RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.2

Arithematic Progressions Ex 19.2 Q1

(i) 10th term of A.P $1, 4, 7, 10, \ldots$

Here, 1st term $= a_1 = 1$

and common difference $d = 4 - 1 = 3$

We know $a_n = a_1 + (n-1)d$

$\therefore \quad a_{10} = a_1 + (10-1)d$

$\quad = 1 + (10-1)3 \Rightarrow 28$

(ii) To find 18th term of A.P $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

Here, 1st term $a_1 = \sqrt{2}$

and $\quad d =$ common difference $= 2\sqrt{2}$

$\therefore \quad a_n = a_1 + (n-1)d$

$a_{18} = \sqrt{2} + 2\sqrt{2}(17) = 35\sqrt{2}$

(iii) Find nth term of A.P $13, 8, 3, -2$

Here, $\quad a_1 = 13$

$d = -5$

$\therefore \quad a_n = a + (n-1)d$

$= 13 + (n-1)(-5)$

$= -5n + 18$

Arithematic Progressions Ex 19.2 Q2

It is given that the sequence $< a_n >$ is an A.P

$\therefore \quad a_n = a + (n-1)d \qquad$ ---(i)

Similarly from (i)

$a_{m+n} = a + (m+n-1)d \qquad$ ---(ii)

$a_{m-n} = a + (m-n-1)d \qquad$ ---(iii)

Adding (ii) and (iii)

$$a_{m+n} + a_{m-n} = (a + (m+n-1)d) + (a + (m-n-1)d)$$
$$= 2a + (m+n-1+m-n-1)d$$
$$= 2a + 2d(m-1)$$
$$= 2(a + (m-1)d)$$
$$= 2a_m$$ Hence proved.

Arithematic Progressions Ex 19.2 Q3

(i) Let nth term of A.P = 248

$\therefore \quad a_n = 248 = a + (n-1)d$

$\Rightarrow \quad 248 = 3 + (n-1)5$

$\therefore \quad n = 50$

$\therefore$ 50th term of the given A.P is 248.

(ii) Which term of A.P 84, 80, 76 is 0?

Let nth term of A.P be 0

Then, $a_n = 0 = a + (n-1)d$

$0 = 84 + (n-1)(-4)$

$\therefore \quad n = 22$

$\therefore$ 22nd term of the given A.P is 0.

(iii) Which term of A.P is 4, 9, 14, ... is 254?

Let nth term of A.P be 254

$a_n = a + (n-1)d$

$254 = 4 + (n-1)5$

$\therefore \quad n = 51$

$\therefore$ 51st term of the given A.P is 254.

Arithematic Progressions Ex 19.2 Q4

(i) Is 68 a term of A.P 7, 10, 13, ...?

Here, $a = 7$

and $x = 10 - 7 = 3$

$\therefore \quad a_n$ term is $= a + (n-1)d$

$= 7 + (n-1)3$

Let 68 be nth temr of A.P

Then,

$68 = 7 + 3(n-1)$

$\Rightarrow \quad 68 = 7 + 3n - 3$

$\Rightarrow \quad 68 - 4 = 3n$

$\Rightarrow \quad 64 = 3n$

$\Rightarrow \quad n = \frac{64}{3}$

Which is not a natural number.

$\therefore$ 68 is nota term of given A.P.

(ii) Is 302 a term of A.P 3, 8, 13

Let 302 be nth ter, pf tje given A.P

Here, $302 = 3 + (n-1)5$

$\frac{299}{5} = (n-1)$

$n = \frac{304}{5}$

Which is not a natural number.

$\therefore$ 302 is not a term of given A.P.

Arithematic Progressions Ex 19.2 Q5

(i) The given sequence is $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, \ldots$

Here, $a = 24$

$$d = 23\frac{1}{4} - 24 = \frac{93 - 96}{4} = \frac{-3}{4}$$

Let nth term be the 1st negative term.

$$a_n < 0$$

$$a + (n-1)d < 0$$

$$24 - \frac{3}{4}(n-1) < 0$$

$$96 - 3n + 3 < 0$$

$$99 < 3n$$

$$33 < n \qquad \text{or} \qquad n > 33$$

$\therefore$ 34th term is 1st negative term.

(ii) The given sequence is $12 + 8i,\ 11 + 6i,\ 10 + 4i, \ldots$

Here, $a = 12 + 8i$

$$d = -1 - 2i$$

Then, $a_n = a + (n-1)d$

$$= 12 + 8i + (n-1)(-1-2i)$$

$$= (13 - n) + i(10 - 2n)$$

Let nth term be purely real the $(10 - 2n) = 0$ or $n = 5$

So, 5th term is purely real.

Let nth term be purely imaginary. Then, $13 - n = 0$

$\therefore \qquad n = 13$

So, 13th term is purely imaginary.

Arithematic Progressions Ex 19.2 Q6

(i) The given A.P is 7, 10, 13, ... 43.

Let there be n terms,

then, n term = 43

or $43 = a_n = a + (n-1)d$

$\Rightarrow 43 = 7 + (n-1)3$

$\Rightarrow n = 13$

Thus, there are 13 terms in the given sequence.

(ii) The given A.P is $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots, \frac{10}{3}$?

Let there be n terms

then, nth term = $\frac{10}{3}$

or $\frac{10}{3} = a_n = a + (n-1)d$

$\Rightarrow \frac{10}{3} = -1 + (n-1)\left(\frac{-5}{6} + 1\right)$

$\Rightarrow n = 27$

Thus, there are 27 terms in the given sequence.

Arithematic Progressions Ex 19.2 Q7

Given: $a = 5$

$d = 3$

a_n = last term = 80

Let there be n terms

$\therefore a_n = 80 = a + (n-1)d$

$80 = 5 + (n-1)3$

$\Rightarrow n = 26$

$\therefore$ Thus, thre are 26 terms in the given sequence.

Arithematic Progressions Ex 19.2 Q8

Given that:

$a_6 = 19 = a + (6-1)d$ ---(i)

$a_{17} = 41 = a + (17-1)d$ ---(ii)

Solving (i) and (ii), we get

$a = 9$ and $d = 2$

$\therefore a_{40} = a + (40-1)d$

$= 9 + (40-1)2$

$= 9 + 39(2)$

$= 87$

40th term of the given sequence is 87.

Arithematic Progressions Ex 19.2 Q9

Given:

$$a_9 = 0$$

$$\therefore \quad a + 8d = 0$$

$$a = -8d \qquad \text{---(i)}$$

$$a_{19} = a + (19 - 1)d$$
$$= a + 18d$$
$$= -8d + 18d \qquad [\because a = -8d \text{ from (i)}]$$
$$= 10d \qquad \text{---(ii)}$$

$$a_{29} = a + (29 - 1)d$$
$$= -8d + 28d \qquad [\because a = -8d \text{ from (i)}]$$
$$= 20d \qquad \text{---(iii)}$$

From (ii) and (iii)

$a_{29} = 2a_{19}$ Hence proved.

Arithematic Progressions Ex 19.2 Q10

Given:

$$10a_{10} = 15a_{15}$$

$$\Rightarrow \quad 10(a + (10 - 1)d) = 15(a + (15 - 1)d)$$

$$\Rightarrow \quad 10a + 90d = 15a + 210d$$

$$\Rightarrow \quad 5a + 120d = 0$$

$$\Rightarrow \quad a + 24d = 0 \qquad \text{---(i)}$$

$$a_{25} = a + (25 - 1)d$$
$$= a + 24d$$
$$= 0 \qquad [\because \text{from (i) } a + 24d = 0]$$

Hence proved.

Arithematic Progressions Ex 19.2 Q11

Given:

$$a_{10} = 41 = a + 9d \qquad \text{---(i)}$$

$$a_{18} = 73 = a + 17d \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$a + 9d = 41$$
$$a + 17d = 73$$

We get $a = 5$ and $d = 4$

$$\therefore \quad a_{26} = a + (26 - 1)d$$
$$= 5 + 25(4)$$
$$= 105$$

26th term of the given A.P is 105.

Arithematic Progressions Ex 19.2 Q12

Given:

$$a_{24} = 2a_{10}$$

$\Rightarrow \quad a + 23d = 2(a + 9d)$

$\Rightarrow \quad a = 5d \qquad$ ---(i)

$a_{72} = a + (72 - 1)d$

$= a + 71d \qquad [\because a = 5d \text{ from (i)}]$

$\Rightarrow \quad = 76d \qquad$ ---(ii)

$a_{34} = a + (34 - 1)d$

$= 5d + 33d \qquad [\because a = 5d \text{ from (i)}]$

$= 38d \qquad$ ---(iii)

From (ii) and (iii)

$a_{72} = 2a_{34}$ Hence proved.

Arithematic Progressions Ex 19.2 Q13

Given:

$$a_{m+1} = 2a_{n+1}$$

$\Rightarrow \quad a + (m + 1 - 1)d = 2(a + (n + 1 - 1)d)$

$\Rightarrow \quad a + md = 2a + 2nd$

$\Rightarrow \quad a = (m - 2n)d \qquad$ ---(i)

Then,

$a_{3m+1} = a + (3m + 1 - 1)d$

$= a + 3md$

$= 3d - 2nd + 3md$

$= 2(2m - n)d \qquad$ ---(ii)

$a_{m+n+1} = a + (m + n + 1 - 1)d$

$= md - 2nd + md + nd$

$= (2m - n)d \qquad$ ---(iii)

From (ii) and (iii)

$a_{2m+1} = 2a_{m+n+1}$ Hence proved.

Arithematic Progressions Ex 19.2 Q14

The given A.P is 9, 7, 5, ... and 15, 12, 9

Here,

$a = 9 \qquad A = 15$

$d = -2 \qquad D = 3$

Let $a_n = A_n$ for same n.

$\Rightarrow \quad a + (n - 1)d = A + (n - 1)d$

$\Rightarrow \quad 9 + (n - 1)(-2) = 15 + (n - 1)3$

$\Rightarrow \quad n = 7$

$\therefore$ 7th term of both the A.P is same.

Arithematic Progressions Ex 19.2 Q15

(i) A.P is 3, 5, 7, 9, ..., 201.

Here, $a = 3$

$d = 2$

nth term from the end is $l - (n-1)d$

i.e. $201 - (n-1)2$ or $203 - 2n$ ---(i)

12th term from end is

$203 - 2(12) = 179$

(ii) A.P is 3, 8, 13, ..., 253.

Then, 12th term from end is $l - (n-1)d$ i.e.,

$= 253 - (12-1)5$

$= 253 - 55$

$= 198$

(iii) A.P is 1, 4, 7, 10, ..., 88

Then, 12th term from end is $l - (n-1)d$

$= 88 - (12-1)3$

$= 88 - 33$

$= 55$

Arithematic Progressions Ex 19.2 Q16

Given,

$a = 3a_1$ ---(i)

$a_7 = 2a_3 + 1$ ---(ii)

Expanding (i) and (ii)

$a + 3d = 2a$

$\therefore$ $2a = 3d$ or $a = \frac{3d}{2}$ ---(iii)

$a + 6d = 2a + 4d + 1$

$a + 1 = 2d$ ---(iv)

From (iii) and (iv)

$a = 3$ and $d = 2$

$\therefore$ 1st term of the given A.P is 3, and common difference is 2.

Arithematic Progressions Ex 19.2 Q17

$a_6 = a + 5d = 12$ ---(i)

$a_8 = a + 7d = 22$ ---(ii)

Solving (i) and (ii)

$a = -13$ and $d = 5$

Then,

$$a_n = a + (n-1)d$$
$$= -13 + (n-1)5$$
$$= 5n - 18$$

and

$$a_2 = a + (2-1)d$$
$$= -13 + 5$$
$$= -8$$

Arithematic Progressions Ex 19.2 Q18

The first two digit number divisible by 3 is 12.
and last two digit number divisible by 3 is 99.

So, the required series is 12, 15, 18, ...99.
Let there be n terms then nth term = 99

$\Rightarrow \quad 99 = a + (n-1)d$

$\Rightarrow \quad 99 = 12 + (n-1)3$

$\Rightarrow \quad n = 30$

30 two digit numbers are divisible by 3.

Arithematic Progressions Ex 19.2 Q19

Given,

$n = 60$

$a = 7$

$l = 125$

$\therefore \quad a + (n-1)d = 125$

$7 + (59)d = 125$

$d = 2$

$$\therefore \quad a_{32} = a + (32-1)d$$
$$= 7 + (31)2$$
$$= 69$$

32nd term is 69.

Arithematic Progressions Ex 19.2 Q20

$$a_4 + a_8 = 24 \qquad \text{[Given]}$$

$$\Rightarrow \quad (a+3d) + (a+7d) = 24$$

$$\Rightarrow \quad a + 5d = 12 \qquad \text{---(i)}$$

$$a_6 + a_{10} = 34$$

$$\Rightarrow \quad (a+5d) + (a+9d) = 34$$

$$\Rightarrow \quad a + 7d = 17 \qquad \text{---(ii)}$$

From (i) and (ii)

$$a = \frac{-1}{2} \text{ and } d = \frac{5}{2}$$

$\therefore$ 1st term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$.

Arithematic Progressions Ex 19.2 Q21

The nth term from starting

$$= a_n = aa + (n-1)d \qquad \text{---(i)}$$

The nth term from end

$$= l - (n-1)d \qquad \text{---(ii)}$$

Adding (i) and (ii), we get

Sum of nth term from begining and nth term from the end

$$= a + (n-1)d + l - (n-1)d$$

$= a + l$ Hence proved.

Arithematic Progressions Ex 19.2 Q22

$$\frac{a_4}{a_7} = \frac{2}{3} \qquad [\text{Given}]$$

$$\Rightarrow \quad \frac{a+3d}{a+6d} = \frac{2}{3} \qquad =$$

$$\Rightarrow \quad 3a + 9d = 2a + 12d$$

$$\Rightarrow \quad a = 3d \qquad \text{---(i)}$$

$$\frac{a_6}{a_8} = \frac{a+5d}{a+7d}$$

$$\Rightarrow \quad = \frac{3d+5d}{3d+7d} \qquad [\because 3d \text{ from (i)}]$$

$$\Rightarrow \quad = \frac{8d}{10d}$$

$$\Rightarrow \quad = \frac{4}{5}$$

$$\frac{a_6}{a_8} = \frac{4}{5}$$

Arithematic Progressions Ex 19.2 Q23

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$$

$$\theta_2 - \theta_1 = \theta_3 - \theta_2 = \ldots\ldots\ldots = d$$

$$\sec\theta_1 \sec\theta_2 = \frac{1}{\cos\theta_1 \cos\theta_2} = \frac{\sin d}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{\sin(\theta_2 - \theta_1)}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1}{(\cos\theta_1 \cos\theta_2)} - \frac{\cos\theta_2 \sin\theta_1}{(\cos\theta_1 \cos\theta_2)}\right]$$

$$= \frac{1}{\sin d}\left[Tan\theta_2 - Tan\theta_1\right]$$

Similarly, $\sec\theta_2 \sec\theta_3 = \frac{1}{\sin d}\left[Tan\theta_3 - Tan\theta_2\right]$

If we add up all terms, we get

$$= \frac{1}{\sin d}\left[Tan\theta_2 - Tan\theta_1 + Tan\theta_3 - Tan\theta_2 + \ldots\ldots\ldots\ldots + Tan\theta_n - Tan\theta_{n-1}\right]$$

$$= \frac{1}{\sin d}\left[Tan\theta_n - Tan\theta_1\right]$$

Hence Proved