

RD Sharma
Solutions
Class 11 Maths
Chapter 16
Ex 16.4

Permutations Ex 16.4 Q1

There are 4 vowels and 3 consonants in the word 'FAILURE'

We have to arrange 7 letters in a row such that consonants occupy odd places. There are 4 odd places $\{1,3,5,7\}$. These consonants can be arranged in these 4 odd places in 4P_3 ways.

Remaining 3 even places $\{2,4,6\}$ are to be occupied by the 4 vowels. This can be done in 4P_3 ways.

Hence, the total number of words in which consonants occupy odd places = ${}^4P_3 \times {}^4P_3$

$$\begin{aligned} &= \frac{4!}{(4-3)!} \times \frac{4!}{(4-3)!} \\ &= 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 \\ &= 24 \times 24 \\ &= 576. \end{aligned}$$

Permutations Ex 16.4 Q2

There are 7 letters in the word 'STRANGE', including 2 vowels (A,E) and 5 consonants (S,T,R,N,G).

(i) Considering 2 vowels as one letter, we have 6 letters which can be arranged in ${}^6P_6 = 6!$ ways
A,E can be put together in $2!$ ways.

Hence, required number of words

$$\begin{aligned} &= 6! \times 2! \\ &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 \\ &= 720 \times 2 \\ &= 1440. \end{aligned}$$

(ii) The total number of words formed by using all the letters of the words 'STRANGE' is ${}^7P_7 = 7!$

$$\begin{aligned} &= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 5040. \end{aligned}$$

So, the total number of words in which vowels are never together

$$\begin{aligned} &= \text{Total number of words} - \text{number of words in which vowels are always together} \\ &= 5040 - 1440 \\ &= 3600 \end{aligned}$$

(iii) There are 7 letters in the word 'STRANGE'. out of these letters 'A' and 'E' are the vowels. There are 4 odd places in the word 'STRANGE'. The two vowels can be arranged in 4P_2 ways. The remaining 5 consonants can be arranged among themselves in 5P_5 ways.

The total number of arrangements

$$\begin{aligned} &= {}^4P_2 \times {}^5P_5 \\ &= \frac{4!}{2!} \times 5! \\ &= 1440 \end{aligned}$$

Permutations Ex 16.4 Q3

There are 6 letters in the word 'SUNDAY'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to ${}^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

If we fix up D in the beginning, then the remaining 5 letters can be arranged in ${}^5P_5 = 5!$ ways.

so, the total number of words which begin with D = 5!

$$= 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Permutations Ex 16.4 Q4

There are 4 vowels and 4 consonants in the word 'ORIENTAL'. We have to arrange 8 letters in a row such that vowels occupy odd places. There are 4 odd places (1,3,5,7). Four vowels can be arranged in these 4 odd places in 4! ways. Remaining 4 even places (2,4,6,8) are to be occupied by the 4 consonants.

This can be done in 4! ways.

Hence, the total number of words in which vowels occupy odd places = $4! \times 4!$

$$= 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1$$

$$= 576.$$

Permutations Ex 16.4 Q5

There are 6 letters in the word 'SUNDAY'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to ${}^6P_6 = 6!$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 720.$$

If we fix up N in the beginning, then the remaining 5 letters can be arranged in ${}^5P_5 = 5!$ ways

so, the total number of words which begin with N = 5!

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120$$

if we fix up N in the beginning and Y at the end, then the remaining 4 letters can be arranged in

$${}^4P_4 = 4! \text{ ways.}$$

So, the total number of words which begin with N and end with Y = $4! = 4 \times 3 \times 2 \times 1 = 24$.

Permutations Ex 16.4 Q6

There are 10 letters in the word 'GANESHPURI'. The total number of words formed is equal to ${}^{10}P_{10} = 10!$

(i) If we fix up G in the beginning, then the remaining 9 letters can be arranged in ${}^9P_9 = 9!$ ways

(ii) If we fix up P in the beginning and I at the end, beginning 8 letters can be arranged in ${}^8P_8 = 8!$.

(iii) There are 4 vowels and 6 consonants in the word 'GANESHPURI'.

Considering 4 vowels as one letter,

We have 7 letters which can be arranged in ${}^7P_7 = 7!$ ways.

A, E, U, I can be put together in $4!$ ways.

Hence, required number of words = $7! \times 4!$.

(iv) We have to arrange 10 letters in a row such that vowels occupy even places. There are 5 even places $(2, 4, 6, 8, 10)$. 4 vowels can be arranged in these 5 even places in 5P_4 ways.

Remaining 5 odd places $(1, 3, 5, 7, 9)$ are to be occupied by the 6 consonants.

This can be done in 6C_5 ways.

Hence, the total number of words in which vowels occupy even places = ${}^5P_4 \times {}^6P_5$

$$= \frac{5!}{(5-4)!} \times \frac{6!}{(6-1)!}$$

$$= 5! \times 6!$$

(i) There are 6 letters in the word 'VOWELS'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to

$${}^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

(ii) If we fix up E in the beginning then the remaining 5 letters can be arranged

in ${}^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways

(iii) If we fix up O in the beginning and L at the end, the remaining 4 letters can be arranged in 4P_4

$$= 4! = 4 \times 3 \times 2 \times 1 = 24.$$

(iv) There are 2 vowels and 4 consonants in the word 'VOWELS'.

Considering 2 vowels as one letter, we have 5 letters which can be arranged in

$${}^5P_5 = 5! \text{ ways.}$$

O, E can be put together in $2!$ ways.

Hence, required number of

$$\begin{aligned} \text{words} &= 5! \times 2! \\ &= 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 \\ &= 120 \times 2 \\ &= 240 \end{aligned}$$

(v) There are 2 vowels and 4 consonants in the word 'VOWELS'.

Considering 4 consonants as one letter, we have 3 letters which can be arranged in ${}^3P_3 = 3!$ ways.

U, W, L, S can be put together in $4!$ ways.

Hence, required number of words in which all consonants come together = $3! \times 4!$

$$\begin{aligned} &= 3 \times 2 \times 4 \times 3 \times 2 \\ &= 144. \end{aligned}$$

Permutations Ex 16.4 Q8

We have to arrange 7 letters in a row such that vowels occupy even places.

There are 3 even places $(2, 4, 6)$. Three vowels can be arranged in these 3 even places in $3!$ ways.

Remaining 4 odd places $(1, 3, 5, 7)$ are to be occupied by the 4 consonants. This can be done in $4!$ ways.

Hence, the total number of words in which vowels occupy even places = $3! \times 4!$

$$= 3 \times 2 \times 4 \times 3 \times 2 = 144$$

Permutations Ex 16.4 Q9

Let two husbands A, B be selected out of seven males in $= {}^7C_2$ ways. excluding their wives, we have to select two ladies C, D out of remaining 5 wives is $= {}^5C_2$ ways. Thus, number of ways of selecting the players for mixed double is $= {}^7C_2 \times {}^5C_2$

$$= 21 \times 10$$

$$= 210$$

Now, suppose A chooses C as partner (B will automatically go to D) or A chooses D as partner (B will automatically go to C)

Thus we have, 4 other ways for teams.

$$\text{Required number of ways} = 210 \times 4 = 840$$

Permutations Ex 16.4 Q10

m men can be seated in a row in ${}^mP_m = m!$ ways.

Now, in the $(m+1)$ gaps n women can be arranged in ${}^{m+1}P_n$ ways.

Hence, the number of ways in which no two women sit together

$$= m! \times {}^{m+1}P_n$$

$$= m! \times \frac{(m+1)!}{(m+1-n)!}$$

$$= m! \times \frac{(m+1)!}{(m-n+1)!}$$

Hence, proved

Permutations Ex 16.4 Q11

(i) M O N D A Y has 6 letters with no repetitions, so

Number of words using 4 letters at a time with no repetitions = 6P_4

$$\begin{aligned} &= \frac{6!}{2!} \\ &= 360 \end{aligned}$$

(ii) Number of words using all 6 letters at a time with no repetitions = 6P_6

$$\begin{aligned} &= \frac{6!}{(6-6)!} \\ &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 720 \end{aligned}$$

(iii) Number of words using all 6 letters, starting with vowels

$$\begin{aligned} &= 2 \cdot {}^5P_5 \\ &= 2 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 240 \end{aligned}$$

Permutations Ex 16.4 Q12

There are 8 letters in the word 'ORIENTAL'. The total number of three letter words is the number of arrangements of 8 items, taken 3 at a time, which is equal to

$$\begin{aligned} {}^8P_3 &= \frac{8!}{(8-3)!} \\ &= \frac{8!}{5!} \\ &= \frac{8 \times 7 \times 6 \times 5!}{8!} \\ &= 336. \end{aligned}$$