

RD Sharma
Solutions
Class 11 Maths
Chapter 19
Ex 19.5

Arithmetic Progressions Ex 19.5 Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

$$\text{if } \frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$$

$$\begin{aligned} \text{LHS} &\Rightarrow \frac{ca+a^2-b^2-cb}{ab} \\ &\Rightarrow \frac{c^2a+a^2c-b^2c-c^2b}{abc} \\ &\Rightarrow \frac{c(a-b)[a+b+c]}{abc} \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{RHS} &\Rightarrow \frac{ab+b^2-c^2-ac}{bc} \\ &\Rightarrow \frac{a^2b+ab^2-ac^2-a^2c}{abc} \\ &\Rightarrow \frac{a(b-c)[a+b+c]}{abc} \quad \text{---(ii)} \end{aligned}$$

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ c(b-a) &= a(b-c) \quad \text{---(iii)} \end{aligned}$$

\therefore LHS = RHS and the given terms are in A.P.

Arithmetic Progressions Ex 19.5 Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

$$\begin{aligned} \text{LHS} &= b(c+a) - a(b+c) \\ &= bc+ab-ab-ac \end{aligned}$$

$$= c(b - a) \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= c(a + b) - b(c + a) \\ &= ca + cb - bc - ba \\ &= a(c - b) \end{aligned} \quad \text{---(ii)}$$

and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\therefore \frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$$

$$\text{or } c(b - a) = a(c - b) \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$a(b + c), b(c + a), c(a + b)$ are in A.P

Arithmetic Progressions Ex 19.5 Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

$$\begin{aligned} \text{LHS} &= \frac{b}{a+c} - \frac{a}{b+c} \\ \Rightarrow & \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)} \\ \Rightarrow & \frac{(b-a)(a+b+c)}{(a+c)(b+c)} \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= \frac{a}{a+b} - \frac{b}{a+c} \\ \Rightarrow & \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)} \end{aligned}$$

$$\Rightarrow \frac{(c-b)(a+b+c)}{(a+b)(b+c)} \quad \text{---(ii)}$$

and a^2, b^2, c^2 are in A.P

$$\therefore b^2 - a^2 = c^2 - b^2 \quad \text{---(iii)}$$

Substituting $b^2 - a^2$ with $c^2 - b^2$

$$(i) = (ii)$$

$$\therefore \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P}$$

Arithmetic Progressions Ex 19.5 Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

$$\text{If } b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(a+c)$$

$$\Rightarrow b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$$

$$\text{Given, } b - a = c - b \quad [a, b, c \text{ are in A.P}]$$

$$c(b^2 - a^2) + ab(b - a) = a(c^2 - b^2) + bc(c - b)$$

$$(b - a)(ab + bc + ca) = (c - b)(ab + bc + ca)$$

Cancelling $ab + bc + ca$ from both sides

$$b - a = c - b$$

$$2b = c + a \text{ which is true}$$

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

Arithmetic Progressions Ex 19.5 Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b) - (b+c-a) = (a+b-c) - (c+a-b)$

$$\begin{aligned} \text{LHS} &\Rightarrow (c+a-b) - (b+c-a) \\ \Rightarrow &2a - 2b \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &\Rightarrow (a+b-c) - (c+a-b) \\ \Rightarrow &2b - 2c \end{aligned} \quad \text{---(ii)}$$

Since,

$$\begin{aligned} &a, b, c \text{ are in A.P} \\ \therefore &b - a = c - b \\ \text{or} &a - b = b - c \end{aligned} \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$$\text{LHS} = \text{RHS}$$

Thus, given numbers

$$b+c-a, c+a-b, a+b-c \text{ are in A.P}$$

Arithmetic Progressions Ex 19.5 Q3(iii)

To prove $bc - a^2, ca - b^2, ab - c^2$ are in A.P

$$(ca - b^2) - (bc - a^2) = (ab - c^2) - (ca - b^2)$$

$$\begin{aligned} \text{LHS} &= (a - b^2 - bc + a^2) \\ &= (a - b)[a + b + c] \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= ab - c^2 - ca + b^2 \\ &= (b-c)[a+b+c] \end{aligned} \quad \text{---(ii)}$$

and since a, b, c are in A.P.

$$b - c = a - b$$

$$\therefore \text{LHS} = \text{RHS}$$

and

Thus, $bc - a^2, ca - b^2, ab - c^2$ are in A.P.

Arithmetic Progressions Ex 19.5 Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{b} - \frac{1}{a} \\ &= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= \frac{1}{c} - \frac{1}{b} \\ &= \frac{a(b-c)}{abc} \end{aligned} \quad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

$$\frac{b+c}{a} - \frac{c+a}{b} = \frac{c+a}{b} - \frac{a+b}{c}$$

$$\frac{b^2 + cb - ac - a^2}{ab} = \frac{c^2 + ac - ab - b^2}{bc}$$

$$\Rightarrow \frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$$

$$\text{or } \frac{a(b-c)}{abc} = \frac{c(a-b)}{abc} \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$$\text{LHS} = \text{RHS}$$

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$$ca - bc = ab - ca$$

$$c(a-b) = a(b-c) \quad \text{---(i)}$$

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow c(a-b) = a(b-c) \quad \text{---(ii)}$$

Thus, the condition necessary to prove bc, ca, ab in A.P is fulfilled.

Thus, bc, ca, ab , are in A.P.

Arithmetic Progressions Ex 19.5 Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$$a^2 + c^2 - 2ac = 4(ab - b^2 - ac + bc)$$

$$\Rightarrow a^2 + c^2 - 4b^2 + 2ac - 4ab - 4bc = 0$$

$$\Rightarrow (a+c-2b)^2 = 0 \quad \left[\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \right]$$

$$\therefore a+c-2b=0$$

$$\text{or } a+c=2b$$

and since,

$$a, b, c \text{ are in A.P}$$

[Given]

$$a+c=2b$$

Hence proved.

$$(a-c)^2 = 4(a-b)(b-c)$$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$$a^2 + c^2 + 2ac - 2ab - 2bc = 0$$

$$\text{or } (a+c-b)^2 - b^2 = 0 \quad \left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \right]$$

$$\text{or } b = a+c-b$$

$$\text{or } 2b = a+c$$

$$b = \frac{a+c}{2}$$

and since,

$$a, b, c \text{ are in A.P}$$

$$b = \frac{a+c}{2}$$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$ Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

$$\text{or } a^3 + c^3 - (2b)^3 + 6abc = 0$$

$$\text{or } a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$$

$$\therefore (a-2b+c) = 0 \quad \left[\because x^3 + y^3 + z^3 + 3xyz = 0 \right]$$

$$\left[\text{or if } x+y+z=0 \right]$$

$$\text{or } a + c = 2b$$

$$a - b = c - b$$

and since, a, b, c are in A.P

$$\text{Thus, } a - b = c - b$$

$$\text{Hence proved. } a^3 + c^3 + 6abc = 8b^3$$

Arithmetic Progressions Ex 19.5 Q6

Here,

$$a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \text{ are in A.P.}$$

$$\Rightarrow \left(\frac{ac + ab + bc}{bc}\right), \left(\frac{ab + bc + ac}{ac}\right), \left(\frac{cb + ac + ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \frac{abc}{bc}, \frac{abc}{ac}, \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow a, b, c \text{ are in A.P.}$$