RD Sharma Solutions Class 11 Maths Chapter 18 Ex 18.2

Binomial Theorem Ex 18.2 Q1

$$T_{r+1} = T_n = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_{11} = T_{10+1} = (-1)^{10}\, {}^{25}C_{10} (2x)^{15} \left(\frac{1}{x^2}\right)^{10} = {}^{25}C_{10} \left(\frac{2^{15}}{x^5}\right) = \frac{25!}{10!5!} 2^{15} x^{15} \times x^{-20}$$

11^{th} term from the end $= (26 - 11 + 1) = 16^{th}$ from beginning.

$$\Rightarrow \quad T_{16} = T_{15+1} = (-1)^{15}\, {}^{25}C_{15} (2x)^{10} \left(\frac{1}{x^2}\right)^{15} = -{}^{25}C_{15} \frac{2^{10}}{x^{20}}$$

Binomial Theorem Ex 18.2 Q2

$$T_n = T_{r+1} = (-1)^r x^{n-r} y^r \times {}^{10}C_r$$

$$n = 7,\ r = 6,\ x = 3x^2,\ y = \frac{1}{x^3}$$

$$T_7 = T_{6+1} = (-1)^6\, {}^{10}C_6 (3x^2)^4 \left(\frac{1}{x^3}\right)^6 = {}^{10}C_6 3^4 x^8 \times \frac{1}{x^{18}} = {}^{10}C_6 \times \frac{81}{x^{10}} = \frac{210 \times 81}{x^{10}} = \frac{17010}{x^{10}}$$

Binomial Theorem Ex 18.2 Q3

Fifth term from the end is

$(11-5+1)=7^{th}$ term from beginning

$T_7=T_{6+1}=(-1)^r\ {}^nC_2x^{n-r}y^r$

$=(-1)^6\ {}^{10}C_6(3x)^4\left(\frac{1}{x^2}\right)^6={}^{10}C_6\times 3^4\times\frac{x^4}{x^{12}}=\frac{210\times 81}{x^8}=\frac{17010}{x^8}$

Binomial Theorem Ex 18.2 Q4

$T_N=T_{r+1}=(-1)^r\ {}^nC_rx^{n-r}y^r$

$N=8,\ r=7,\ x=x^{3/2}y^{1/2},\ y=x^{1/2}y^{3/2},\ n=10$

$T_8=T_{7+1}=(-1)^7\ {}^{10}C_7\left(x^{3/2}y^{1/2}\right)^3\left(x^{1/2}y^{3/2}\right)^7=-{}^{10}C_7x^{9/2}\times x^{7/2}\times y^{3/2}y^{21/2}=-120x^8y^{12}$

Binomial Theorem Ex 18.2 Q5

$T_N=T_{r+1}={}^nC_rx^{n-r}y^r$

$N=7,\ r=6,\ n=8,\ x=\frac{4x}{5},\ y=\frac{5}{2x}$

$T_7=T_{6+1}={}^8C_6\left(\frac{4x}{5}\right)^2\left(\frac{5}{2x}\right)^6=28\times\frac{4^2}{5^2}\times x^4\times\frac{5^6}{2^6\times x^6}=\frac{28}{4}\times\frac{5^4}{x^4}=\frac{7\times 5\times 125}{x^4}=\frac{4375}{x^4}$

Binomial Theorem Ex 18.2 Q6

Term from the beginning

$T_N=T_{r+1}={}^nC_rx^{n-r}y^r$ ---(i)

$N=4,\ r=3,\ n=9,\ x=x,\ y=\frac{2}{x}$

$T_4=T_{3+1}={}^9C_3x^6\left(\frac{2}{x}\right)^3=\frac{9\times 7\times 8}{3\times 2}x^3\times 8=\ 672x^3$

4^{th} term from the end $=7^{th}$ term from beginning

Using (i)

$N=7,\ r=6,\ n=9,\ x=x,\ y=\frac{2}{x}$

$T_7=T_{6+1}={}^9C_6x^3\left(\frac{2}{x}\right)^6=\frac{9\times 8\times 7}{3\times 2}\times\frac{2^6}{x^3}=\frac{5376}{x^3}$

Binomial Theorem Ex 18.2 Q7

$T_N=T_{r+1}=(-1)^r\ {}^nC_2x^{n-r}y^r$

4^{th} term from the end $=7^{th}$ term from beginning

$N=7,\ r=6,\ n=9,\ x=\frac{4x}{5},\ y=\frac{5}{2x}$

$T_7=T_{6+1}=(-1)^6\ {}^9C_6\left(\frac{4x}{5}\right)^3\left(\frac{5}{2x}\right)^6=\frac{9\times 8\times 7}{3\times 2}\times\frac{4^3\times 5^6}{5^3\times 2^6}\times\frac{x^3}{x^6}=\frac{9\times 8\times 7\times 5^3}{6\times x^3}=\frac{9\times 8\times 7\times 125}{6\times x^3}=\frac{10500}{x^3}$

Binomial Theorem Ex 18.2 Q8

7th term from the end = 3[rd] term from beginning

$$T_N = T_{r+1} = (-1)^r \, {}^nC_2 x^{n-r} y^r$$

$$N = 3,\ r = 2,\ n = 8,\ x = 2x^2,\ y = \frac{3}{2x}$$

$$T_3 = T_{2+1} = (-1)^2 \, {}^8C_2 \left(2x^2\right)^6 \left(\frac{3}{2x}\right)^2 = \frac{8 \times 7}{2} \times \frac{2^6 \times 3^2 \times x^{12}}{2^2 \times x^2} = 8 \times 7 \times 9 \times 8 \times x^{10} = 4032x^{10}$$

Binomial Theorem Ex 18.2 Q9(i)

x^{10} in $\left(2x^2 - \frac{1}{x}\right)^{20}$

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$(-1)^r \, {}^{20}C_r \left(2x^2\right)^{20-r} \left(\frac{1}{x}\right)^r$$

Coefficient of x^{10} is

$$(-1)^r \, {}^{20}C_r 2^{20-r} x^{40-2r} x^{-r} \quad \text{—(i)}$$

$\Rightarrow \quad x^{40-3r} = x^{10}$

$\Rightarrow \quad 10 - 3r = 10$

$3r = 30$

$r = 10$

Substituting $r = 10$ in(i)

$$(-1)^{10} \, {}^{20}C_{10} 2^{10}$$

$$= {}^{20}C_{10} 2^{10}$$

Binomial Theorem Ex 18.2 Q9(ii)

x^7 in $\left(x - \frac{1}{x^2}\right)^{40}$

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$= (-1)^r \, {}^{40}C_r x^{40-r} \left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r \, {}^{40}C_r x^{40-r-2r}$$

$\Rightarrow \quad x^7 = x^{40-3r}$

$7 = 40 - 3r$

$3r = 33$

$r = 11$

$= (-1)^{11} \, {}^{40}C_{11}$ is coeff of x^7

$= -{}^{40}C_{11}$

Binomial Theorem Ex 18.2 Q9(iii)

x^{-15} in $\left(3x^2 - \frac{a}{3x^3}\right)^{10}$

$$(-1)^r\ {}^{10}C_r \left(3x^2\right)^{10-r} \left(\frac{a}{3x^3}\right)^r$$

$$(-1)^r\ {}^{10}C_r \frac{3^{10-r} a^r}{3^r} x^{20-2r-3r}$$

$$\Rightarrow \quad x^{20-5r} = x^{-15}$$

$$20 - 5r = -15$$

$$35 = 5r$$

$$r = 7$$

$$(-1)^7\ {}^{10}C_7 \frac{3^3 a^7}{3^7}$$

$$-\frac{40}{27} a^7$$

Binomial Theorem Ex 18.2 Q9(iv)

x^9 in expansion of $\left(x^2 - \frac{1}{3x}\right)^9$

$$T_n = T_{r+1} = (-1)^r\ {}^nC_r x^{n-r} y^r$$

$$= (-1)^r\ {}^9C_r \left(x^2\right)^{9-r} \left(\frac{1}{3x}\right)^r$$

$$= (-1)^r\ {}^9C_r \times \frac{1}{3^r} \times x^{18-2r-r}$$

$$\Rightarrow \quad x^{18-3r} = x^9$$

$$18 - 3r = 9$$

$$3r = 9$$

$$r = 3$$

$$= (-1)^3\ {}^9C_3 \frac{1}{3^3}$$

$$= -\frac{9 \times 8 \times 7}{3 \times 2 \times 9 \times 3}$$

$$= \frac{-28}{9}$$

Binomial Theorem Ex 18.2 Q9(v)

x^m in expansion of $\left(x + \frac{1}{x}\right)^n$

$$T_n = {}^nC_r x^{n-r} y^r$$

$$= {}^nC_r x^{n-r} \left(\frac{1}{x}\right)^r$$

$$x^{n-2r} = x$$

$$n - 2r = m$$

$$r = \frac{n-m}{2}$$

$${}^nC_{\frac{n-m}{2}} = \frac{n!}{\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)!}$$

Binomial Theorem Ex 18.2 Q9(vi)

$$(1-2x^3+3x^5)\left(1+\frac{1}{x}\right)^8 = (1-2x^3+3x^5)\left(\begin{array}{l} {}^8C_0 + {}^8C_1\frac{1}{x} + {}^8C_2\left(\frac{1}{x}\right)^2 + {}^8C_3\left(\frac{1}{x}\right)^3 + {}^8C_4\left(\frac{1}{x}\right)^4 + \\ {}^8C_5\left(\frac{1}{x}\right)^5 + {}^8C_6\left(\frac{1}{x}\right)^6 + {}^8C_7\left(\frac{1}{x}\right)^7 + {}^8C_8\left(\frac{1}{x}\right)^8 \end{array}\right)$$

$$= -(2x^3)\left({}^8C_3\left(\frac{1}{x}\right)^3\right) + \left(3x^5 \times {}^8C_4\left(\frac{1}{x}\right)^4\right)$$

$$= -(56) + (210)$$

$$= -112 + 168$$

$$= 154$$

Binomial Theorem Ex 18.2 Q9(vii)

$$(a-2b)^{12} = {}^{12}C_0a^{12} - {}^{12}C_1a^{11}(2b)^1 + {}^{12}C_2a^{10}(2b)^2 - {}^{12}C_3a^9(2b)^3 + \_ - {}^{12}C_7a^5(2b)^7 + \_$$

$$= -\frac{12!}{7!5!} \times 128$$

$$= -\frac{12\times11\times10\times9\times8}{5\times4\times3\times2} \times 128$$

$$= -101376$$

Binomial Theorem Ex 18.2 Q9(viii)

$$(1-3x+7x^2)(1-x)^{16} = (1-3x+7x^2)\left({}^{16}C_0 - {}^{16}C_1x + {}^nC_2x^2 + \ldots\ldots + {}^{16}C_{16}x^{16}\right)$$

$\therefore$ Coefficient of x in $(1-3x+7x^2)(1-x)^{16}$

$$= 1 \times (-{}^{16}C_1) - 3 \times (-{}^{16}C_0)$$

$$= -16 - 3$$

$$= -19$$

Binomial Theorem Ex 18.2 Q10

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$= {}^{21}C_r \left(\left(\frac{x}{\sqrt{y}}\right)^{\frac{1}{3}}\right)^{21-r} \left(\left(\frac{y}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\right)^r$$

$$= {}^{21}C_r \left(\frac{x^{7-\frac{r}{3}}}{y^{\frac{7}{2}-\frac{r}{6}}}\right) \frac{y^{\frac{r}{2}}}{x^{\frac{r}{6}}}$$

$$\frac{x^{7-\frac{r}{3}-\frac{r}{6}}}{y^{\frac{7}{2}-\frac{r}{6}-\frac{r}{2}}}$$

$$\Rightarrow \quad x^{\frac{42-2r-r}{6}} = y^{\frac{21-r-3r}{6}}$$

Since x and y have same power

$$\frac{42-3r}{6} = \frac{-(21-4r)}{6}$$

$$42 + 21 = 4r + 3r$$

$$63 = 7r$$

$$r = 9$$

Term is 10^{th}

$$(t_n = t_{r+1})$$

Binomial Theorem Ex 18.2 Q11

$(-1)^r\ {}^{20}C_r\left(2x^2\right)^{20-r}\left(\frac{1}{x}\right)^r$

$x^{40-2r}x^{-r}=x^9$

$40-3r=9$

$31=3r$

$r=\frac{31}{3}$

r can not be in fraction

$\therefore$ There is no term involving x^9.

Binomial Theorem Ex 18.2 Q12

Any term in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$ is

$T_N=T_{r+1}={}^nC_rx^{n-r}y^r$

$={}^{12}C_r\left(x^2\right)^{12-r}\left(\frac{1}{x}\right)^{12}$

$={}^{12}C_rx^{24-2r}x^{-12}$

$x^{12-2r}=x^{-1}$

$12-2r=-1$

$2r=13$

$r=\frac{13}{2}$

r can not be a fraction, therefore there is no term in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$

having the term x^{-1}.

Binomial Theorem Ex 18.2 Q13(i)

$\left(\frac{2}{3}x-\frac{3}{2x}\right)^{20}$

Here, $n=20$ which is an even number so, $\left(\frac{20}{2}+1\right)^{th}$ i.e., 11^{th} term is the middle term.

We know that,

$T_n=T_{r+1}=(-1)^r\ {}^nC_rx^{n-r}y^r$

$n=20,\ r=10,\ x=\frac{2}{3}x,\ Y=\frac{2}{3X}$

$T_{11}=T_{10+1}=(-1)^{10}\ {}^{20}C_{10}\left(\frac{2}{3}x\right)^{10}\left(\frac{3}{2x}\right)^{10}$

$={}^{20}C_{10}\frac{2^{10}}{3^{10}}\times\frac{3^{10}}{2^{10}}\times\frac{x^{10}}{x^{10}}$

$={}^{20}C_{10}$

Binomial Theorem Ex 18.2 Q13(ii)

Here, $n = 12$, which is even number.

So, $\left(\frac{12}{2}+1\right)$th term i.e., 7th term is the middle term.

Hence, the middle term $= T_7 = T_{6+1}$

$$\therefore \quad T_7 = T_{6+1} = {}^{12}C_6 \times \left(\frac{a}{x}\right)^{12-6} \times (bx)^6$$

$$= {}^{12}C_6 \left(\frac{a}{x}\right)^6 \times (bx)^6$$

$$= \frac{12!}{(12-6)!\,6!} \times \frac{a^6}{x^6} \times b^6 x^6$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} \times a^6 b^6$$

$$= 924 \times a^6 b^6$$

$\therefore$ The middle term $= 924 \times a^6 b^6$.

Binomial Theorem Ex 18.2 Q13(iii)

$$\left(x^2 - \frac{2}{x}\right)^{10}$$

Here, $n = 10$

$$\therefore \left(\frac{n}{2}+1\right)^{th} = \left(\frac{10}{2}+1\right)^{th} = 6^{th} \text{ term is the middle term.}$$

The term formula is

$$T_n\_T_{r+1} = (-1)^r \, {}^nC_r x^{r-n} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \, {}^{10}C_5 \left(x^2\right)^{10-5} \left(\frac{2}{x}\right)^5$$

$$= \quad -{}^{10}C_5 x^{20-10} \frac{2^5}{x^5}$$

$$= \quad \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 2^5 x^5$$

$$= \quad -8064 x^5$$

Binomial Theorem Ex 18.2 Q13(iv)

$$\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$$

Here $n = 10$, which is even, therefore it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right) = 6^{th}$ term

$$T_n = T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \, {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{a}{x}\right)^5$$

$$= -\frac{10!}{5!5!} \times \frac{x^5}{a^5} \times a^5 \times x^{-5}$$

$$= -252$$

Binomial Theorem Ex 18.2 Q14(i)

$$\left(3x-\frac{x^3}{6}\right)^9$$

Here, $n=9$, which is odd number

$\therefore \left(\frac{9+1}{2}\right)^{th}$ and $\left(\frac{9+1}{2}+1\right)^{th}$ i.e., 5^{th}, 6^{th} term are the middle term.

Here, the term formula is

$$T_5 = T_{4+1} = (-1)^4\, {}^9C_4 (3x)^5 \left(\frac{x^3}{6}\right)^4$$

$$= {}^9C_4 \frac{3^5}{6^4} \times x^5 \times x^{12}$$

$$= \frac{9\times8\times7\times6\times3^5}{4\times3\times2\times3^4\times2^4} x^{17}$$

$$= \frac{189}{8} x^{17}$$

$$T_6 = T_{5+1} = (-1)^5\, {}^9C_5 (3x)^4 \left(\frac{x^3}{6}\right)^5$$

$$= -\frac{9\times8\times7\times6}{5\times4\times3\times2} \times \frac{3^4}{6^5} \times x^4 \times x^{15}$$

$$= -\frac{9\times8\times7\times6\times3^4}{5\times4\times3\times2\times3^5\times2^5} x^{19}$$

$$= \frac{-21}{16} x^{19}$$

Binomial Theorem Ex 18.2 Q14(ii)

$$\left(3x^2-\frac{1}{x}\right)^7$$

Here, $n=7$, which is odd

$\therefore \left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th} = 4^{th}$, 5^{th} term are middle term or $\left(2x^2-\frac{1}{x}\right)^7$

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_4 = T_{3+1} = (-1)^3\, {}^7C_3 \left(2x^2\right)^{7-3} \left(\frac{1}{x}\right)^3$$

$$= -{}^7C_3 \frac{2^4 x^8}{x^3}$$

$$= -560x^5$$

$$T_5 = T_{4+1} = (-1)^4\, {}^7C_4 \left(2x^2\right)^{7-4} \left(\frac{1}{x}\right)^4$$

$$= {}^7C_4 \frac{2^3 x^6}{x^4}$$

$$= {}^7C_4 \frac{7\times6\times5\times8}{3\times2} x^2$$

$$= 280x^2$$

Binomial Theorem Ex 18.2 Q14(iii)

$$\left(3x-\frac{2}{x^2}\right)^{15}$$

7^{th} and 8^{th} terms are middle terms

$$\binom{15}{7}(3x)^8\left(-\frac{2}{x^2}\right)^7, \binom{15}{8}(3x)^7\left(-\frac{2}{x^2}\right)^8$$

$$\frac{-6435\times 3^8\times 2^7}{x^6}, \frac{6437\times 3^7\times 2^8}{x^9}$$

Binomial Theorem Ex 18.2 Q14(iv)

$$\left(x^4-\frac{1}{x^3}\right)^{11}$$

Here, $n=11$, which is odd number

$\therefore \left(\frac{11+1}{2}\right)^{th}$ and $\left(\frac{11+1}{2}+1\right)^{th}=6^{th}, 7^{th}$ term are the middle terms in $\left(x^4-\frac{1}{x^3}\right)^{11}$

The term formula is

$$T_n=T_{r+1}=(-1)^r\,{}^nC_r x^{n-r}y^r$$

$$T_6=T_{5+1}=(-1)^5\,{}^{11}C_5\left(x^4\right)^{11-5}\left(\frac{1}{x^3}\right)^5$$

$$=-{}^{11}C_5 x^{24}\frac{1}{x^{15}}$$

$$=\frac{-11\text{x}10\text{x}9\text{x}8\text{x}7}{5\text{x}4\text{x}3\text{x}2\text{x}1}x^9$$

$$=-11\text{x}3\text{x}2\text{x}7\ \ x^9$$

$$=-462\,x^9$$

$$T_7=T_{6+1}=(-1)^6\,{}^{11}C_6\left(x^4\right)^{11-6}\left(\frac{1}{x^3}\right)^6$$

$$=462\frac{x^{20}}{x^{18}}$$

$$=462x^2$$

Binomial Theorem Ex 18.2 Q15(i)

$$\left(x-\frac{1}{x}\right)^{10}$$

Here, $n=10$, which is even, $\therefore$ it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right)=6^{th}$ term

$$T_n=T_{r+1}=(-1)^r\,{}^nC_r x^{n-r}y^r$$

$$T_6=T_{5+1}=(-1)^5\,{}^{10}C_5(x)^{10-5}\left(\frac{1}{x}\right)^5$$

$$=\frac{-10\text{x}9\text{x}8\text{x}7\text{x}6}{5\text{x}4\text{x}3\text{x}2}\text{x}\frac{x^5}{x^5}$$

$$=-3\text{x}2\text{x}7\text{x}6$$

$$=-252$$

Binomial Theorem Ex 18.2 Q15(ii)

$\left(1-2x+x^2\right)^n$

Here, n is odd, $\therefore$ $\left(1-2x+x^2\right)$ has $n+1$ = even term

$\therefore$ middle term is $\left(\frac{n+1}{2}\right)^{th}$ term

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{\frac{n+1}{2}} = T_{\frac{n}{2}} = {}^nC_{\frac{n}{2}} (1-2x)^{n-\frac{n}{2}} \left(x^2\right)^{\frac{n}{2}}$$

$$= \frac{n!}{\frac{n}{2}!\frac{n}{2}!}(1-2x)^{\frac{n}{2}} x^{\frac{2n}{2}}$$

$$= \frac{(2n)!}{(n!)^2}(-1)^n x^n \qquad \left[\because (1-x)^n = 1-nx\right]$$

Binomial Theorem Ex 18.2 Q15(iii)

$\left(1+3x+3x^2+x^3\right)^{2n}$

This expansion is $\left((1+x)^3\right)^{2n} = (1+x)^{6n}$

Since $6n$ is even $\therefore$ it has $6n+1$ = odd terms has middle term is

$$\left(\frac{6n}{2}+1\right)^{th} = (4n)^{th} \text{ term}$$

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{4n} = T_{3n+1} = {}^{6n}C_{3n} (1)^{6n-3n} (x)^{3n}$$

$$= \frac{(6n)!}{(3n)!(3n)!} x^{3n} \qquad \left[\because 1^{6n-3n} = 1\right]$$

Binomial Theorem Ex 18.2 Q15(iv)

$\left(2x - \frac{x^2}{4}\right)^9$

4^{th} and 5^{th} terms are middle terms

$$\binom{9}{4}(2x)^5\left(-\frac{x^2}{4}\right)^4 + \binom{9}{5}(2x)^4\left(-\frac{x^2}{4}\right)^5$$

$$\frac{63}{4}x^{13}, -\frac{63}{32}x^{14}$$

Binomial Theorem Ex 18.2 Q15(v)

$$\left(x-\frac{1}{x}\right)^{2n+1}$$

$2n+1$ is odd hence this expansion will have $2n+2$ = even terms.

Hene, middle terms is $\frac{2n+1}{2} = n+1, n+2$

Term formula is

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_{n+1} = T_{n+1} = (-1)^n\, {}^{2n+1}C_n (x)^{2n+1-n}\left(\frac{1}{x}\right)^n$$

$$= (-1)^n\, {}^{2n+1}C_n x^{n+1-n}$$

$$= (-1)^n\, {}^{2n+1}C_n x$$

$$T_{n+2} = T_{n+1+1} = (-1)^{n+1}\, {}^{2n+1}C_{n+1} (x)^{2n+1-n-1}\left(\frac{1}{x}\right)^{n+1}$$

$$= (-1)^{n+1}\, {}^{2n+1}C_{n+1} x^{-1}$$

$$= (-1)^{n+1}\, {}^{2n+1}C_{n+1} \frac{1}{x}$$

$$= (-1)^{n+1}\, {}^{2n+1}C_n \frac{1}{x} \qquad \left[\because {}^nC_r = {}^nC_{r-1}\right]$$

Binomial Theorem Ex 18.2 Q15(vi)

$$\left(3-\frac{x^3}{6}\right)^7$$

Here $n=7$, which is odd

$\therefore$ middle term is $\left(\frac{7+1}{2}\right)$ and $\left(\frac{7+1}{2}+1\right) = 4^{th}, 5^{th}$ terms

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_4 = T_{3+1} = (-1)^3\, {}^7C_3 (3)^{7-3}\left(\frac{x^3}{6}\right)^3$$

$$= -\frac{7!}{3!4!}\times 3^4 \times \frac{x^9}{6^3}$$

$$= -\frac{7\times 6\times 5}{3\times 2\times 1}\times 81\times\frac{x^9}{216}$$

$$= -\frac{105}{8}x^9$$

And

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_5 = T_{4+1} = (-1)^4\, {}^7C_4 (3)^{7-4}\left(\frac{x^3}{6}\right)^4$$

$$= \frac{7!}{4!3!}\times 3^3 \times \frac{x^{12}}{6^4}$$

$$= \frac{7\times 6\times 5}{3\times 2\times 1}\times 27\times\frac{x^{12}}{1296}$$

$$= \frac{35}{48}x^{12}$$

Binomial Theorem Ex 18.2 Q15(vii)

$$\left(\frac{x}{3}+9y\right)^{10}$$

Here $n=10$, which is even, therefore it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right)=6^{th}$ term

$$T_n = T_{r+1} = (-1)^r\, {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5\, {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$$

$$= -\frac{10!}{5!5!} \times \frac{x^5}{3^5} \times 9^5 \times y^5$$

$$= 61236x^5y^5$$

Binomial Theorem Ex 18.2 Q15(viii)

For the given binomial expansion n = 12.

So middle term is $\left(\frac{12}{2}+1\right) = 7^{th}$ term.

$$T_7 = {}^{12}C_6 (2ax)^{12-6} \left(-\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2ax)^6 \left(\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2^6 a^6 x^6) \left(\frac{b^6}{x^{12}}\right)$$

$$T_7 = {}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right)$$

Middle term is ${}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right)$.

Binomial Theorem Ex 18.2 Q15(ix)

For the given binomial expansion n = 9.

So middle terms are $\left(\frac{9+1}{2}\right) = 5^{th}$ term and $\left(\frac{9+3}{2}\right) = 6^{th}$ term.

$$T_5 = {}^9C_4\left(\frac{p}{x}\right)^{9-4}\left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4\left(\frac{p}{x}\right)^5\left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4\left(\frac{p}{x}\right)$$

$$T_6 = {}^9C_5\left(\frac{p}{x}\right)^{9-5}\left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5\left(\frac{p}{x}\right)^4\left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5\left(\frac{x}{p}\right)$$

The middle terms are ${}^9C_4\left(\frac{p}{x}\right)$ and ${}^9C_5\left(\frac{x}{p}\right)$.

Binomial Theorem Ex 18.2 Q15(x)

For the given binomial expansion n = 10.

So middle term is $\left(\frac{10}{2}+1\right) = 6^{th}$ term.

$$T_6 = {}^{10}C_5\left(\frac{x}{a}\right)^{10-5}\left(-\frac{a}{x}\right)^5$$

$$T_6 = -\,{}^{10}C_5\left(\frac{x}{a}\right)^5\left(\frac{a}{x}\right)^5$$

$$T_6 = -\,{}^{10}C_5 = -252$$

Middle term is -252.

Binomial Theorem Ex 18.2 Q16(i)

$$\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$$

In expansion

$$T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(x^{18-2r}\right)\left(\frac{-1}{3}\right)^r x^{-r}$$

Let T_{r+1} be independent of x

$18-3r=0$ or $r=6$

$\therefore$ Required term

$$\Rightarrow \quad T_{r+1}=T_{6+1}=T_7={}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(\frac{-1}{3}\right)^6 x^{18-3(6)}$$

$$=84\left(\frac{27}{8}\right)\left(\frac{1}{179}\right)x^0=\frac{7}{18}$$

Binomial Theorem Ex 18.2 Q16(ii)

$$\left(2x+\frac{1}{3x^2}\right)^9$$

4th term is independent of x

$$\binom{9}{3}(2x)^6\left(\frac{1}{3x^2}\right)^3=\binom{9}{3}\frac{64}{27}$$

Binomial Theorem Ex 18.2 Q16(iii)

$$T_{r+1}=(-1)^r\,{}^nC_r\left(2x^2\right)^{25-r}\left(\frac{3}{x^3}\right)^r=(-1)^r\,{}^nC_r 2^{25-r}3^r x^{50-2r-3r}$$

Term independent of $x = x^0$

$$\Rightarrow \quad x^{50-5r}=x^0 \Rightarrow 50-5r=0 \Rightarrow r=10$$

$$\therefore\ t_{11}=(-1)^{10}\,{}^{25}C_{10}2^{15}\times 3^{10}={}^{25}C_{10}2^{15}3^{10}$$

Binomial Theorem Ex 18.2 Q16(iv)

$$\left(3x-\frac{2}{x^2}\right)^{15}$$

$$T_{r+1}=(-1)^r\,{}^{15}C_r(3x)^{15-r}\left(\frac{2}{x^2}\right)^r$$

$$=(-1)^r\,{}^{15}C_r 3^{15-r}2^r x^{15-r-2r}$$

Term independent of $x \Rightarrow x^0$

$$\Rightarrow \quad x^{15-3r}=x^0$$

$$15-3r=0 \Rightarrow r=5$$

$$\therefore\ t_6=(-1)^5\,{}^{15}C_5 3^{10}2^5$$

$$=-\frac{15!}{5!10!}3^{10}2^5=-\frac{15\times14\times13\times12\times11}{120}3^{10}2^5$$

$$=-3003\times3^{10}\times2^5$$

Binomial Theorem Ex 18.2 Q16(v)

$$\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r x^{5-\frac{r}{2}-2r} 3^r \times 3^{-5+\frac{r}{2}} \times 2^{-r}$$

Independent of $x \Rightarrow x^0$

$$x\,\frac{10-r-4r}{r} = x^0$$

$$10-5r=0$$

$$r=2$$

$$t_3 = {}^{10}C_2 3^{2-5+1} 2^{-2}$$

$$= {}^{10}C_2 3^{-2} 2^{-2}$$

$$= \frac{10!}{2!8!} \times \frac{1}{36} = \frac{10\times 9}{2\times 36} = \frac{5}{4}$$

Binomial Theorem Ex 18.2 Q16(vi)

$$\left(x-\frac{1}{x^2}\right)^{3n}$$

$$T_{r+1} = (-1)^r\, {}^{3n}C_r x^{3n-r}\left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r\, {}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r} = x^0 \Rightarrow r = n$$

$$= (-1)^n\, {}^{3n}C_r$$

Binomial Theorem Ex 18.2 Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}}+x^{\frac{-1}{5}}\right)^{8}$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-r} \left(x^{\frac{-1}{5}}\right)^{r}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times \left(x^{\frac{1}{3}}\right)^{8-r} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)^{r}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3}-\frac{r}{5}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-5r-3r}{15}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-8r}{15}}$$

If it is independent of x, we must have

$$\frac{40-8r}{15} = 0$$

$\Rightarrow \quad 8r = 40$

$\Rightarrow \quad r = 5$

$\therefore \quad$ The term independet of $x = T_6$

Now,

$$T_6 = {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-5} \left(x^{\frac{-1}{5}}\right)^{5}$$

$$= 56 \times \left(\frac{1}{2}\right)^{3}$$

$$= 56 \times \frac{1}{8}$$

$$= 7$$

Hence, required term = 7

Binomial Theorem Ex 18.2 Q16(viii)

$$\left(1+x+2x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$$

$$=\left(1+x+2x^3\right)\left[\left(\frac{3}{2}x^2\right)^9-{}^9C_1\left(\frac{3}{2}x^2\right)^8\frac{1}{3x}\ldots\ldots+{}^9C_6\left(\frac{3}{2}x^2\right)^3\left(\frac{1}{3x}\right)^6-{}^9C_7\left(\frac{3}{2}x^2\right)^2\left(\frac{1}{3x}\right)^7\right]$$

In the second bracket, we have to search the term so x^0 and $\frac{1}{x^3}$ which when multiplying by 1 and $2x^3$ is first bracket will give the term in dependent of x. The term containing $\frac{1}{x}$ will not occur is second bracket.

The term independent of x

$$=1\left[{}^9C_6\frac{3^3}{2^3}\times\frac{1}{3^6}\right]-2x^3\left[{}^9C_7\frac{3^3}{2^3}\times\frac{1}{3^7}\times\frac{1}{x^3}\right]$$

$$=\left[\frac{9\times8\times7}{1\times2\times3}\times\frac{1}{8\times27}\right]-2\left[\frac{9\times8}{1\times2}-\frac{1}{4\times243}\right]$$

$$=\frac{7}{18}-\frac{2}{27}$$

$$=\frac{17}{54}$$

Required term $=\frac{17}{54}$

Binomial Theorem Ex 18.2 Q16(ix)

We have,

$$\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}, x>0$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{18}C_r \left(\sqrt[3]{x}\right)^{18-r} \times \left(\frac{1}{2\sqrt[3]{x}}\right)^r$$

$$= {}^{18}C_r \left((x)^{\frac{1}{3}}\right)^{18-r} \times \left(\frac{1}{2}\right)^r \times \left(\frac{1}{x^{\frac{1}{3}}}\right)^r$$

$$= {}^{18}C_r (x)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{r}{3}}}\right) \times \left(\frac{1}{2}\right)^r$$

$$= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{2}\right)^r$$

$$= {}^{18}C_r (x)^{\frac{18-r}{3}} \times \left(\frac{1}{2}\right)^r$$

If it is independent of x, we must have

$$\frac{18-2r}{3} = 0$$

$\Rightarrow \quad 18 = 2r$

$\Rightarrow \quad r = 9$

$\therefore$ Term independet of x = $T_{9+1} = T_{10}$

Now,

$$T_{10} = {}^{18}C_9 \left(\sqrt[3]{x}\right)^{18-9} \left(\frac{1}{2\sqrt[3]{x}}\right)^9$$

$$= {}^{18}C_9 \left(\sqrt[3]{x}\right)^9 \times \frac{1}{2^9} \times \left(\frac{1}{\sqrt[3]{x}}\right)^9$$

$$= \frac{{}^{18}C_9}{2^9}$$

Hence, required term = $\frac{{}^{18}C_9}{2^9}$.

Binomial Theorem Ex 18.2 Q16(x)

$$\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^6$$

In expansion

$$T_{r+1} = {}^6C_r \left(\frac{3x^2}{2}\right)^{6-r} \left(-\frac{1}{3x}\right)^r$$

$$= {}^6C_r \left(\frac{3}{2}\right)^{6-r} \left(x^{12-3r}\right)\left(-\frac{1}{3}\right)^r$$

Let T_{r+1} be independent of x,

$12-3r=0$ or $r=4$

$\therefore$ Required term

$$\Rightarrow T_{r+1} = T_{4+1} = T_5 = {}^6C_4 \left(\frac{3}{2}\right)^{6-4} \left(\frac{-1}{3}\right)^4 x^{12-3(4)}$$

$$= 15\left(\frac{9}{4}\right)\left(\frac{1}{81}\right)x^0 = \frac{5}{12}$$

Binomial Theorem Ex 18.2 Q17

We know that the coefficient of rth term in the expansion of $(1+x)^n$ is ${}^nC_{r-1}$

$\therefore$ Coefficient of $(2r+4)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{2r+4-1} = {}^{18}C_{2r+3}$

and, coefficient of $(r-2)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{r-2-1} = {}^{18}C_{r-3}$

It is given that these coefficients are equal.

$$\therefore \quad {}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$\Rightarrow \quad 2r+3 = r-3 \text{ or, } 2r+3+r-3 = 18 \qquad \left[\because {}^nC_r = {}^nC_s \Rightarrow r = s \text{ or, } r+s = n\right]$$

$$\Rightarrow \quad r = -6 \text{ or, } 3r = 18$$

$$\Rightarrow \quad r = -6 \text{ or, } r = 6$$

$$\Rightarrow \quad r = 6 \qquad [\because r = -6 \text{ is not possible}]$$

Binomial Theorem Ex 18.2 Q18

$$(1+x)^{43}$$

$$\binom{43}{2r} = \binom{43}{r+1}$$

$$2r + r + 1 = 43$$

$$3r = 42$$

$$r = 14$$

Binomial Theorem Ex 18.2 Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(4+x)^{n+1} = {}^{n+1}C_{r+1-1} = {}^{n+1}C_r$

and, Coefficient of rth term in $(1+x)^n$ + Coefficient of $(r+1)$ th term in $(1+x)^n$

$$= {}^nC_{r-1} + {}^nC_{r+1-1}$$

$$= {}^nC_{r-1} + {}^nC_r$$

$$= \frac{n!}{\{n-(r-1)\}!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$$

$$= \frac{n!}{\{n-r+1\}!\,(r-1)!} + \frac{n!}{(n-r)!\,r!}$$

$$= \frac{n!}{(n-r+1)(n-r)!\,(r-1)!} + \frac{n!}{(n-r)!\,r\,(r-1)!}$$

$$= \frac{n!}{(n-r+1)(n-r)!\,(r-1)!} + \frac{n!}{(n-r)!\,(r-1)!\,r}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{1}{n-r+1} + \frac{1}{r}\right]$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{r+n-r+1}{(n-r+1)r}\right]$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{n+1}{(n-r+1)r}\right]$$

$$= \frac{n!\,(n+1)}{(n-r)!\,(n-r+1)\,(r-1)!\,r}$$

$$= \frac{(n+1)!}{(n-r+1)!\,r!}$$

$$= \frac{(n+1)!}{(n+1-r)!\,r!}$$

$$= {}^{n+1}C_r$$

$\therefore \quad {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of rth and $(r+1)$ th terms in the expansion of $(1+x)^n$.

Binomial Theorem Ex 18.2 Q20

We have,

$$\left(x+\frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{2n}C_r\,(x)^{2n-r}\left(\frac{1}{x}\right)^r$$

$$= {}^{2n}C_r\,(x)^{2n-r-r}$$

$$= {}^{2n}C_r x^{2n-2r}$$

If it is independent of x, we must have,

$$2n-2r=0$$

$$\Rightarrow \quad 2n=2r$$

$$\Rightarrow \quad r=n$$

$\therefore$ Term independent of $x = T_{n+1}$

Now,

$$T_{n+1} = {}^{2n}C_n\,(x-1)^{2n-n}\left(\frac{1}{x}\right)^n$$

$$= {}^{2n}C_n$$

$$= \frac{(2n)!}{(2n-n)!\,n!}$$

$$= \frac{(2n)!}{n!\,n!}$$

$$= \frac{(2n)(2n-1)(2n-2)\ldots 5\times 4\times 3\times 2\times 1}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\{2\times 4\times 6\times\ldots 2n\}}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\times 2^n\{1\times 2\times 3\times\ldots n\}}{n!\,n!}$$

$$= \frac{\{1\times 3\times 5\times\ldots(2n-1)\}\times 2^n\times n!}{n!\,n!}$$

$$= 2^n\times\frac{\{1\times 3\times 5\times\ldots(2n-1)\}}{n!}$$

$\therefore$ The term independent to $x = \dfrac{\{1\times 3\times 5\times\ldots(2n-1)\}}{n!}\times 2^n$ Hence proved.

Binomial Theorem Ex 18.2 Q21

We have,

$$(1+x)^n$$

Now,

$$\text{Coefficient of 5th term} = {}^nC_{5-1} = {}^nC_4$$

$$\text{Coefficient of 5th term} = {}^nC_{6-1} = {}^nC_5$$

and, $$\text{Coefficient of 5th term} = {}^nC_{7-1} = {}^nC_6$$

It is given that these coefficients are in A.P.

$\therefore$ $$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \quad 2\left[\frac{n!}{(n-5)!5!}\right] = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)(n-6)!5\times4!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6\times5\times4!}$$

$$\Rightarrow \quad \frac{2}{(n-5)\times5} = \frac{1}{(n-4)(n-5)} + \frac{1}{6\times5}$$

$$\Rightarrow \quad \frac{2}{5(x-5)} - \frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-(n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-n+5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{17-n}{30} = \frac{1}{n-4}$$

$$\Rightarrow \quad 17n - 68 - n^2 + 4n = 30$$

$$\Rightarrow \quad 21n - 68 - m^2 - 30 = 0$$

$$\Rightarrow \quad 21n - n^2 - 98 = 0$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$\Rightarrow \quad n^2 - 7n - 14n + 98 = 0$$

$$\Rightarrow \quad n(n-7) - 17(n-7) = 0$$

$$\Rightarrow \quad (n-7)(n-14) = 0$$

$$\Rightarrow \quad n = 7 \text{ or, } n = 14$$

Binomial Theorem Ex 18.2 Q22

We have,

$$(1+x)^{2n}$$

Now,

Coefficient 2nd term = ${}^{2n}C_{2-1} = {}^{2n}C_1$

Coefficient 3rd term = ${}^{2n}C_{3-1} = {}^{2n}C_2$

and, Coefficient 4th term = ${}^{2n}C_{4-1} = {}^{2n}C_3$

It is given that these coefficients are in A.P.

$\therefore \quad 2\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow \quad 2 = \dfrac{{}^{2n}C_1}{{}^{2n}C_2} + \dfrac{{}^{2n}C_3}{{}^{2n}C_2}$

$\Rightarrow \quad 2 = \dfrac{2}{2n-2+1} + \dfrac{2n-3+1}{3} \qquad \left[\because \dfrac{{}^{n}C_r}{{}^{n}C_{r-1}} = \dfrac{n-r+1}{r}\right]$

$\Rightarrow \quad 2 = \dfrac{2}{2n-1} + \dfrac{2n-2}{3}$

$\Rightarrow \quad 2 = \dfrac{6+(2n-1)(2n-2)}{3(2n-1)}$

$\Rightarrow \quad 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$

$\Rightarrow \quad 12n - 6 = 8 + 4n^2 - 6n$

$\Rightarrow \quad 4n^2 - 6n - 12n + 8 + 6 = 0$

$\Rightarrow \quad 4n^2 - 18n + 14 = 0$

$\Rightarrow \quad 2(2n^2 - 9n + 7) = 0$

$\Rightarrow \quad 2n^2 - 9n + 7 = 0$ Hence proved.

Binomial Theorem Ex 18.2 Q23

We have,

$$(1+x)^n$$

Let the three consecutive terms are rth $(r+1)^{th}$ and $(r+2)^{th}$ i.e., T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term = ${}^nC_{r-1} = 220$

Coefficients of $(r+1)^{th}$ term = ${}^nC_{r+1-1} = {}^nC_r = 495$

and, Coefficients of $(r+2)^{th}$ term = ${}^nC_{r+2-1} = {}^nC_{r+1} = 792$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{792}{495}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{792}{495} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{792}{495} = \frac{72}{45} = \frac{8}{5}$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{8}{5}$$

$\Rightarrow$ $5n - 5r = 8r + 8$

$\Rightarrow$ $5n - 5r - 8r = 8$

$\Rightarrow$ $5n - 13r = 8$ ---(i)

and, $$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{495}{220}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{495}{220} = \frac{45}{20} = \frac{9}{4}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{9}{4}$$

$\Rightarrow$ $4n - 4r + 4 = 9r$

$\Rightarrow$ $4n - 4r - 9r = -4$

$\Rightarrow$ $4n - 13r = -4$ ---(ii)

Subtracting equation (ii) from equation (i),

$n = 8 + 4$

$\Rightarrow$ $n = 12$

Binomial Theorem Ex 18.2 Q24

We have,

$(1+x)^n$

$\therefore$ Coefficients of 2nd term = ${}^nC_{2-1} = {}^nC_1$

Coefficients of 3rd term = ${}^nC_{3-1} = {}^nC_2$

and, Coefficients of 4th term = ${}^nC_{4-1} = {}^nC_3$

It is given that these coefficents are in A.P.

$\therefore \quad 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow \quad 2 = \frac{{}^nC_1}{{}^nC_2} + \frac{{}^nC_3}{{}^nC_2}$

$\Rightarrow \quad 2 = \frac{2}{n-2+1} + \frac{n-3+1}{3}$ $\left[\because \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right]$

$\Rightarrow \quad 2 = \frac{2}{n-1} + \frac{n-2}{3}$

$\Rightarrow \quad 2 = \frac{6+(n-1)(n-2)}{3(n-1)}$

$\Rightarrow \quad 6(n-1) = 6+n^2-2n-n+2$

$\Rightarrow \quad 6n-6 = 8+n^2-3n$

$\Rightarrow \quad n^2-3n-6n+8+6 = 0$

$\Rightarrow \quad n^2-9n+14 = 0$

$\Rightarrow \quad n^2-7n-2n+14 = 0$

$\Rightarrow \quad n(n-7)-2(n-7) = 0$

$\Rightarrow \quad (n-2)(n-7) = 0$ $[\because n-2 \neq 0]$

$\Rightarrow \quad n = 7$

Binomial Theorem Ex 18.2 Q25

We have,

$(1+x)^n$

Coefficients of pth term = ${}^nC_{p-1}$

and, Coefficients of qth term = ${}^nC_{q-1}$

It is given that, these coefficients are equal.

$\therefore \quad {}^nC_{p-1} = {}^nC_{q-1}$

$\Rightarrow \quad p-1 = q-1 \text{ or, } p-1+q-1 = n$ $\left[\begin{array}{l}\because {}^nC_r = {}^nC_s \\ \Rightarrow r = s \text{ or, } r+s = n\end{array}\right]$

$\Rightarrow \quad p-q = 0 \text{ or, } p+q = n+2$

$\therefore \quad p+q = n+2$ Hence proved.

Binomial Theorem Ex 18.2 Q26

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of $T_r = {}^nC_{r-1} = 56$

Coefficients of $T_{r+1} = {}^nC_{r+1-1} = {}^nC_r = 70$

and, Coefficients of $T_{r+2} = {}^nC_{r+2-1} = {}^nC_{r+1} = 56$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{56}{70}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{4}{5} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{70}{56}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4 = 8$$

Put $n = 8$ in equation (i), we get

$$5 \times 8 - 9r = 4$$

$$\Rightarrow \quad -9r = 4 - 40$$

$$\Rightarrow \quad r = 4$$

$\therefore$ Three consecutive terms are 4th, 5th and 6th.

Binomial Theorem Ex 18.2 Q27

We are given,

$$T_3 = a,\ T_4 = b,\ T_5 = c,\ T_6 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{5}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{5}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{5}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \qquad \text{---(i)}$$

Now we know,

$$a = {}^nC_2 x^{n-2}\alpha^2$$

$$b = {}^nC_3 x^{n-3}\alpha^3$$

$$c = {}^nC_4 x^{n-4}\alpha^4$$

$$d = {}^nC_5 x^{n-5}\alpha^5$$

Putting these values in equation (i), we get

$$\frac{{}^nC_3 x^{n-3}\alpha^3}{{}^nC_2 x^{n-2}\alpha^2} - \frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} = \frac{5}{3}\left[\frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} - \frac{{}^nC_5 x^{n-5}\alpha^5}{{}^nC_4 x^{n-4}\alpha^4}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_3}{{}^nC_2} - \frac{{}^nC_4}{{}^nC_3}\right]\frac{\alpha}{x} = \frac{5\alpha}{3x}\left[\frac{{}^nC_4}{{}^nC_3} - \frac{{}^nC_5}{{}^nC_4}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n - r + 1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-2}{3} - \frac{n-3}{4}\right] = \frac{5}{3}\left[\frac{n-3}{4} - \frac{n-4}{5}\right]$$

$$\Rightarrow \quad \frac{4n - 8 - 3n + 9}{3 \times 4} = \frac{5n - 15 - 4n + 16}{3 \times 4}$$

$$\Rightarrow \quad \frac{n+1}{12} = \frac{n+1}{12}$$

Which is true.

Hence proved.

Binomial Theorem Ex 18.2 Q28

Suppose the binomial is $(x+\alpha)^n$

We are given,

$$T_6 = a,\ T_7 = b,\ T_8 = c,\ T_9 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{4}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{4}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{4}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \qquad \text{---(i)}$$

Now we know,

$$a = {}^nC_5 x^{n-5}\alpha^5$$

$$b = {}^nC_6 x^{n-6}\alpha^6$$

$$c = {}^nC_7 x^{n-7}\alpha^7$$

$$d = {}^nC_8 x^{n-8}\alpha^8$$

Putting these values in equation (i), we get

$$\frac{{}^nC_6 x^{n-6}\alpha^6}{{}^nC_5 x^{n-5}\alpha^5} - \frac{{}^nC_7 x^{n-7}\alpha^7 v}{{}^nC_6 x^{n-6}\alpha^6} = \frac{4}{3}\left[\frac{{}^nC_7 x^{n-7}\alpha^7}{{}^nC_6 x^{n-6}\alpha^6} - \frac{{}^nC_8 x^{n-8}\alpha^8}{{}^nC_7 x^{n-7}\alpha^7}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_6}{{}^nC_5} - \frac{{}^nC_7}{{}^nC_6}\right]\frac{\alpha}{x} = \frac{4\alpha}{3x}\left[\frac{{}^nC_7}{{}^nC_6} - \frac{{}^nC_8}{{}^nC_7}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-5}{6} - \frac{n-6}{7}\right] = \frac{4}{3}\left[\frac{n-6}{7} - \frac{n-7}{8}\right]$$

$$\Rightarrow \quad \frac{7n - 35 - 6n + 36}{6 \times 7} = \frac{8n - 48 - 7n + 49}{3 \times 7 \times 2}$$

$$\Rightarrow \quad \frac{n+1}{42} = \frac{n+1}{42}$$

Which is true.

Hence proved.

Binomial Theorem Ex 18.2 Q29

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term = ${}^nC_{r-1} = 76$

Coefficients of $(r+1)$ th term = ${}^nC_{r+1-1} = {}^nC_r = 95$

and, Coefficients of $(r+2)$ th term = ${}^nC_{r+2-1} = {}^nC_{r+1} = 76$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{76}{95}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{76}{95} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r-1}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 5r - 4r = 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{95}{76}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - 9r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4$$

$$\Rightarrow \quad n = 8$$

Binomial Theorem Ex 18.2 Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$\therefore$ $$T_6 = {}^nC_{n-5}x^{n-5} \times a^5 = 112$$

$$T_7 = {}^nC_{n-6}x^{n-6} \times a^6 = 7$$

and, $$T_8 = {}^nC_{n-7}x^{n-7} \times a^7 = \frac{1}{4}$$

Now,

$$\frac{T_7}{T_6} = \frac{{}^nC_{n-6}x^{n-6} \times a^6}{{}^nC_{n-5}x^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \quad \frac{{}^nC_{n-6}}{{}^nC_{n-5}} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{x} = \frac{1}{16} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-5}{6} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{3}{8} \times \frac{1}{(n-5)} \qquad \text{---(i)}$$

and,

$$\frac{T_8}{T_7} = \frac{{}^nC_{n-7}x^{n-7} \times a^7}{{}^nC_{n-6}x^{n-6} \times a^6} = \frac{\frac{1}{4}}{7}$$

$$\Rightarrow \quad \frac{T_8}{T_7} = \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-6}{7} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{1}{4(n-6)} \qquad \text{---(ii)}$$

Comparing equation (i) and (ii), we get

$$\frac{3}{8}\times\frac{1}{(n-5)}=\frac{1}{4(n-6)}$$

$$\Rightarrow \quad \frac{3}{2}\times\frac{1}{(n-5)}=\frac{1}{(n-6)}$$

$$\Rightarrow \quad 3(n-6)=2(n-5)$$

$$\Rightarrow \quad 3n-18=2n-10$$

$$\Rightarrow \quad 3n-2n=18-10$$

$$\Rightarrow \quad n=8$$

Putting $n = 8$ in equation (ii), we get

$$\frac{a}{x}=\frac{1}{4(8-6)}$$

$$\Rightarrow \quad \frac{a}{x}=\frac{1}{8}$$

$$\Rightarrow \quad x=8a$$

Now,

$$T6=112$$

$$\Rightarrow \quad {}^{n}C_{n-5}\times x^{n-5}\times a^{5}=112$$

$$\Rightarrow \quad {}^{8}C_{3}\times x^{3}\times a^{5}=112 \qquad [\because n=8]$$

$$\Rightarrow \quad {}^{8}C_{3}\times(8a)^{3}a^{5}=112 \qquad [\because x=8a]$$

$$\Rightarrow \quad \frac{8!}{(8-3)!\,3!}\times 8^{3}\times a^{8}=112$$

$$\Rightarrow \quad \frac{8!}{5!\,3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad \frac{8\times7\times6\times5!}{5!\,3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad a^{8}=\frac{112}{56\times512}$$

$$\Rightarrow \quad a^{8}=\frac{2}{512}$$

$$\Rightarrow \quad a^{8}=\frac{1}{256}$$

$$\Rightarrow \quad a^{8}=\left(\frac{1}{2}\right)^{8}$$

$$\Rightarrow \quad a=\frac{1}{2}$$

Putting $a=\frac{1}{2}$ in $x = 8a$, we get

$$x=8\times\frac{1}{2}=4$$

Hence, $x = 4$, $a=\frac{1}{2}$ and $n = 8$.

Binomial Theorem Ex 18.2 Q31

It is given that

$$T_2 = 240$$
$$T_3 = 720$$
$$T_4 = 1080$$

$\therefore \quad T_2 = {}^nC_1 \times x^{n-1} \times a = 240$

$T_3 = {}^nC_2 \times x^{n-2} \times a^2 = 720$

and, $\quad T_4 = {}^nC_3 \times x^{n-3} \times a^3 = 1080$

Now,

$$\frac{T_4}{T_3} = \frac{{}^nC_3 \times x^{n-3} \times a^3}{{}^nC_2 \times x^{n-2} \times a^2} = \frac{1080}{720}$$

$$\Rightarrow \quad \frac{{}^nC_3 a}{{}^nC_2 x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{n-3+1}{2+1} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{n-2}{3} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{9}{2(n-2)} \qquad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_2 \times x^{n-2} \times a^2}{{}^nC_1 \times x^{n-1} \times a} = \frac{720}{240}$$

$$\Rightarrow \quad \frac{{}^nC_2}{{}^nC_1} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{n-2+1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{n-1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \quad \frac{a}{x} = \frac{6}{n-1} \qquad \text{---(ii)}$$

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$\Rightarrow \quad 12(n-2) = 9(n-1)$

$\Rightarrow \quad 12n - 24 = 9n - 9$

$\Rightarrow \quad 3n = 24 - 9$

$\Rightarrow \quad 3n = 15$

$\Rightarrow \quad n = 5$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$\Rightarrow \quad \frac{a}{x} = \frac{6}{4}$

$\Rightarrow \quad \frac{a}{x} = \frac{3}{2}$

$\Rightarrow \quad a = \frac{3}{2}x$

Now,

$$T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$\Rightarrow \quad {}^5C_1 \times x^4 \times \left(\frac{3}{2}x\right) = 240 \qquad \left[\because n = 5 \text{ and } a = \frac{3}{2}x\right]$

$\Rightarrow \quad x^5 = \frac{240 \times 2}{5 \times 3}$

$\Rightarrow \quad x^5 = 32$

$\Rightarrow \quad x^5 = 2^5$

$\Rightarrow \quad x = 2$

Putting $x = 2$ in $a = \frac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

Binomial Theorem Ex 18.2 Q32

It is given that

$$T_1 = 729$$

$$T_2 = 7290$$

and, $T_3 = 30375$

$$\therefore \quad T_1 = {}^nC_0 \times a^n = 729$$

$$T_2 = {}^nC_{n-1} \times a^{n-1} \times b = 7290$$

and, $T_3 = {}^nC_{n-2} \times a^{n-2} \times b^2 = 30375$

Now,

$$\frac{T_2}{T_1} = \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = \frac{7290}{729}$$

$$\Rightarrow \quad \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = 10$$

$$\Rightarrow \quad \frac{{}^nC_{n-1}}{1} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n!}{(n-n+1)!\,(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{n\,(n-1)!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \quad \frac{b}{a} = \frac{10}{n} \qquad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_{n-2} \times a^{n-2} \times b^2}{{}^nC_{n-1} \times a^{n-1} \times b} = \frac{30375}{7290}$$

$$\Rightarrow \quad \frac{{}^nC_{n-2}}{{}^nC_{n-1}} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{n-2+1}{n-(n-1)+1} \times \frac{b}{a} = \frac{26}{6} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

$\Rightarrow \quad \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$

$\Rightarrow \quad \frac{b}{a} = \frac{25}{3(n-1)}$ ---(ii)

Comparing equation (i) and equation (ii), we get

$\frac{10}{n} = \frac{25}{3(n-1)}$

$\Rightarrow \quad 30(n-1) = 25n$

$\Rightarrow \quad 30n - 30 = 25n$

$\Rightarrow \quad 5n = 30$

$\Rightarrow \quad n = 6$

Now,

$T_1 = {}^nC_0 \times a^n = 729$

$\Rightarrow \quad a^n = 729$

$\Rightarrow \quad a^6 = 729$ $[\because n = 6]$

$\Rightarrow \quad a^6 = 3^6$

$\Rightarrow \quad a = 3$

Putting $a = 3$ in $n = 6$ in equation (i), we get

$\frac{b}{3} = \frac{10}{6}$

$\Rightarrow \quad b = \frac{10}{2} = 5$

Hence, $a = 3$, $b = 5$ and $n = 6$.

Binomial Theorem Ex 18.2 Q33

We have,

$(3+ax)^9 = {}^9C_0 \times 3^9 + {}^9C_1 \times 3^8 \times (ax)^1 + {}^9C_2 \times 3^7 \times (ax)^2 + {}^9C_3 \times 3^6 \times (ax)^3 + \dots$

$\therefore$ Coefficient of $x^2 = {}^9C_2 \times 3^7 \times a^2$

and, Coefficient of $x^3 = {}^9C_3 \times 3^6 \times a^3$

Now, Coefficient of x^2 = Coefficient of x^3

$\Rightarrow \quad {}^9C_2 \times 3^7 \times a^2 = {}^9C_3 \times 3^6 \times a^3$

$\Rightarrow \quad 36 \times 3^7 \times a^2 = 84 \times 3^6 \times a^3$

$\Rightarrow \quad a = \frac{36 \times 3^7}{84 \times 3^6} = \frac{9}{7}$

Binomial Theorem Ex 18.2 Q34

We have,

$$(1+2a)^4(2-a)^5$$

Now,

$$(1+2a)^4 = {}^4C_0 + {}^4C_1 2a + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4$$

and, $$(2-5)^5 = {}^5C_0\times 2^5 + {}^5C_2\times 2^4(-a) + {}^5C_2\times 2^3(-a)^3 + {}^5C_3\times 2^2(-a)^3 + {}^5C_4\times 2(-a)^4 + {}^5C_5(-a)^5$$

$$= {}^5C_0\times 2^5 - {}^5C_1\times 2^4\times a + {}^5C_2\times 2^3\times a^2 - {}^5C_3\times 2^2\times a^3 + {}^5C_4\times 2\times a^4 - {}^5C_5\times a^5$$

$\therefore$ $$(1+2a)^4(2-a)^5 = \left[{}^4C_0 + {}^4C_1 2a + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4\right]\left[\begin{matrix}{}^5C_0\times 2^5 - {}^5C_1\times 2^4\times a + {}^5C_2\times 2^3\times a^2 \\ -{}^5C_3\times 2^2\times a^3 + {}^5C_4\times 2\times a^4 - {}^5C_5\times a^5\end{matrix}\right]$$

$\therefore$ $$\text{Coefficients of } a^4 = 2\,{}^5C_4 - {}^4C_1\times 2\times {}^5C_3\times 2^2 + {}^4C_2(2)^2\times {}^5C_2\times 2^3 - {}^4C_3(2)^3\times {}^5C_1\times 2^4 + {}^4C_4(2)^4\times {}^5C_0\times 2^5$$

$$= 2\times 5 - 8\times 4\times 10 + 32\times 6\times 10 - 128\times 4\times 5 + 512\times 1\times 1$$

$$= 10 - 320 + 1920 - 2560 + 512$$

$$= 2442 - 2880$$

$$= -438$$

$\therefore$ Coefficients of $a^4 = -438$.

Binomial Theorem Ex 18.2 Q35

$$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

$$\binom{10}{2}\left(\sqrt{x}\right)^8\left(-\frac{k}{x^2}\right)^2 = 405$$

$$45k^2 = 405$$

$$k^2 = 9$$

$$k = 3$$

Binomial Theorem Ex 18.2 Q36

$$\left(y^{1/2} + x^{1/3}\right)^n$$

$$\binom{n}{n-2}\left(y^{1/2}\right)^2\left(x^{1/3}\right)^{n-2}$$

$$\binom{n}{n-2} = 45$$

$$n(n-1) = 90$$

$$n^2 - 10n + 9n - 90$$

$$n(n-10) + 9(n-10) = 0$$

$$n = -9 \text{ or } 10$$

$$n \text{ cannot be negative. So, } n = 10$$

$$6th\,term\binom{10}{5}\left(y^{1/2}\right)^5\left(x^{1/3}\right)^5 = 252y^{\frac{5}{3}}x^{\frac{5}{3}}$$

Binomial Theorem Ex 18.2 Q37

$$\left(\frac{p}{2}+2\right)^8$$

$$\binom{8}{4}\left(\frac{p}{2}\right)^4 2^4 = 1120$$

$$70p^4 = 1120$$

$$p^4 = 16$$

$$p = 2$$

Binomial Theorem Ex 18.2 Q38

$$\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$$

7th term from begining is

$$\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6$$

7th term from end is

$$\binom{n}{n-6}\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$\text{Given } \frac{\text{7th term from begining}}{\text{7th term from end}} = \frac{\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-12}\left(\frac{1}{\sqrt[3]{3}}\right)^{12-n}}{\binom{n}{n-6}}$$

$$= \frac{\binom{n}{6}\left(\sqrt[3]{2}\right)^{n-12}\left(\sqrt[3]{3}\right)^{n-12}}{\binom{n}{n-6}}$$

$$= \frac{\binom{n}{6}(6)^{\frac{n-12}{3}}}{\binom{n}{n-6}} = \frac{1}{6}$$

$$\frac{n-12}{3} = -1$$

$$n = 12 - 3 = 9$$

Binomial Theorem Ex 18.2 Q39

Seventh term from the beginning and end in the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{2}}\right)^n$ are equal,

$$\Rightarrow T_7 = T_{n-6}$$

$$\Rightarrow \ {}^nC_6\left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = {}^nC_{n-6}\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\Rightarrow \left(\sqrt[3]{2}\right)^6\left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = \left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\Rightarrow \left(\frac{1}{\sqrt[3]{2}}\right)^{2n-12} = \left(\frac{1}{\sqrt[3]{2}}\right)^{12}$$

$$\Rightarrow 2n-12 = 12$$

$$\Rightarrow n = 12$$