RD Sharma Solutions Class 11 Maths Chapter 16 Ex 16.2

Permutations Ex 16.2 Q1

Here the teacher is to perform two jobs.

(i) selecting a boy among 27 boys, and

(ii) selecting a girl among 14 girls.

The first of these can be performed in 27 ways and the second in 14 ways.
Therefore by the fundamental principle of multiplication, the required number of ways is

$$27 \times 14 = 378$$

Hence, the teacher can make the selection of a boy a girl in 378 ways.

Permutations Ex 16.2 Q2

Here the person is to perform three jobs.

(i) selecting a ball pen from 12 ball pens

(ii) selecting a fountain pen from 10 fountain pens, and

(iii) selecting a pencil from 5 pencils.

The first of these can be performed in 12 ways, the second in 10 ways and the third in 5 ways.

Therefore by the fundamental principle of multiplication, the required number of ways is

$$12 \times 10 \times 5 = 600$$

Hence, the person can make the selection of a fountain pen, ball pen and pencil in 600 ways.

Permutations Ex 16.2 Q3

From Goa to Bombay there are two roots; air and sea.
From Bombay to Delhi there are three routs; air rail and road.

Therefore by the fundamental principle of multiplication, the required number of ways are $2 \times 3 = 6$

Hence, total number of different kinds routes are 6.

Permutations Ex 16.2 Q4

The mint has to perform two jobs,
(i) selecting the number of days in the february month (there can be 28 days or 29 days), and
(ii) selecting the first day of february.

The first job can be compeleted in 2 ways the second can be performed in 7 ways by selecting any one of the seven days of a week.

Thus, the required number of plates $= 2 \times 7 = 14$

Hence, total number of calendars $= 7 \times 2 = 14$

Permutations Ex 16.2 Q4

Total number of letters = 7
Total number of letter boxes = 4.

$\therefore$ Total number of ways in which 7 letters be posted in 4 letter boxes

$$= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^7$$

Permutations Ex 16.2 Q5

Total number of parcels = 4
Total number of post-offices = 5

Since a parcel can be sent to any one of the five post offices.
So, the required number of ways $= 5 \times 5 \times 5 \times 5$

$$= 5^4$$
$$= 625$$

Hence, total number of ways is 625.

Permutations Ex 16.2 Q6

Since toss of each coin can result in 2 ways.

When coin is tossed five times, the total number of outcomes

$$= 2 \times 2 \times 2 \times 2 \times 2$$
$$= 32$$

Hence, required number of ways is 32

Permutations Ex 16.2 Q7

The number of ways to examinee answer a true/false type question is 2.

$\therefore$ the number of ways for an examinee to answer a set of ten true/false type questions $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$= 1024.$$

Hence, the required number of ways is 1024.

Permutations Ex 16.2 Q8

The total number of ways to make attempt to open the lock = $10 \times 10 \times 10 = 1000$.

The number of ways to successfuly open the lock = 1

$\therefore$ The number of ways to make an unsuccessful attempt to open the lock = $1000 - 1 = 999$.

Hence,required number of ways to make an unsuccessfuly attempt to the open the lock is 999.

Permutations Ex 16.2 Q9

Each one of the first three questions can be answered in 4 ways.

$\therefore$ The total number of ways to answered the first three question = $4 \times 4 \times 4$
$= 64$

Each of the next three question can be answered in 2 ways.

$\therefore$ The total number of ways the answered the next three questions = $2 \times 2 \times 2 = 8$

so, total number of sequences at answers = $64 \times 8 = 512$

Permutations Ex 16.2 Q10

There are 5 books on mathematics and 6 books on physics in a book shop.

The number of ways to select a mathematics book = 5
The number of ways to select a physics book = 6

Now,

(i) Number of ways in which a student can buy a mathematics book and a physics book = $5 \times 6 = 30$

(ii) Number of ways in which a student buy either a mathematics book or a physics book = $5 + 6 = 11$

Permutations Ex 16.2 Q11

Since there are 7 flags of different colours, therefore, first flag can be selected in 7 ways.

Now, the second flag can be selected from any one of the remaining flags in 6 ways.

Hence, by the fundamental principle of multiplication, the number of flag is $7 \times 6 = 42$

Permutations Ex 16.2 Q12

A boy can be selected from the first team in 6 ways, and from the second in 5 ways.

so, number of single matches between the boys of two teams = $6 \times 5 = 30$.

similarly, the number of single matches between the girls of two teams = $4 \times 3 = 12$.
so, total number of matches = $30 + 12 = 42$.

Permutations Ex 16.2 Q13

Clearly, the total number of ways to select first three prizes is equal to the 3 students from 12 students.

$\therefore$ number of ways to select the three prizes
$= 12 \times 11 \times 10$
$= 1320$

Permutations Ex 16.2 Q14

There are 3 ways to choose the first form and corresponding to each such way there are 5 ways of selecting the common difference.

So, required number of A.P.'s

$= 3 \times 5$

$= 15$

Permutations Ex 16.2 Q15

Clearly the number of ways to appoint one principal, one vice-principal and the teacher- incharge is equal to the number of ways to select the three teachers from the 36 teachers.

$\therefore$ Number of ways to appointed 3 teachers $= 36 \times 35 \times 34 = 42840$

Hence, the number of ways to appoint one principal, one vice-principal and the teacher-incharge is equal to 42840.

Permutations Ex 16.2 Q16

We have to form all possible 3-digit numbers with distinct digits.

we cannot have 0 at the hundred's place. so, the hundred's place can be filled with any of the 9 digits 1,2,3,4....,9.

so, there are 9 ways of filling the hundred's place.

Now, 9 digits are left including 0, so, ten's place can be filled with any of the remaining 9 digits in 9 ways. now, the unit's place can be filled which in any of the remaining 8 digits. so, there are 8 ways of filling the unit's place.

Hence, the total number of required numbers $= 9 \times 9 \times 8 = 648$

Permutations Ex 16.2 Q17

We cannot have a 0 at the hundred's place. So, the hundred's place can be filled with any of the 9 digits 1,2,3.....,9.

So, there are 9 ways of filling the hundred's place.

Ten's place can be filled with any 10 digits in 10 ways.

Now, the unit's place can be filled with any 10 digits in 10 ways.

Hence, the total number of required numbers $= 9 \times 10 \times 10 = 900$

Permutations Ex 16.2 Q18

The three digit numbers are 100 to 999 inclusive so there are

$999 - 100 + 1 = 999 - 99 = 900$

So, 900 three digit numbers

If half of all numbers is odd then half of 900 is 450, there are 450 odd positive 3 digit numbers

Permutations Ex 16.2 Q19(i)

Zero cannot be first digit of the license plates.

This means the first digit can be selected from the 9 digits 1,2,3,4....,9
So, there are 9 ways of filling the first digit of the license plates.

Now, 9 digits are left including 0. So, second place can be filled with any of the remaining 9 digits in 9 ways.

The third place of the license plates can be filled with in any of the remaining 8 digits.
So, there are 8 ways of filling the third place.

The fourth place of the license plates can be filled with in any of the remaining 7 digits.
So, there are 7 ways at filling the fourth place.

The last place of the license plates can be filled with in any of the remaining 6 digits.
So, there are 6 ways of filling the fourth place.

Hence, the total number of ways = $9 \times 9 \times 8 \times 7 \times 6 = 27216$

Permutations Ex 16.2 Q19(ii)

Zero cannot be first digit of the license plates.

$\therefore$ first digit can be selected from the 9 digits 1,2,3.....,9
So, there are 9 ways at filling the first digit of the licence plates.

The repetition of digits is allowed to made a license plates number.

$\therefore$ the number of ways to fill the remaining places of the number plates = $10 \times 10 \times 10 \times 10$.

Hence, the total number of ways = $9 \times 10 \times 10 \times 10 \times 10 = 90,000$

Permutations Ex 16.2 Q20

The required numbers are greater than 7000.

$\therefore$ the thousand's place can be filled with any of the 3 digits 7,8,9.

so, there are 3 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's, ten's and one's places can be filled in 4,3, and 2 ways respectively.

Hence, the required number of numbers = $3 \times 4 \times 3 \times 2 = 72$

Permutations Ex 16.2 Q21

Since the required numbers are greater than 8000.

$\therefore$ the thousand's place can be two digits 8 or 9
So, there are 2 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's, ten's and one's places can be filled in 4,3 and 2 ways respectively.

Hence, the required number of number = $2 \times 4 \times 3 \times 2 = 48$

Permutations Ex 16.2 Q22

First person can be seated in a row in 6 ways.

Second person can be seated in a row in 5 ways.

Third person can be seated in a row in 4 ways.

Fourth person can be seated in a row in 3 ways.

Fifth person can be seated in a row in 2 ways.

And, sixth person can be seated in a row in 1 ways.

Hence, total number of ways in which six persons can be seated in a row
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

Permutations Ex 16.2 Q23

In a nine-digit number 0 cannot appear in the first digit. So, the number of ways of filling up the first-digit = 9.

Now, 9 digits are left including 0. So, second digit can be filled with any of the remaining 9 digits in 9 ways.

Similarly, remaining digits can be filled in 8,7,6,5,4,3 and 2 ways.

Hence, the total number of required numbers

$$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$$
$$= 9 \times (9!)$$

Permutations Ex 16.2 Q24

Any number less than 1000 may be any of a number from one-digit number, two-digit number and three-digit number.

One-digit odd number:

3 possible ways are there. These numbers are 3 or 5 or 7.

Two-digit odd number:

Tens place can be filled up by 3 ways (using any of the digit among 3, 5 and 7) and then the ones place can be filled in any of the remaining 2 digits.

So, there are $3 \times 2 = 6$ such 2-digit numbers.

Three-digit odd number:

Ignore the presence of zero at ones place for some instance.

Hundreds place can be filled up in 3 ways (using any of any of the digit among 3, 5 and 7), then tens place in 3 ways by using remaining 3 digits (after using a digit, there will be three digits) and then the ones place in 2 ways.

So, there are a total of $3 \times 3 \times 2 = 18$ numbers of 3-digit numbers which includes both odd and even numbers (ones place digit are zero). In order to get the odd numbers, it is required to ignore the even numbers i.e. numbers ending with zero.

To obtain the even 3-digit numbers, ones place can be filled up in 1 way (only 0 to be filled), hundreds place in 3 ways (using any of the digit among 3, 5, 7) and then tens place in 2 ways (using remaining 2 digits after filling up hundreds place).

So, there are a total of $1 \times 3 \times 2 = 6$ even 3-digit numbers using the digits 0, 3, 5 and 7 (repetition not allowed)

So, number of three-digit odd numbers using the digits 0, 3, 5 and 7 (repetition not allowed) $= 18 - 6 = 12$.

Therefore, odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed are $3 + 6 + 12 = 21$.

Permutations Ex 16.2 Q25

The odd digits are 1,3,5,7,9

$\therefore$ Total number of odd digits = 5

Clearly, the hundred's place can be filled with any of the 5 digits 1,3,5,7 or 9
So, there are 5 ways of filling the hundred's place.

Now, 4 digits are left. So, ten's place can be filled with any of the remaining 4 digits in 4 ways.

Now, the unit's place can be filled with in any of the remaining 3 digits. So, there are 3 ways of filling the unit's place.

Hence, the total number of required number = $5 \times 4 \times 3 = 60$

Permutations Ex 16.2 Q26

First digit of six-digit numbers can be selected in 6 ways.

Second digit of six-digit numbers can be selected in 5 ways

Third digit of six-digit numbers can be selected in 4 ways.

Fourth digit of six-digit numbers can be selected in 3 ways.

Fifth digit of six-digit numbers can be selected in 2 ways.

Last digit of six-digit numbers can be selected in 1 ways.

Hence, total number of numbers = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Permutations Ex 16.2 Q27

We cannot have 0 at the first digit of six-digit numbers.

So, the first digit of six-digit numbers can be selected in 5 ways.

Now, 5 digits are left including 0. So, second digit of six-digit numbers can be selected in 5 ways.

Third digit of six-digit numbers can be selected in 4 ways.

Fourth digit of six-digit numbers can be selected in 3 ways.

Fifth digit of six-digit numbers can be selected in 2 ways.

Last digit of six-digit numbers can be selected in 1 ways.

Hence, total number of numbers = $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Permutations Ex 16.2 Q28

Since the required numbers are greater than 5000.

$\therefore$ the thousand's place can be filled with any of two digits 5 or 9.

So, there are 2 ways of filling the thousand's place.

Since repetition of digits is not allowed, so the hundred's ten's and one's places can be filled in 4,3 and 2 ways respectively.

Hence, the required number of numbers $= 2 \times 4 \times 3 \times 2 = 48$

Permutations Ex 16.2 Q29

Each serial number of the product consists of six components. First two are letters and remaining four are numbers.
So all the serial numbers will look as shown below.

L	L	N	N	N	N

For the first position of serial number we can have one of the 6 letters. As repetition is not allowed first position of serial number we can have one of the 5 letters. For the third position of serial number we can have one of the 10 numbers. Similarly for the remaining position we can have 9, 8 and 7 possible ways.

L	L	N	N	N	N
$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
6	5	10	9	8	7

So the required number of serial number is
$6 \times 5 \times 10 \times 9 \times 8 \times 7$.

Permutations Ex 16.2 Q30

Total number of digits = 10
The digits is not repeats in a sequence of three digits.

$\therefore$ required number of sequences $= 10 \times 9 \times 8 = 720$

$\therefore$ total number of unsuccessful attempts $= 720 - 1 = 719$

Permutations Ex 16.2 Q31

Total number of digits = 4.

$\therefore$ the largest possible number of trials to obtain the correct code $= 4 \times 3 \times 2 \times 1$ [$\because$ digits are not repeated]

$= 24$

Permutations Ex 16.2 Q32

Total number of jobs = 3

$\therefore$ the number of ways to assined these job is to three persons $= 3 \times 2 \times 1$

$= 6$

Permutations Ex 16.2 Q33

The given digits are 1, 2, 3 and 4. These digits can be repeated while forming the numbers. So, number of required four digit natural numbers can be found as follows.

Consider four digit natural numbers whose digit at thousandths place is 1.

Here, hundredths place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Similarly, tens place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Ones place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Number of four digit natural numbers whose digit at thousandths place is $1 = 4 \times 4 \times 4 = 64$

Similarly, number of four digit natural numbers whose digit at thousandths place is $2 = 4 \times 4 \times 4 = 64$

Now, consider four digit natural numbers whose digit at thousandths place is 4:

Here, if the digit at hundredths place is 1, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 2, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 1, then ones place can be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 2, then ones place can be filled only in 1 way so that the number formed is not exceeding 4321.

Number of four digit natural numbers not exceeding 4321 and digit at thousandths place is $3 = 4 \times 4 + 4 \times 4 + 4 + 1 = 37$

Thus, required number of four digit natural numbers not exceeding 4321 is $64 + 64 + 64 + 37 = 229$.

Permutations Ex 16.2 Q34

Total number of digits = 6

we cannot have 0 at the first digit of the required six-digit numbers.

The digits cannot repeat in the six digits number.

$\therefore$ total number of six digit number are $= 5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Now, the six digit number can be divided by 10, if its last digit is 0

$\therefore$ Total numbers which are divisible by $10 = 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$

Permutations Ex 16.2 Q35

Total numbers of faces in each die = 6

$\therefore$ The total number of possible outcomes of three six faced die

$= 6 \times 6 \times 6$

$= 216$

Permutations Ex 16.2 Q36

Since a toss of a coin can result in a head or a tail.

$\therefore$ Total number of possible outcomes in each tossed = 2

$\therefore$ Total number of possible outcomes in four tossed = $2 \times 2 \times 2 = 2^3 = 8$

$\therefore$ Total number of possible outcomes in four tossed = $2 \times 2 \times 2 \times 2 = 2^4 = 16$

$\therefore$ total number of possible outcomes in five tossed = $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

$\therefore$ total number of possible outcomes in n tossed = $2 \times 2 \times 2 \ldots\ldots n$ times = 2^n

Permutations Ex 16.2 Q37

Total number of digits = 5

Since, the digits can be repeated in the same number.

$\therefore$ Total numbers of four digits numbers = $5 \times 5 \times 5 \times 5 = 625$

Permutations Ex 16.2 Q38

Total number of digits = 5

We cannot have 0 at the hundred's place so, the hundred's place can be digits with any of the 4 digits 1, 3, 5 or 7, So, there are 4 ways of filling the hundred's place.

Since, the digit may be repeated in three digit numbers.

$\therefore$ Ten's place can be filled with any of the 5 digits in 5 ways

A

nd unit's place can be filled with any of the 5 digits in 5 ways

Hence, the total number of required numbers = $4 \times 5 \times 5 = 100$

Permutations Ex 16.2 Q39

Total number of digits = 6

Clearly, the natural numbers ten's than 1000 can be 3 digits, 2 digits and 1 digit numbers.

Now, 0 cannot be a first digit of the three digit numbers.
So, the hundred's place can be filled with any of the 5 digits 1,2,3....5. So, there are 5 ways of filling the hundred place.

The ten's place can be filled with in any of the 6 digits 0,1,2.....5. So, there are 6 ways of filling the ten's place.

The unit's place can be filled with in any of the 6 digits 0,1,2.....5. So, there are 6 ways of filling the ten's place.

$\therefore$ The total number of 3 digit numbers $= 5 \times 6 \times 6 = 180$
Similarly, the total number of 2 digit numbers $= 5 \times 6 = 30$

Now, 0 is not a natural number
$\therefore$ the total number of 1digit numbers = 5

$\therefore$ Total number of natural numbers tens than 1000
$= 180 + 30 + 5 = 215.$

Permutations Ex 16.2 Q40

Total number of digits = 10
each number starts with 67 and no digit appears more than once.

$\therefore$ total number of five digit telephone numbers
$= 1 \times 1 \times 8 \times 7 \times 6 = 336$

Permutations Ex 16.2 Q41

Total numbers of toys = 8
Total number of children = 5

$\therefore$ The total number ways in which 8 distinct toys can be distributed among 5 children.
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$

Permutations Ex 16.2 Q42

Total numbers of letters = 5
Total number of letters boxes = 7

$\therefore$ The number ways in which one can post 5 letters in 7 letter boxes
$= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

Permutations Ex 16.2 Q43

Total numbers of dice = 3

$\therefore$ The number of possible outcomes
$= 6 \times 6 \times 6 = 216$

$\therefore$ Total number of possible outcomes in which 5 dose not appear on any dice
$= 5 \times 5 \times 5 = 125$

$\therefore$ *R*equired number of possible
outcomes $= 216 - 125 = 91$

Permutations Ex 16.2 Q44

Total numbers of balls = 20
Total number of boxes = 5

One ball can be put in first box in 20 ways because we can put any one of the twenty balls in first box.

Now, remaining 19 balls are to be but into remaining 4 boxes.
This can be done in 4^{19} ways; because there are 4 choices for each ball

Hence, the required number of ways = 20×4^{19}.

Permutations Ex 16.2 Q45

Total number of balls = 5
Total number of boxes = 3

$\therefore$ Total number of ways to distributed 5 different balls in three boxes
$= 3 \times 3 \times 3 \times 3 \times 3 = 243$

Permutations Ex 16.2 Q46

Total number of ball = n = 5
Number of boxes = r = 3

5 different balls can be distributed among three boxes in ${}^5P_3$ ways.

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60.$$

In 60 ways 5 different balls can be distributed among three boxes.

Permutations Ex 16.2 Q47(i)

4 prizes be distributed among 5 students so that no student gets more than one prize can be done in

$${}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{(1)!} = 5!\text{ ways.}$$

Permutations Ex 16.2 Q47(ii)

The first prize can be given away in 5 ways as it may be given to anyone of the 5 students.
The second prize can also be given away in 5 ways, since if may be obtained by the student who has already received a prize. Similarly, third and fourth prize can be given away in 5 ways.

Hence, the number of ways in which all the prize can be given away = $5 \times 5 \times 5 \times 5 = 625$

Permutations Ex 16.2 Q47(iii)

Since any of the 5 students may get all the prizes. So, the number of ways in which a student gets all the 4 prizes is 5.

So, the number of ways in which a student does not get all the prizes = 625 – 5 = 620

Permutations Ex 16.2 Q48

Each lamps has two possibilities either it can be switched on or off.

There are 10 lamps in the hall.

So the total numbers of possibilities are 2^{10}.

To illuminate the hall we require at least one lamp is to be switched on.

There is one possibility when all the lamps are switched off. If all the bulbs are switched off then hall will not be illuminated.

So the number of ways in which the hall can be illuminated is $2^{10}-1$.

Permutations Ex 16.2 Q1(i)

We have,

$$^8P_3 = \frac{8!}{(8-3)!} \quad \left[\because\ ^nP_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!}$$

$$= 336$$

Hence, $^8P_3 = 336$

Permutations Ex 16.2 Q1(ii)

We have,

$$^{10}P_4 = \frac{10!}{(10-4)!} \qquad \left[\because\ ^nP_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{10!}{6!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= 5040$$

$\therefore\ ^{10}P_4 = 5040$

Permutations Ex 16.2 Q1(iii)

We have,

$$^6P_6 = \frac{6!}{(6-6)!} \qquad \left[\because\ ^nP_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{6!}{0!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{1} \quad [\because\ 0! = 1]$$

$$= 720$$

Hence, $^6P_6 = 720$

Permutations Ex 16.2 Q1(iv)

We have,

$$P(6,4) = \frac{6!}{(6-4)!} \qquad \left[\because \ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$= \frac{6!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!}$$

$$= 360$$

Hence, $P(6,4) = 360$

Permutations Ex 16.2 Q2

We have,

$$P(5,r) = P(6,r-1)$$

$$\Rightarrow \quad \frac{5!}{(5-r)!} = \frac{6!}{[6-(r-1)]!} \qquad \left[\because \ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{[7-r]!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)\times(7-r-1)(7-r-2)!}$$

$$\Rightarrow \quad \frac{1}{(5-r)!} = \frac{6}{(7-r)\times(6-r)(5-r)!}$$

$$\Rightarrow \quad 1 = \frac{6}{(7-r)\times(6-r)}$$

$$\Rightarrow \quad (6-r)\times(7-r) = 6$$

$$\Rightarrow \quad 42 - 6r - 7r + r^2 = 6$$

$$\Rightarrow \quad r^2 - 12r + 42 - 6 = 0$$

$$\Rightarrow \quad r^2 - 13r + 36 = 0$$

$$\Rightarrow \quad r^2 - 9r - 4r + 36 = 0$$

$$\Rightarrow \quad r(r-9) - 4(r-9) = 0$$

$$\Rightarrow \quad (r-9)(r-4) = 0$$

$$\Rightarrow \quad r = 4 \qquad \left[\begin{array}{l}\because \ r \le n \\ \therefore \ r - 9 \ne 0\end{array}\right]$$

Hence, $r = 4$

Permutations Ex 16.2 Q3

We have,

$$5P(4,n) = 6.\ P(5,n-1)$$

$$\Rightarrow \quad 5 \times \frac{4!}{(4-n)!} = 6 \times \frac{5!}{[5-(n-1)]!} \qquad \left[\because\ ^{n}P_{r} = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad 5 \times \frac{4!}{(4-n)!} = \frac{6 \times 5 \times 4!}{[5-n+1]!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{[6-n]!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{(6-n)(6-n-1)(6-n-2)!}$$

$$\Rightarrow \quad \frac{1}{(4-n)!} = \frac{6}{(6-n)(5-n)(4-n)!}$$

$$\Rightarrow \quad \frac{(6-n)(5-n)(4-n)!}{(4-n)!} = 6$$

$$\Rightarrow \quad (6-n)(5-n) = 6$$

$$\Rightarrow \quad 30 - 6n - 5n + n^2 = 6$$

$$\Rightarrow \quad n^2 - 11n + 30 = 6$$

$$\Rightarrow \quad n^2 - 11n + 24 = 0$$

$$\Rightarrow \quad n^2 - 8n - 3n + 24 = 0$$

$$\Rightarrow \quad n(n-8) - 3(n-8) = 0$$

$$\Rightarrow \quad (n-8)(n-3) = 0$$

$$\Rightarrow \quad n - 3 = 0 \qquad \begin{bmatrix} \because n \le 4 \\ \therefore n \ne 8 \end{bmatrix}$$

$$\Rightarrow \quad n = 3$$

Hence, $n = 3$

Permutations Ex 16.2 Q4

We have,

$$P(n,5) = 20.\ P(n,3)$$

$$\Rightarrow \quad \frac{n!}{(n-5)!} = 20 \times \frac{n!}{(n-3)!}$$

$$\Rightarrow \quad \frac{1}{(n-5)!} = \frac{20}{(n-3)(n-3-1)(n-3-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-5)!} = \frac{20}{(n-3)(n-4)(n-5)!}$$

$$\Rightarrow \quad \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 20$$

$$\Rightarrow \quad (n-3)(n-4) = 20$$

$$\Rightarrow \quad n^2 - 4n - 3n + 12 = 20$$

$$\Rightarrow \quad n^2 - 7n - 8 = 0$$

$$\Rightarrow \quad n^2 - 8n + 1n - 8 = 0$$

$$\Rightarrow \quad n(n-8) + 1(n-8) = 0$$

$$\Rightarrow \quad (n-8)(n+1) = 0$$

$$\Rightarrow \quad n - 8 = 0 \qquad [\because\ n \neq -1]$$

$$\Rightarrow \quad n = 8$$

Hence, $n = 8$

Permutations Ex 16.2 Q5

We have,

$$^nP_4 = 360$$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 360$$

$$\Rightarrow \quad \frac{n(n-1)(n-2)(n-3)(n-4)! = 360}{(n-4)!}$$

$$\Rightarrow \quad n(n-1)(n-2)(n-3) = 6 \times 5 \times 4 \times 3$$

$$\Rightarrow \quad n = 6 \qquad [13y\ \text{comparing}]$$

Hence, $n = 6$

Permutations Ex 16.2 Q6

We have,

$$P(9, r) = 3024$$

$$\Rightarrow \quad \frac{9!}{(9-r)!} = 3024 \qquad \left[\because {}^{n}P_{r} = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{3024}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{336}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{42}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow \quad \frac{1}{(9-r)!} = \frac{1}{5!}$$

$$\Rightarrow \quad (9-r)! = 5!$$

$$\Rightarrow \quad 9 - r = 5$$

$$\Rightarrow \quad 9 - 5 = r$$

$$\Rightarrow \quad 4 = r$$

$$\Rightarrow \quad r = 4$$

Hence, $r = 4$

Permutations Ex 16.2 Q7

We have,

$$P(11, r) = P(12, r-1)$$

$$\Rightarrow \quad \frac{11!}{(11-r)!} = \frac{12!}{[12-(r-1)]!} \qquad \left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{11!}{(11-r)!} = \frac{12 \times 11!}{[12-r+1]!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{[13-r]!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{(13-r)(13-r-1)(13-r-2)!}$$

$$\Rightarrow \quad \frac{1}{(11-r)!} = \frac{12}{(13-r)(12-r)(11-r)!}$$

$$\Rightarrow \quad \frac{(13-r)(12-r)(11-r)}{(11-r)!} = 12$$

$$\Rightarrow \quad (13-r)(12-r) = 12$$

$$\Rightarrow \quad 156 - 13r - 12r + r^2 = 12$$

$$\Rightarrow \quad r^2 - 25r + 156 - 12 = 0$$

$$\Rightarrow \quad r^2 - 25r + 144 = 0$$

$$\Rightarrow \quad r^2 - 16r - 9r + 144 = 0$$

$$\Rightarrow \quad r(r-16) - 9(r-16) = 0$$

$$\Rightarrow \quad (r-16)(r-9) = 0$$

$$\Rightarrow \quad r - 9 = 0 \qquad \left[\begin{array}{l} \because r \le 11 \\ \therefore r \ne 16 \end{array}\right]$$

$$\Rightarrow \quad r = 9$$

Permutations Ex 16.2 Q8

We have,

$$P(n,4) = 12.\quad P(n,2)$$

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!} \qquad \left[\because\ {}^nP_r = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)(n-2-1)(n-2-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{12}{(n-2)(n-3)(n-4)!}$$

$$\Rightarrow \quad \frac{(n-2)(n-3)(n-4)!}{(n-4)!} = 12$$

$$\Rightarrow \quad (n-2)(n-3) = 12$$

$$\Rightarrow \quad n^2 - 3n - 2n + 6 = 12$$

$$\Rightarrow \quad n^2 - 5n + 6 - 12 = 0$$

$$\Rightarrow \quad n^2 - 5n - 6 = 0$$

$$\Rightarrow \quad n^2 - 6n + 1n - 6 = 0$$

$$\Rightarrow \quad n(n-6) + 1(n-6) = 0$$

$$\Rightarrow \quad (n-6)(n+1) = 0$$

$$\Rightarrow \quad n - 6 = 0 \qquad [\because n \neq -1]$$

$$\Rightarrow \quad n = 6$$

Hence, $n = 6$

Permutations Ex 16.2 Q9

We have,

$$P(n-1,3) : \quad P(n,4) = 1 : 9$$

$$\Rightarrow \quad \frac{P(n-1,3)}{P(n,4)} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{\frac{(n-1)!}{(n-1-3)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)! \times (n-4)!}{(n-4)! \times n!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)}{n!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{(n-1)!}{n \times (n-1)!} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow \quad n = 9$$

Hence, $n = 9$

Permutations Ex 16.2 Q10

We have,

$$P(2n-1,n) : \quad P(2n+1,n-1) = 22 : 7$$

$$\Rightarrow \quad \frac{P(2n-1,n)}{P(2n+1,n-1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{\frac{(2n-1)!}{(2n-1-n)!}}{\frac{(2n+1)!}{[2n+1-(n-1)]!}} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)!\times(n+2)!}{(n-1)!(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)!\times(n+2)(n+2-1)(n+2-2)(n+2-3)!}{(n-1)!(2n+1)(2n+1-1)(2n+1-2)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)!\times(n+2)(n+1).n.(n-1)!}{(n-1)!(2n+1).2n.(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{n(n+2)(n+1)}{2n(2n+1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{n^2+n+2n+2}{4n+2} = \frac{22}{7}$$

$$\Rightarrow \quad 7(n^2+3n+2) = 22\times(4n+2)$$

$$\Rightarrow \quad 7n^2+21n+14 = 88n+44$$

$$\Rightarrow \quad 7n^2+21n-88n+14-44 = 0$$

$$\Rightarrow \quad 7n^2-67n-30 = 0$$

$$\Rightarrow \quad 7n^2-70n+3n-30 = 0$$

$$\Rightarrow \quad 7n(n-10)+3(n-10) = 0$$

$$\Rightarrow \quad (n-10)(7n+3) = 0$$

$$\Rightarrow \quad n-10 = 0 \qquad [\because 7n+3 \neq 0]$$

$$\Rightarrow \quad n = 10$$

Permutations Ex 16.2 Q11

We have,

$$P(n,5) : P(n,3) = 2 : 1$$

$$\Rightarrow \quad \frac{P(n,5)}{P(n,3)} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{\frac{n!}{(n-5)!}}{\frac{n!}{(n-3)!}} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{n! \times (n-3)!}{(n-5)! \times n!} = 2$$

$$\Rightarrow \quad \frac{(n-3)!}{(n-5)!} = 2$$

$$\Rightarrow \quad \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 2$$

$$\Rightarrow \quad (n-3)(n-4) = 2$$

$$\Rightarrow \quad n^2 - 4n - 3n + 12 = 2$$

$$\Rightarrow \quad n^2 - 7n + 12 - 2 = 0$$

$$\Rightarrow \quad n^2 - 7n + 10 = 0$$

$$\Rightarrow \quad n^2 - 5n - 2n + 10 = 0$$

$$\Rightarrow \quad n(n-5) - 2(n-5) = 0$$

$$\Rightarrow \quad (n-5)(n-2) = 0$$

$$\Rightarrow \quad n = 5 \qquad \begin{bmatrix} \because n \geq 5 \\ \therefore n \neq 2 \end{bmatrix}$$

Hence, $n = 5$

Permutations Ex 16.2 Q12

We have,

$$\text{LHS} = 1.P(1,1) + 2.\ P(2,2) + 3.\ P(3,3)\ldots\ldots + n.\ P(n,n)$$

$$= 1.1 + 2.2! + 3.3!\ldots\ldots\ldots n.n! \qquad [\because\ P(n,n) = n!]$$

$$= \sum_{r=1}^{n} r.r!$$

$$= \sum_{r=1}^{n} [(r+1)r! - r!]$$

$$= \sum_{r=1}^{n} [(r+1)! - r!] \qquad [\because\ (r+1)r! = (r+1)!]$$

$$= [(2! - !) + (3! - 2!) + (4! - 3!)\ldots\ldots\ldots + (n+1)! - n!]$$

$$= (n+1)! - 1!$$

$$= {}^{n+1}P_{n+1} - 1! \qquad [\because\ {}^{n}P_{n} = n!]$$

$$= P(n+1, n+1) - 1$$

$\Rightarrow$ LHS = RHS

Hence proved.

Permutations Ex 16.2 Q13

We have,

$$P(15, r-1) = P(16, r-2) = 3 : 4$$

$$\Rightarrow \quad \frac{P(15, r-1)}{P(16, r-2)} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[15-(r-1)]!}}{\dfrac{16!}{[16-(r-2)]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[16-r]!}}{\dfrac{16!}{[18-r]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15! \times (18-r)(17-r)(16-r)!}{(16-r)! \times 16 \times 15!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$\Rightarrow \quad 306 - 18r - 17r + r^2 = \frac{3}{4} \times 16$$

$$\Rightarrow \quad r^2 - 35r + 306 = 12$$

$$\Rightarrow \quad r^2 - 35r + 306 - 12 = 0$$

$$\Rightarrow \quad r^2 - 35r + 294 = 0$$

$$\Rightarrow \quad r^2 - 21r - 14r + 294 = 0$$

$$\Rightarrow \quad r(r-21) - 14(r-21) = 0$$

$$\Rightarrow \quad (r-21)(r-14) = 0$$

$$\Rightarrow \quad r - 14 = 0 \qquad [\because r = 21 \neq 0]$$

$$\Rightarrow \quad r = 14$$

Hence, $r = 14$

Permutations Ex 16.2 Q14

We have,

$$^{n+5}P_{n+1} = \frac{11(n-1)}{2}\ ^{n+3}P_{n}$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-(n+1)]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{[n+3-n]!}$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-n-1]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)(n+3)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)}{4 \times 3!} = \frac{11(n-1)}{2 \times 3!}$$

$$\Rightarrow \quad (n+5)(n+4) = \frac{11(n-1) \times 4}{2}$$

$$\Rightarrow \quad (n+5)(n+4) = 22(n-1)$$

$$\Rightarrow \quad n^2 + 4n + 5n + 20 = 22n - 22$$

$$\Rightarrow \quad n^2 + 9n - 22n + 20 + 22 = 0$$

$$\Rightarrow \quad n^2 - 13n + 42 = 0$$

$$\Rightarrow \quad n^2 - 6n - 7n + 42 = 0$$

$$\Rightarrow \quad n(n-6) - 7(n-6) = 0$$

$$\Rightarrow \quad n = 6 \quad \text{or,} \quad n = 7$$

Hence, $n = 6$ or,7

Permutations Ex 16.2 Q15

The total number of ways

$$= \text{Number of arrangements of 5 things, taken all at a time} = {}^{5}P_{5}$$

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{0!} \qquad [\because \ 0! = 1]$$

$$= 120$$

Hence, the total number of ways in which children stand in a queue is 120.

Permutations Ex 16.2 Q16

The total number of teachers in a school = 36

One principal and one uice-principal are to be appointed.

$\therefore$ Total of ways

= Number of arrangement of 36 things taken two at a time

$$= {}^{36}P_2$$

$$= \frac{36!}{(36-2)!}$$

$$= \frac{36!}{34!}$$

$$= \frac{36 \times 35 \times 34!}{34!}$$

$$= 36 \times 35$$
$$= 1260$$

Hence, Total number of ways to oppoint one principal and one vice-principal are 1260.

Permutations Ex 16.2 Q17

Total number of letters = 4

$\therefore$ The total number of ordred

paris = Number of arrangements of 4 letters, taken two at a time

$$= {}^{4}P_2$$

$$= \frac{4!}{(4-2)!}$$

$$= \frac{4!}{2!}$$

$$= \frac{4 \times 3 \times 2!}{2!}$$

$$= 12$$

Hence, the total number of ordered paris = 12

Permutations Ex 16.2 Q18

Total number of books = 4

∴ Total number of ways

$= \text{Number of arrangments of 4 books, taken all at a time}$

$= {}^{4}P_{4}$

$= \dfrac{4!}{(4-4)!} \qquad \left[\because {}^{n}P_{r} = \dfrac{n!}{(n-r)!}\right]$

$= \dfrac{4!}{0!}$

$= 4! \qquad [\because 0! = 1]$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Hence, the total number of ways to arrange the books in a shelf = 24

Permutations Ex 16.2 Q19

Total number of letters = 6

∴ Total number of words

$= \text{Number of arrangements of 6 letters, taken 4 at a time} = {}^{6}P_{4}$

$= \dfrac{6!}{(6-4)!}$

$= \dfrac{6!}{2!}$

$= \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2!}$

$= 360$

Hence, the total number of 4 letter words are 360.

Permutations Ex 16.2 Q20

The odd number digits are 1,3,5,6,9.

Total number of odd digits = 5

∴ Required number of 3 digit numbers

$= \text{number of arrangenments of 5 digits by taking 3 at a time}$

$= {}^{5}P_{3}$

$= \dfrac{5!}{(5-3)!}$

$= \dfrac{5!}{2!}$

$= \dfrac{5 \times 4 \times 3 \times 2!}{2!}$

$= 60$

Hence, total number of 3 digit numbers are 60

Permutations Ex 16.2 Q21

Total number of letters = 5

$\therefore$ Total number of words

$= \text{Number of arrangement of 5 letters, taken 5 at a time}$

$= {}^{5}P_{5}$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

$= 5! \qquad [\because 0! = 1]$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Hence, the number of words are 120

Permutations Ex 16.2 Q22

Total number of letters = 8

$\therefore$ Total number of words

$= \text{Number of arrangements of 8 letters, taken 8 at a time}$

$= {}^{8}P_{8}$

$= \frac{8!}{(8-8)!}$

$= \frac{8!}{0!}$

$= 8! \qquad [\because 0! = 1]$

Hence, total number of words are 8!

Permutations Ex 16.2 Q23

Let, w_1, w_2, w_3 and w_4 be 4 words, where w_1, w_2 have 3 volumes each and w_3, w_4 have 2 volume each.

These 4 works can be arranged in 4! ways.

Now,

volumes of w_1 can be arranged in 3! ways.

volumes of w_2 can be arranged in 3! ways.

volumes of w_3 can be arranged in 2! ways.

And volumes of w_4 can be arranged in 2! ways

$\therefore$ Total number of ways to arrange

all books $= 4!(3! \times 3! \times 2! \times 2!)$

$= 24 \times 6 \times 6 \times 2 \times 2$

$= 3456.$

Permutations Ex 16.2 Q24

There are 6 items in column A and 6 items in column B.

Now,

Each answer to the given question is an arrangement of the 6 items of column B keeping the order of items in column A fixed.

Hence, the total number of answers

= Number of arrangements of 6 items in column B

$$= {}^{6}P_{6}$$

$$= \frac{6!}{(6-6)!}$$

$$= \frac{6!}{0!}$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \qquad [\because\ 0! = 1]$$

$$= 720$$

Permutations Ex 16.2 Q25

Total number of digits = 10

Total number of 3 digit numbers = ${}^{10}P_{3}$

But these arrangements also include those numbers which have 0 at hundred's place.

such numbers are not 3-digit numbers.

When 0 is fixed at hundred's place, we have to arrange remaining 9 digits by taking 2 at a time.

The number of such arrangements is ${}^{9}P_{3}$.

So, the total of numbers having 0 at hundred's place = ${}^{9}P_{2}$

Hence, total number of 3 digit numbers which distinct = ${}^{10}P_{3} - {}^{9}P_{2}$

$$= \frac{10!}{(10-3)!} - \frac{9!}{(9-2)!}$$

$$= \frac{10!}{7!} - \frac{9!}{7!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7!} - \frac{9 \times 8 \times 7!}{7!}$$

$$= 720 - 72$$

$$= 648.$$

Permutations Ex 16.2 Q26

Total number of digits = 10

The first two digits of telephone is 35 and no digit appears more than once.

$\therefore$ Total number of remaining digits = 10 – 2 = 8
And, Total number of remaining digits of telephone number = 6 – 2 = 4.

$\therefore$ Required number of telephone numbers = ${}^{8}P_{4}$

$$= \frac{8!}{(8-4)!}$$

$$= \frac{8!}{4!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$$

$$= 1680$$

Permutations Ex 16.2 Q27

Total number of boys = 6
Total number of girls = 5

Now,

Five girls can sit on chairs in a row in ${}^{5}P_{5} = 5!$ ways.

and 6 boys can stand behind them in a row in ${}^{6}P_{6} = 6!$ ways.

Hence, the total number of ways

$$= 5! \times 6!$$
$$= 5 \times 4 \times 3 \times 2 \times 1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 120 \times 720$$

$$= 86400$$

Permutations Ex 16.2 Q28

$'a'$ denotes the number of permutations of $(x+2)$ things taken all at a time.

$\therefore\ a = {}^{x+2}P_{x+2}$

$'b'$ is the number of permutations of x things taken 11 at a time.

$\therefore\quad b = {}^{x}P_{11}$

and, C is the number of permutations of $x-11$ things taken all at a time.

$\therefore\quad C = {}^{x-11}P_{x-11}$

Now,

$$a = 182bc \qquad [\text{given}]$$

$$\Rightarrow \quad {}^{x+2}P_{x+2} = 182 \times {}^{x}P_{11} \times {}^{x-11}P_{x-11}$$

$$\Rightarrow \quad (x+2)! = 182 \times \frac{x!}{(x-11)!} \times (x-11)! \qquad \left[\because\ {}^{n}P_{n} = n! \text{ and } {}^{n}P_{r} = \frac{n!}{(n-r)!}\right]$$

$$\Rightarrow \quad (x+2)! = 182 \times x!$$

$$\Rightarrow \quad (x+2)(x+1)x! = 182 \times x!$$

$$\Rightarrow \quad (x+2)(x+1) = 182$$

$$\Rightarrow \quad x^2 + x + 2x + 2 = 182$$

$$\Rightarrow \quad x^2 + 3x + 2 - 182 = 0$$

$$\Rightarrow \quad x^2 + 3x - 180 = 0$$

$$\Rightarrow \quad x^2 + 15x - 12x - 180 = 0$$

$$\Rightarrow \quad x(x+15) - 12(x+15) = 0$$

$$\Rightarrow \quad (x-12)(x+15) = 0$$

$$\Rightarrow \quad x - 12 = 0 \qquad [\because\ x \neq -15]$$

$$\Rightarrow \quad x = 12$$

Hence, $x = 12$

Permutations Ex 16.2 Q29

There are 9 ways to pick the 1st digit.

For each of those 9 ways there are 8 ways to choose the second digit.
That's 9×8 or 72 ways to pick the first two digits.

For each of those 72 ways there are 7 ways to choose the third digit.
That's 72×7 ways or 504 ways to pick all three digits.

Permutations Ex 16.2 Q30

The even number so last digit must be even .We can so number patterns are

1)odd, odd, even
2)odd, even, even
3)even, odd, even
4)even, even, even

For the pattern 1 - number of ways of choosing 1st digit is 3
2nd digit (already one is gone) is 2
3rd is 3

Therefore, the no of ways is 3x2x3 .
Similarly for pattern 2, the no. of ways is 3x3x2
for pattern 3, the no. of ways is 3x3x2
for pattern 4, the no. of ways is 3x2x1

Total no of ways is 3x2x3 + 3x3x2 + 3x3x2 + 3x2x1
18x3 + 6 = 60

Permutations Ex 16.2 Q31

We can take the digits one at a time, starting at either end.
Let's start from the right.
d c b a = the digits to be chosen.
For a we have 5 choices (1,2,3,4,5)
For b we only have 4 (having used one for a, and repeats not allowed)
For c we have 3
For d we have 2.
5 * 4 * 3 * 2 = 120 choices overall

If we want the number to be even,
we don't have 5 choices for a, we are limited to the set {2, 4}
there are only two digits available.

But for the remaining digits the calculation is the same.

2/5 of the numbers are even $= \frac{2}{5} \times 120 = 48 = 2 \times 4 \times 3 \times 2$

Permutations Ex 16.2 Q32

There are 6 letters in the word 'EAMCOT'. Out of these letters 'E','A' and 'O' are the three vowels.

The remaining three consonants can be arranged in ${}^3P_3$ ways. In each of these arrangements 4 places are created, shown by the cross marks.

×	V	×	V	×	V	×

Since no two vowels are to be placed adjacent to each other, so we may arrange 3 vowels in 4 places in ${}^4P_3$ ways.

The total number of arrangements

$= {}^3P_3 \times {}^4P_3$

$= 3! \times 4!$

$= 144$