$
$$\Rightarrow \quad 2 + x + 3 < 0$$
$$\Rightarrow \quad x + 5 < 0$$
$$\Rightarrow \quad x < -5 \qquad ...(iii)$$

Combining (ii) and (iii) we get $(-\infty, -5) \cup (-3, 3) \cup (5, \infty)$ as the solution set.

Linear Inequations Ex 15.3 Q6

We have,

$$\frac{|x+2|-x}{x} < 0$$

$$\frac{|x+2|-x}{x} - 2 < 0$$

$$\frac{|x+2|-x-2x}{x} < 0$$

$$\frac{|x+2|-3x}{x} < 0 \qquad \ldots(i)$$

Case I: when $|x+2| \geq 0$

$i.e, \quad x \geq -2$

$\Rightarrow \qquad \frac{x+2-3x}{x} < 0$

$\Rightarrow \qquad -2x+2 < 0$

$\Rightarrow \qquad -2x < -2 \qquad$ and $\qquad x > 0$

$\Rightarrow \qquad x > 1 \quad \ldots(ii)$

Case II: $|x+2| < 0$

$i.e, \; x < -2$

$\Rightarrow \qquad -(x+2) - 3x < 0$

$\Rightarrow \qquad -x-2-3x < 0$

$\Rightarrow \qquad -4x-2 < 0$

$\Rightarrow \qquad -4x < 2$

$\Rightarrow \qquad x > \frac{-1}{2} \; \ldots\ldots\ldots\ldots(iii)$

and $\quad x < 0$

Combining (ii) and (iii) we get $(-\infty, 0) \cup (1, \infty)$ as the solution set.

Linear Inequations Ex 15.3 Q7

We have,

$$\frac{|2x-1|}{x-1} - 2 > 0$$

$$\frac{|2x-1| - 2(x-1)}{x-1} > 0$$

$$\frac{|2x-1| - 2x + 2}{x-1} > 0 \quad \ldots(i)$$

Case I: when $|2x-1| \geq 0$

$$i.e,\ 2x - 1 \geq 0$$
$$2x \geq 1$$
$$x \geq \frac{1}{2}$$

$\Rightarrow \quad |2x-1| - 2x + 2 > 0$ and $x - 1 > 0$

$\Rightarrow \quad 2x - 1 - 2x + 2 > 0$ and $x > 1$

$\Rightarrow \quad x > 1 \quad \ldots(ii)$

Case II: when $|2x-1| < 0$

$$i.e,\ 2x - 1 < 0$$
$$2x < 1$$
$$x < \frac{1}{2}$$

$\Rightarrow \quad -(2x-1) - 2x + 2 > 0$ and $x < 1$

$\Rightarrow \quad -4 + 3 > 0$

$\Rightarrow \quad -x > -\frac{3}{4}$

$\Rightarrow \quad x < \frac{3}{4}$ and $x < 1$

$\Rightarrow \quad x \in \left(\frac{3}{4}, 1\right) \quad \ldots(iii)$

Combining (ii) and (iii) we get $\left(\frac{3}{4}, 1\right) \cup (1, \infty)$ as the solution set.

Linear Inequations Ex 15.3 Q8

We have,

$|x-1|+|x-2|+|x-3|-6 \geq 0 \qquad \ldots(i)$

Case I: $|x-1| \geq 0$

$x \geq 1$

$\Rightarrow \quad x-1-(x-2)-(x-3)-6 \geq 0$

$\Rightarrow \quad -x+4-6 \geq 0$

$\Rightarrow \quad -x \geq 2$

$\Rightarrow \quad x \leq -2$

$\Rightarrow \quad (-\infty,-2] \qquad \ldots(ii)$

Case II: $|x-2| \geq 0$

$x \geq 2$

$\Rightarrow \quad x-1+x-2-(x-3)-6 \geq 0$

$x-6 \geq 0$

$x \geq 6$

$\Rightarrow \quad [6,\infty) \ldots\ldots\ldots\ldots(iii)$

case III: When $|x-3| \geq 0$

$x \geq 3$

$\Rightarrow \quad x-1+x-2+x-3-6 \geq 0$

$\Rightarrow \quad 3x-12 \geq 0$

$\Rightarrow \quad 3x \geq 12$

$\Rightarrow \quad x \geq 4$

$\Rightarrow \quad \therefore\ x \in [4,\infty)$

also

$\Rightarrow \quad |x-1| < 0$

$\Rightarrow \quad x < 1$

$\Rightarrow -(x-1)-(x-2)-(x-3)-6 \geq 0$

$\Rightarrow \quad -3x \geq 0$

$\Rightarrow \quad x \leq 0$

$\Rightarrow \quad |x-2| < 0$

$x < 2$

$\Rightarrow (x-1)-(x-2)-(x-3)-6 \geq 0$

$\Rightarrow x-1-x+2-x+3-6 \geq 0$

$\Rightarrow \quad -x-2 \geq 0$

$\Rightarrow \quad -x \geq 2$

$\Rightarrow \quad x \leq -2$

$\Rightarrow \quad |x-3| < 0$

$\Rightarrow \quad x < 3$

$\Rightarrow (x-1)+(x-2)-(x-3)-6 \geq 0$

$\Rightarrow \quad x-6 \geq 0$

$\Rightarrow \quad x \geq 6$

Combining all cases we get $(-\infty,0] \cup [4,\infty)$ as the solution set.

Linear Inequations Ex 15.3 Q9

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

Let $y = |x-2|$

$$\Rightarrow \frac{y-1}{y-2} \leq 0$$

$$\Rightarrow 1 \leq y < 2$$

$$\Rightarrow 1 \leq |x-2| < 2$$

$$\Rightarrow x \in [-2+2, -1+2] \cup [1+2, 2+2]$$

$$\Rightarrow x \in [0,1] \cup [3,4]$$

The solution set is $[0,1] \cup [3,4]$.

Linear Inequations Ex 15.3 Q10

$$\frac{1}{|x|-3} \leq \frac{1}{2}$$

$$\Rightarrow \frac{1}{|x|-3} - \frac{1}{2} \leq 0$$

$$\Rightarrow \frac{2-|x|+3}{2(|x|-3)} \leq 0$$

$$\Rightarrow \frac{5-|x|}{2(|x|-3)} \leq 0$$

$$\Rightarrow \frac{|x|-5}{2(|x|-3)} \geq 0$$

$$\Rightarrow \frac{|x|-5}{|x|-3} \geq 0$$

$$\Rightarrow |x| \geq 5 \text{ or } |x| < 3$$

$$\Rightarrow x \in (-\infty, -5] \cup [5, \infty) \text{ or } x \in (-3, -3)$$

$$\Rightarrow x \in (-\infty, -5] \cup (-3, -3) \cup [5, \infty)$$

The solution set is $(-\infty, -5] \cup (-3, -3) \cup [5, \infty)$.

Linear Inequations Ex 15.3 Q11

$|x+1|+|x|>3$

CASE1: When $-\infty<x<-1$

$|x+1|=-(x+1)$ and $|x|=-x$

$\therefore |x+1|+|x|>3$

$\Rightarrow -(x+1)-x>3$

$\Rightarrow -2x>4$

$\Rightarrow x<-2$

But, $-\infty<x<-1$.

$\therefore$ The solution set of the given inequation is $(-\infty,-2)$.

CASE2: When $-1\leq x<0$

$|x+1|=(x+1)$ and $|x|=-x$

$\therefore |x+1|+|x|>3$

$\Rightarrow (x+1)-x>3$

$\Rightarrow 1>3$

Which is not true.

CASE3: When $0<x<\infty$

$|x+1|=(x+1)$ and $|x|=x$

$\therefore |x+1|+|x|>3$

$\Rightarrow (x+1)+x>3$

$\Rightarrow 2x>2$

$\Rightarrow x>1$

But, $0<x<\infty$.

$\therefore$ The solution set of the given inequation is $(1,\infty)$.

Combining Case1, Case2 and Case3,

we obtain that the solution set of given in equality is $(-\infty,-2)\cup(1,\infty)$

Linear Inequations Ex 15.3 Q12

$$1 \le |x-2| \le 3$$
$$\Rightarrow x \in [-3+2, -1+2] \cup [1+2, 3+2]$$
$$\Rightarrow x \in [-1, 1] \cup [3, 5]$$

$\therefore$ The solution set for given inequality is $[-1,1] \cup [3,5]$.

Linear Inequations Ex 15.3 Q13

$$|3-4x| \ge 9$$
$$\Rightarrow 4\left|\frac{3}{4} - x\right| \ge 9$$
$$\Rightarrow \left|\frac{3}{4} - x\right| \ge \frac{9}{4}$$

CASE1: When $-\infty < x \le -\frac{3}{4}$

$$\left|\frac{3}{4} - x\right| = \left(\frac{3}{4} - x\right)$$
$$\therefore \left|\frac{3}{4} - x\right| \ge \frac{9}{4}$$
$$\Rightarrow \left(\frac{3}{4} - x\right) \ge \frac{9}{4}$$
$$\Rightarrow -\frac{6}{4} \ge x$$
$$\Rightarrow -\frac{3}{2} \ge x$$

But, $-\infty < x < -1$.

$\therefore$ The solution set of the given inequation is $\left(-\infty, -\frac{3}{2}\right]$.

CASE2: When $-\frac{3}{4} < x < \infty$

$$\left|\frac{3}{4} - x\right| = -\left(\frac{3}{4} - x\right)$$

$$\therefore \left|\frac{3}{4} - x\right| \geq \frac{9}{4}$$

$$\Rightarrow -\left(\frac{3}{4} - x\right) \geq \frac{9}{4}$$

$$\Rightarrow x \geq 3$$

But, $-\frac{3}{4} < x < \infty$

$\therefore$ The solution set of the given inequation is $[3, \infty)$.

Combining Case1 and Case2,

we obtain that the solution set of given in equality is $\left(-\infty, -\frac{3}{2}\right] \cup (3, \infty)$.