RD Sharma Solutions Class 11 Maths Chapter 17 Ex 17.1

Combinations Ex 17.1 Q1(i)

$^{14}C_3$

$= \frac{14!}{3!(14-3)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{14!}{3!11!}$

$= \frac{14 \times 13 \times 12 \times 11!}{3 \times 2 \times 1 \times 11!}$

$= \frac{14 \times 13 \times 12}{6}$

$= 364$

Combinations Ex 17.1 Q1(ii)

$^{12}C_{10}$

$= \frac{12!}{10!(12-10)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{12 \times 11 \times 10!}{10! \times 2 \times 1}$

$= 66$

Combinations Ex 17.1 Q1(iii)

$^{35}C_{35}$

$= \frac{35!}{35!(35-35)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= 1$

Combinations Ex 17.1 Q1(iv)

$^{n+1}C_n$

$= \frac{(n+1)!}{(n!)(n+1-n)!}$ $\left(\because\ ^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{(n+1) \times n!}{n! \times 1!}$

$= n + 1$

Combinations Ex 17.1 Q1(v)

$$\sum_{r=1}^{5} {}^5C_r$$

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$

$$= \frac{5!}{1!4!} + \frac{5!}{2!3!} + \frac{5!}{3!2!} + \frac{5!}{4!1!} + \frac{5!}{5!0!} \qquad \left(\because\ {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$= 5 + \frac{5 \times 4}{2} + \frac{5 \times 4}{2} + 5 + 1$$

$$= 5 + 10 + 10 + 5 + 1$$
$$= 31$$

Combinations Ex 17.1 Q2

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Hence $n = n$

$r = 12$ and 5

Applying formula

$${}^nC_p = {}^nC_q = n$$

Then $P + q = n$

$$\Rightarrow\ {}^nC_{12} = {}^nC_5$$
$$12 + 5 = n$$

$$\Rightarrow\quad n = 17$$

Combinations Ex 17.1 Q3

If ${}^{n}C_{p} = {}^{n}C_{q}$

Then $p + q = n$

Also ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ....(i)

$\Rightarrow$ ${}^{n}C_{4} = {}^{n}C_{6}$

$4 + 6 = n$

$\Rightarrow$ $n = 10$

then ${}^{12}C_{n} = {}^{12}C_{10}$

Applying (i)

$$ {}^{12}C_{10} = \frac{12!}{10!\ 2!} $$

$$ = \frac{12 \times 11 \times 10!}{10! \times 2 \times 1} $$

$$ = \frac{12 \times 11}{2 \times 1} = 66 $$

Combinations Ex 17.1 Q4

If ${}^{n}C_{p} = {}^{n}C_{q}$

Then $p + q = n$

$\Rightarrow$ ${}^{n}C_{10} = {}^{n}C_{12}$

$10 + 12 = n$

$\Rightarrow$ $n = 22$

Find ${}^{23}C_{n}$

$\Rightarrow$ ${}^{23}C_{22}$

$$ = \frac{23!}{22!\ 1!} $$

$$ = \frac{23 \times 22!}{22!} $$

$= 23$

Combinations Ex 17.1 Q5

If ${}^{n}C_{p} = {}^{n}C_{r}$ then $p + r = n$

$\therefore$ $x + 2x + 3 = 24$

$3x = 21$

$x = 7$

Combinations Ex 17.1 Q6

If ${}^{n}C_{p} = {}^{n}C_{q}$

$\Rightarrow \quad p + q = n$

also $C_{x} = {}^{18}C_{x+2}$

$$\Rightarrow \quad x + x + 2 = 18$$
$$2x + 2 = 18$$
$$2x = 18 - 2 = 16$$
$$2x = 16$$

$x = 8$

Combinations Ex 17.1 Q7

If ${}^{n}C_{p} = {}^{n}C_{q}$

Then $p + q = n$

$\Rightarrow \quad {}^{15}C_{3r} = {}^{15}C_{r+3}$

$$\Rightarrow \quad 3r + r + 3 = 15$$
$$4r + 3 = 15$$
$$4r = 15 - 3 = 12$$

$r = 3$

Combinations Ex 17.1 Q8

$^8C_r = {}^7C_2 + {}^7C_3$

Applying formula $^nC_r = \dfrac{n!}{r!(n-r)!}$

$$\frac{8!}{r!(8-r)!} = \frac{7!}{2!\ 5!} + \frac{7!}{3!\ 4!}$$

$$\frac{8 \times 7!}{r!\ (8-r)!} = \frac{7!}{2 \times 5 \times 4!} + \frac{7!}{3 \times 2 \times 4!}$$

$$\frac{8 \times 7!}{r!(8-r)!} = \frac{7!}{2 \times 4!}\left(\frac{1}{5} + \frac{1}{3}\right)$$

Cancelling 7! from both sides

$$\frac{8}{r!\ (8-r)!} = \frac{8}{2 \times 15 \times 4!}$$

Cancelling 8 on both sides

$$2 \times 5 \times 3 \times 4 \times 3 \times 2 \times 1 = r!(8-r)!$$

$$(3 \times 2)(5 \times 4 \times 3 \times 2 \times 1) = r!(8-r)!$$

$$\Rightarrow r! = 3!$$

$$r = 3$$

or $\quad r! = 5!$

$$r = 5$$

Combinations Ex 17.1 Q9

$$\frac{\dfrac{15!}{(15-r)!\ r!}}{\dfrac{15!}{(15-r+1)!(r-1)!}} = \frac{11}{5}$$

$$\frac{\dfrac{15!}{(15-r)(16-r)!\ r(r-1)!}}{\dfrac{15!}{(16-r)!(r-1)!}} = \frac{11}{5}$$

$$\Rightarrow \quad \frac{16-r}{r} = \frac{11}{5}$$

$$80 - 5r = 11r$$

$$80 = 16r$$

$$r = \frac{80}{16}$$

$$= 5$$

$$r = 5$$

Combinations Ex 17.1 Q10

$^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$$\frac{\dfrac{(n+2)!}{8!(n-6)!}}{\dfrac{(n-2)!}{(n-6)!}} = \frac{57}{16}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)(n)(n-1)(n-2)!}{8!(n-2)!} = \frac{57}{16}$$

Cancelling $(n-2)!$ from numerator and denominator

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = \frac{57 \times 7 \times 6 \times 5 \times 4 \times 3 \times 1 \times 16}{16}$$

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = 21 \times 20 \times 19 \times 18$$

comparing both sides $n = 19$

Combinations Ex 17.1 Q11

$$\frac{\dfrac{28!}{(2r)!(28-2r)!}}{\dfrac{24!}{(2r-4)!(24-(2r-4))!}} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28 \times 27 \times 26 \times 25 \times 24!\,(2r-4)!(28-2r)!}{(2r)!\,(28-2r)!\quad 24!} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28 \times 27 \times 26 \times 25}{2r \times (2r-1) \times (2r-2)(2r-3)} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28 \times 27 \times 26 \times 25 \times 11}{15 \times 15} = 2r(2r-1)(2r-2)(2r-3)$$

$$\Rightarrow \quad 11 \times 12 \times 13 \times 14 = 2r(2r-1)(2r-2)(2r-3)$$

Composing both sides $r = 7$

Combinations Ex 17.1 Q12

$$\frac{\frac{4n!}{(2n)!(2n)!}}{\frac{2n!}{n!\ n!}} \qquad \left(\because\ {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$= \frac{(4n)!}{(2n)!(2n)!}\frac{\times(n!)^2}{\times(2n)!^2}$$

$$= \frac{[1.2.3.4\ldots(4n-1)(4n)](n!^2)}{(2n)!\ [1.2.3.4\ldots\ (2n-2)(2n-1)(2n)]^2}$$

$$= \frac{[1.3.5\ldots(4n-1)]\times[2.4.6\ldots 4n]\times(n!)^2}{(2n)!\,[1.3.5\ldots\ (2n-1)]^2\times[2.4.6\ldots\ (2n-2)(2n)]^2}$$

$$= \frac{[1.3.5\ldots(4n-1)]\times 2^{2n}\times[1.2.3\ldots 2n]\times n!^2}{(2n)!\times\left[1.3.5\ldots(2n-1)^2\times 2^{2n}\times n!^2\right]}$$

$$= \frac{[1.3.5\ldots.(4n-1)]}{[1.3.5\ldots.(2n-1)]^2}$$

Hence Proved

Combinations Ex 17.1 Q13

$$\frac{\frac{2n!}{3!\ (2n-3)!}}{\frac{n!}{2!\ (n-2)!}} = \frac{44}{3}$$

$\Rightarrow \qquad \frac{2n!\ 2!(n-2)!}{3!\ (2n-3)!\,n!} = \frac{44}{3}$

$\Rightarrow \qquad \frac{2n!}{3n!\ (n-1)(2n-3)!} = \frac{44}{3}$

$\Rightarrow \qquad 2n(2n-1)(2n-2) = 44n(n-1)$

$\Rightarrow \qquad (2n-1)(n-1) = 11(n-1)$

Q $\qquad n = 6$

$\therefore \qquad n = 6$

Combinations Ex 17.1 Q14

If ${}^nC_r = {}^nC_p$

then $r + p = n$

$\therefore \quad 16 = r + r + 2$

$r = 7$

then ${}^rC_4 = {}^7C_4 \qquad (\because r = 7)$

$$\Rightarrow \frac{7!}{4!(7-4)!} \qquad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$\Rightarrow \frac{7 \times 5 \times 6}{3 \times 2}$$

$= 35$

Combinations Ex 17.1 Q15

$${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4$$

$$\Rightarrow \left({}^{20}C_5 + {}^{20}C_4\right) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4$$

$$\Rightarrow \left({}^{21}C_5 + {}^{21}C_4\right) + {}^{22}C_4 + {}^{23}C_4 \qquad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right)$$

$$\Rightarrow \left({}^{22}C_5 + {}^{22}C_4\right) + {}^{23}C_4 \qquad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right)$$

$$\Rightarrow {}^{23}C_5 + {}^{23}C_4 \qquad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r\right)$$

$$\Rightarrow {}^{24}C_5$$

$$\Rightarrow 42504$$

Combinations Ex 17.1 Q16

Product $= \left[(2n+1)(2n+3)(2n+5)\ldots(2n+r)\right]$

$$= \frac{(2n)!\left[(2n+1)(2n+3)\ldots(2n+r)\right]}{(2n)!}$$

$$= \frac{(2n)\left[(2n-1)(2n-2)\ldots4.2\,(2n+1)\,(2n+3)\right]}{(2n)!}$$

$$= \frac{(2n+r)!}{(2n)!}$$

Hence $r = 2n$

$$= \frac{(2n+2n)!}{2n}$$

$$= \frac{(4n)!}{(2n)!}$$

$$= (2n)!$$

Combinations Ex 17.1 Q17

L.H.S, $= {}^{2n}C_n + {}^{2n}C_{n-1}$

$$\frac{2n!}{n!\,n!} + \frac{2n!}{(n-1)!(n-1)!}$$

$$= (2n)!\left[\frac{1}{n\,(n-1)!(n)\,(n-1)!} + \frac{1}{(n-1)!(n-1)!}\right]$$

$$= \frac{(2n)!}{(n-1)!(n-1)!}\left[\frac{1+n^2}{n^2}\right] \ldots\ldots\ldots\ldots(i)$$

$${}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{(2n+2)(2n+1)(2n)!}{n\,(n+1)\,(n-1)!(n+1)\,n\,(n-1)!} \ldots\ldots\ldots(ii)$$

$$\Rightarrow \quad \frac{(2n)!}{(n-1)!(n-1)!} \times \frac{(n+1)^2\,(n)^2\,(n-1)!(n-1)!}{(2n+2)(2n+1)\,(2n)!} \times \left(\frac{1+n^2}{n^2}\right)$$

$$= \frac{(n+1)^2\,(1+n^2)}{(2n+2)\,(2n+1)} = \frac{(n+1)\,(n+1)\,(n^2+1)}{2\,(n+1)\,(2n+1)}$$

$$= \frac{(n+1)\,(n^2+1)}{(2n+1)} \times \frac{1}{2}$$

Combinations Ex 17.1 Q18

${}^nC_4, {}^nC_5,$ and ${}^nC_6$ are in A.P

$\therefore \quad {}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$

$$\frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{6!(n-5)!}$$

$$\Rightarrow \quad \frac{n!}{4!(n-5)!}\left[\frac{1}{5} - \frac{1}{n-4}\right] = \frac{n!}{5!(n-6)!}\left[\frac{1}{6} - \frac{1}{n-5}\right]$$

$$\Rightarrow \quad \frac{1}{n-5}\left[\frac{n-4-5}{5(n-4)}\right] = \frac{1}{5}\left[\frac{n-5-6}{6(n-5)}\right]$$

$$\Rightarrow \quad \frac{n-9}{n-4} = \frac{n-11}{6}$$

$$\Rightarrow \quad 6n - 54 = n^2 - 15n + 44$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$n = 7, 14$$

n is 7 or 14

Combinations Ex 17.1 Q19

We have $\alpha = {}^mC_2 = \frac{m(m-1)}{2}\left({}^nC_r = \frac{n!}{r!(n-r)!}\right)$

Now ${}^{\alpha}C_2 = \frac{\alpha(\alpha-1)}{2}$

$$= \frac{\left(\frac{m(m-1)}{2}\right)\left(\frac{m(m-1)}{2} - 1\right)}{2}$$

$$= \frac{m(m-1)(m^2-m-2)}{2\times 2\times 2} = \frac{m(m-1)(m+1)(m-2)}{8}$$

$$= \frac{m(m-1)(m+1)(m-2)}{4\times 2}$$

multiplying with 3, numerator and denominator to make 4:

Or $$= \frac{m(m+1)m(m-1)(m-2)}{4.3.2.1}$$

$$= \frac{3(m+1)m(m-1)(m-2)}{4!}$$

$$= 3.\ {}^{m+1}C_4 \qquad \left(\because\ {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

Combinations Ex 17.1 Q20(i)

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nC_{r-1} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n!(r-1)!(n-r+1)!}{r!(n-r)!\ n!}$$

$$= \frac{(r-1)!(n-r+1)\times(n-r)!}{r_2\times(r-1)!(n-r)!}$$

$$= \frac{n-r+1}{r}$$

Hence Proved

Combinations Ex 17.1 Q20(ii)

$$n \times {}^{n-1}C_{r-1}$$

$$= n \times \frac{(n-1)!}{(r-1)!\left[(n-1)-(r-1)\right]!}$$

$$= \frac{n! \times (n-r+1)}{(r-1)!(n-r)!(n-r+1)}$$

multiplying numerator and denominator by $(n-r+1)]$

$$= \frac{(n-r+1) \times n!}{(r-1)!(n-r+1)!}$$

$$= (n-r+1)^{n} C_{r-1}$$

Hence Proved

Combinations Ex 17.1 Q20(iii)

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$${}^{n-1}C_{r-1} = \frac{(n-1)!}{(r-1)!(n-1)-(r-1)!}$$

Or $$\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n!(r-1)!(n-r)!}{r!(n-r)!(n-1)!}$$

$$= \frac{n \times (n-1)!(r-1)! \times (n-r)!}{r \times (n-1)! \times (r-1)! \times (n-r)!}$$

$$= \frac{n}{r}$$

Hence Proved

Combinations Ex 17.1 Q20(iv)

L.H.S $\Rightarrow {}^{n}C_{r} + 2\,{}^{n}C_{r-1} +^{n} C_{r-2}$

$$= \left({}^{n}C_{r} +^{n} C_{r-1}\right) + \left({}^{n}C_{r-2} +^{n} C_{r-1}\right)$$

$$=^{n+1} C_{r} +^{n+1} C_{r-1} \qquad \left[\because\ {}^{n}C_{r} +^{n} C_{r-1} =^{n+1} C_{r}\right]$$

$$= (n+1) +^{1} C_{r}$$

$$=^{n+2} C_{r}$$